A Quaternion Sensor Based Controller for Homing of Underactuated AUVs

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Abstract— **A new quaternion sensor based homing integrated guidance and control law is presented to drive an underactuated autonomous underwater vehicle (AUV) towards a fixed target, in 3D, using the information provided by an Ultra-Short Baseline (USBL) positioning system. The guidance and control law is firstly derived using quaternions to express the vehicle's attitude kinematics, which are directly obtained from the time differences of arrival (TDOA) measured by the USBL sensor. The dynamics are then included resorting to backstepping techniques. The proposed Lyapunov based control law yields global asymptotic stability (GAS) in the absence of external disturbances and is further extended, keeping the same properties, to the case where constant known ocean currents affect the vehicle's dynamics. Finally, a globally exponentially stable (GES) TDOA and range based nonlinear observer is introduced to estimate the ocean current and uniform asymptotic stability is obtained for the overall closed loop system. Simulations are presented illustrating the performance of the proposed solutions.**

I. Introduction

In the recent past several sophisticated Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) have been developed, affording the marine science community with not only advanced but also cost-effective ocean research tools [1], [2], and [3]. The control of these vehicles has naturally been subject of intense work but while the control of fully actuated vehicles is nowadays fairly well established, as evidenced by the large body of publications, see [4], [5], [6], and the references therein, the control of underactuated vehicles is still an active field of research. To address the problem of stabilization of an underactuated vehicle a variety of solutions has been proposed in the literature, [7], [8], [9], and [10]. In [11] and [12] two solutions are offered to solve the trajectory tracking problem. In [13] a solution to the problem of following a straight line is presented and in [14] a way-point tracking controller for an underactuated AUV is introduced. It turns out that all the aforementioned references share a common approach: the vehicle position is computed in the inertial coordinate frame and the control laws are developed in the body frame. Sensor based control has been a hot topic in the field of computer vision where the so-called visual servoing techniques have been subject of intensive research effort during the last years, see [15], and [16] for further information.

This paper addresses the design of an integrated guidance and control law to drive an underactuated AUV to a fixed target, in 3D. The solution to this problem, usually denominated as homing in the literature, is central to drive the vehicle to the proximity of a base station or support vessel. Once the vehicle is close enough to the base station a different control strategy should be adopted. In this paper it is assumed that an acoustic transponder is installed on a predefined fixed position in the mission scenario, denominated as target in the sequel, and an Ultra-Short Baseline (USBL) sensor, composed by an array of hydrophones and an acoustic emitter, is rigidly mounted on the vehicle's nose, as depicted in Fig. 1. During the homing phase the USBL sensor interrogates the transponder and synchronizes, detects and records the time of arrival as measured by each receiver. The implementation of the control laws requires the vehicle's linear velocity relative to the water, as provided by a Doppler velocity log, and the vehicle attitude and angular velocity measured by an Attitude and Heading Reference System (AHRS).

The paper is organized as follows. In Section II the homing problem is introduced and the dynamics of the AUV are briefly described, whereas the USBL model is presented in Section III. A Lyapunov based guidance and control law is firstly derived, in Section IV, using quaternions to express the vehicle's attitude kinematics, which are directly obtained from the USBL data. This control law is then extended to include the dynamics of the vehicle resorting to backstepping techniques and, in Section V, it is further extended to the case where known constant ocean currents affect the vehicle's dynamics. Global asymptotic stability (GAS) is achieved in both cases. Afterwards, a globally exponentially stable (GES) TDOA and range based nonlinear observer is proposed to estimate the ocean current and uniform asymptotic stability is guaranteed for the overall closed loop system. Simulation results are presented and discussed in Section VI and finally Section VII summarizes the main results of the paper.

II. Problem statement

Let $\{I\}$ be an inertial coordinate frame, and $\{B\}$ the body-fixed coordinate frame, whose origin is located at the

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center of mass of the vehicle. Consider $\mathbf{p} = [x,\ y,\ z]^T$ as the position of the origin of $\{B\}$, described in $\{I\}$, $\mathbf{v} = [u,\ v,\ w]^T$ the linear velocity of the vehicle relative to $\{I\}$, expressed in body-fixed coordinates, $\boldsymbol{\omega} = [p,\ q,\ r]^T$ the angular velocity, also expressed in body-fixed coordinates, and $\boldsymbol{\lambda} = [\phi,\ \theta,\ \psi]^T$ the vector of the Euler angles of roll, pitch and yaw. The vehicle kinematics can be written as

$$\dot{\mathbf{p}} = \mathbf{R}\mathbf{v} \quad \dot{\boldsymbol{\lambda}} = \mathbf{Q}(\boldsymbol{\lambda})\boldsymbol{\omega}, \tag{1}$$

where $\mathbf{R} = {}^I_B\mathbf{R} = ({}^B_I\mathbf{R})^T$ is the rotation matrix from $\{B\}$ to $\{I\}$, verifying $\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}(\boldsymbol{\omega})$, and $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x})\mathbf{y} = \mathbf{x} \times \mathbf{y}$ with $\times$ denoting the cross product. The vehicle's dynamic equations of motion, can be written in a compact form as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} = -\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} - \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v} - \mathbf{g}_{\mathbf{v}}(\mathbf{R}) + \mathbf{b}_{\mathbf{v}}u_{\mathbf{v}} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} - \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} - \mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R}) + \mathbf{u}_{\boldsymbol{\omega}}, \end{cases} \tag{2}$$

where $\mathbf{M} = \text{diag}\{m_u, m_v, m_w\}$ is a positive definite diagonal mass matrix, $\mathbf{J} = \text{diag}\{J_{xx}, J_{yy}, J_{zz}\}$ is a positive definite inertia matrix, $u_{\mathbf{v}} = \tau_u$ is the force control input that acts along the x_B axis, $\mathbf{u}_{\boldsymbol{\omega}} = [\tau_p,\ \tau_q,\ \tau_r]^T$ is the vector of torque control inputs that affect the rotation of the vehicle about the x_B, y_B and z_B axes, respectively, $\mathbf{D}_{\mathbf{v}}(\mathbf{v}) = \text{diag}\{X_u + X_{|u|u}|u|, Y_v + Y_{|v|v}|v|, Z_w + Z_{|w|w}|w|\}$ is the positive definite matrix of the linear motion drag coefficients, $\mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega}) = \text{diag}\{K_p + K_{|p|p}|p|, M_q + M_{|q|q}|q|, N_r + N_{|r|r}|r|\}$ is the matrix of the rotational motion drag coefficients, $\mathbf{b}_{\mathbf{v}} = [1,\ 0,\ 0]^T$, $\mathbf{g}_{\mathbf{v}}(\mathbf{R}) = \mathbf{R}^T[0,\ 0,\ W - B]^T$ represents the gravitational and buoyancy effects, W and B respectively, on the vehicle's linear motion, and $\mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R}) = \mathbf{S}(\mathbf{r}_B)\mathbf{R}^T[0,\ 0,\ B]^T$ accounts for the effect of the center of buoyancy displacement relatively to the center of mass on the vehicle rotational motion. Assume that the vehicle is neutrally buoyant, i.e., $W = B$ and therefore $\mathbf{g}_{\mathbf{v}}(\mathbf{R}) = \mathbf{0}$.

The homing problem considered in this paper can be stated as follows:

Problem Statement. *Consider an underactuated AUV with kinematics and dynamics given by* (1) *and* (2)*, respectively. Assume that a target equipped with an acoustic transponder is placed in a fixed position. Design a sensor based integrated guidance and control law to drive the vehicle towards a well defined neighborhood of the target using the time differences of arrival and range to the target as measured by an USBL sensor installed on the AUV.*

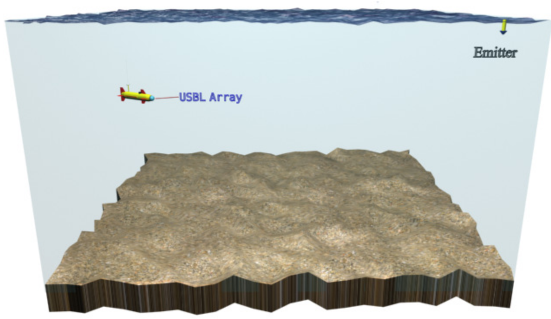

Fig. 1. Mission Scenario

III. USBL MODEL

During the homing phase the vehicle is assumed to be far away from the acoustic emitter, that is, the distance from the vehicle to the target is much larger than the distance between any pair of receivers. Therefore, the plane-wave approximation is valid. Let $\mathbf{r}_i = [x_i,\ y_i,\ z_i]^T \in \mathbb{R}^3,\ \ i = 1,\ 2,\ \ldots,\ N$, denote the positions of the N acoustic receivers installed on the USBL sensor and consider a plane-wave traveling along the opposite direction of the unit vector $\mathbf{d} = [d_x,\ d_y,\ d_z]^T$. Notice that both $\mathbf{r}_i$ and $\mathbf{d}$ are expressed in the body frame and the later corresponds to the direction of the target. Let t_i be the time of arrival of the plane-wave at the i^{th} receiver and V_S the velocity of propagation of the sound in water, assumed to be constant and known. Then, assuming that the medium is homogeneous and neglecting the velocity of the vehicle, which is a reasonable assumption since $\|\mathbf{v}\| << V_S$, the time difference of arrival between receivers i and j satisfies

$$t_i - t_j = -\left[d_x\left(x_i - x_j\right) + d_y\left(y_i - y_j\right) + d_z\left(z_i - z_j\right)\right]/V_S. \tag{3}$$

Denote by $\Delta_1 = t_1 - t_2$, $\Delta_2 = t_1 - t_3$, $\ldots$, $\Delta_M = t_{N-1} - t_N$ all the possible combinations of TDOA, where $M = N\left(N-1\right)/2$, and let $\boldsymbol{\Delta} = \left[\Delta_1,\ \Delta_2,\ \cdots,\ \Delta_M\right]^T$. Define

$$\begin{aligned} \mathbf{r}_x &:= \left[x_1 - x_2,\ x_1 - x_3,\ \cdots,\ x_{N-1} - x_N\right]^T, \\ \mathbf{r}_y &:= \left[y_1 - y_2,\ y_1 - y_3,\ \cdots,\ y_{N-1} - y_N\right]^T, \\ \mathbf{r}_z &:= \left[z_1 - z_2,\ z_1 - z_3,\ \cdots,\ z_{N-1} - z_N\right]^T, \end{aligned}$$

and $\mathbf{H}_R \in \mathbb{R}^{M\times 3}$ as $\mathbf{H}_R = [\mathbf{r}_x,\ \ \mathbf{r}_y,\ \ \mathbf{r}_z]$. Then, the generalization of (3) for all TDOA yields $\boldsymbol{\Delta} = -\mathbf{H}_R\mathbf{d}/V_S$. Define also $\mathbf{H}_Q := \mathbf{H}_R^T\mathbf{H}_R/V_S \in \mathbb{R}^{3\times 3}$, which is assumed to be non-singular. This turns out to be a weak hypothesis as it is always true if, at least, 4 receivers are mounted in noncoplanar positions. In those conditions $\mathbf{H}_R$ has maximum rank and so does $\mathbf{H}_Q$. Then,

$$\mathbf{d} = -\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}, \tag{4}$$

which directly relates the direction of the target, as seen from the AUV, to the TDOA vector.

IV. CONTROLLER DESIGN

In this section an integrated nonlinear closed loop guidance and control law is derived for the homing problem stated earlier in Section II. Assuming that there are no ocean currents the idea behind the control strategy proposed here is to steer the vehicle directly towards the emitter. The synthesis of the guidance and control law resorts extensively to the Lyapunov's direct method and backstepping techniques whereas the kinematic error takes the form of a quaternion directly obtained from the TDOA provided by the USBL sensor.

To drive the vehicle with constant forward speed towards the target, define a first error variable as

$$z_1 := [1,\, 0,\, 0]\mathbf{v} - V_d,$$

where $V_d > 0$ is the desired vehicle velocity during the homing phase. When z_1 converges to zero, the surge speed converges to V_d. However, this single error variable is not sufficient to ensure that the vehicle is driven towards the target as the attitude of the vehicle is not constrained. Using (4), an attitude error can be defined in terms of a rotation matrix $\mathbf{R}_e$ implicitly defined by

$$\mathbf{R}_e[1,0,0]^T := -\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}. \tag{5}$$

When $\mathbf{R}_e$ is the identity matrix, the vehicle's x axis is aligned with the direction of the target. Expressing $\mathbf{R}_e$ as $\mathbf{R}_e(\overline{q})$, where $\overline{q}$ is a unit quaternion corresponding to the same rotation, then the direction of the target is aligned with the body-fixed frame x axis when $\overline{q} = \pm(1,0,0,0)$. Define $\mathbf{q} = [q_0, \mathbf{q}_v^T]^T$ as the vector representation of $\overline{q}$, where q_0 and $\mathbf{q}_v$ are the so-called scalar part and vector part, respectively. It is now possible to define two new error variables to represent the attitude error,

$$z_2 := q_0 - 1$$

and

$$\mathbf{z}_3 := \mathbf{q}_v.$$

Driving z_1, z_2 and $\mathbf{z}_3$ to zero is still insufficient to ensure the correct behavior of the vehicle during the homing phase as the sway and heave velocities are left free. However, it will be shown that, with the control law based upon these three error variables, the sway and heave velocities will also converge to zero, which completes a set of sufficient conditions to drive the vehicle towards the target. The quaternion dynamics are given by

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{D}(\boldsymbol{\omega}_g)\mathbf{q}, \tag{6}$$

where

$$\mathbf{D}(\boldsymbol{\omega}_g) = \begin{bmatrix} 0 & -\boldsymbol{\omega}_g^T \\ \boldsymbol{\omega}_g & \mathbf{S}(\boldsymbol{\omega}_g) \end{bmatrix}$$

and $\boldsymbol{\omega}_g = -\boldsymbol{\omega} + \boldsymbol{\omega}_l$, with $\boldsymbol{\omega}_l = \mathbf{v} \times \mathbf{d}/\rho$. Notice that the first term represents the vehicle rotation velocity while the second term $\boldsymbol{\omega}_l$ denotes the induced rotation velocity due to the linear vehicle displacement. The range to the target, as measured by the USBL sensor, is represented by ρ. The rotation matrix $\mathbf{R}_e$ is chosen to preserve smoothness over time, which is always possible as the right side of (5) is continuous and continuously differentiable.

To synthesize the control law, consider the Lyapunov function

$$V_1 := \frac{1}{2}z_1^2 + z_2^2 + \mathbf{z}_3^T\mathbf{z}_3.$$

After some computations it can be shown that the time derivative $\dot{V}_1$ can be written as

$$\begin{aligned}\dot{V}_1 \;=\; & z_1\left([1,\,0,\,0]\mathbf{M}^{-1}\mathbf{b}_\mathbf{v}u_\mathbf{v} - [1,\,0,\,0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v}\right.\right.\\ & \left.\left.+\mathbf{D}_\mathbf{v}(\mathbf{v})\mathbf{v} + \mathbf{g}_\mathbf{v}(\mathbf{R})\right]\right) + \mathbf{z}_3^T\left(-\boldsymbol{\omega} + \boldsymbol{\omega}_l\right).\end{aligned}$$

Setting $u_\mathbf{v}$ as

$$u_\mathbf{v} = \frac{[1,\,0,\,0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} + \mathbf{D}_\mathbf{v}(\mathbf{v})\mathbf{v} + \mathbf{g}_\mathbf{v}(\mathbf{R})\right] - k_1z_1}{[1,\,0,\,0]\mathbf{M}^{-1}\mathbf{b}_\mathbf{v}}, \tag{7}$$

where k_1 is a positive scalar control gain, and $\boldsymbol{\omega} = \boldsymbol{\omega}_d$, with $\boldsymbol{\omega}_d := k_2\mathbf{z}_3 + \boldsymbol{\omega}_l$ where k_2 is a second positive scalar control gain, the time derivative $\dot{V}_1$ becomes $\dot{V}_1 = -k_1z_1^2 - k_2\mathbf{z}_3^T\mathbf{z}_3$, which is strictly non-positive.

Although $u_\mathbf{v}$ is an actual control input, the same cannot be said about $\boldsymbol{\omega}$, which was regarded here as a virtual control input. Following the standard backstepping technique, define a fourth error variable

$$\mathbf{z}_4 := \boldsymbol{\omega} - \boldsymbol{\omega}_d$$

and the augmented Lyapunov function

$$V_2 := V_1 + \frac{1}{2}\mathbf{z}_4^T\mathbf{z}_4 = \frac{1}{2}z_1^2 + z_2^2 + \mathbf{z}_3^T\mathbf{z}_3 + \frac{1}{2}\mathbf{z}_4^T\mathbf{z}_4.$$

The time derivative of V_2 can be written, after some more computations, as

$$\begin{aligned}\dot{V}_2 \;=\; & -k_1z_1^2 - k_2\mathbf{z}_3^T\mathbf{z}_3 + \mathbf{z}_4^T\left(\mathbf{J}^{-1}\left[-\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v}\right.\right.\\ & \left.\left.-\mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} - \mathbf{D}_\omega(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_\omega(\mathbf{R}) + \mathbf{u}_\omega\right] - \dot{\boldsymbol{\omega}}_d - \mathbf{z}_3\right).\end{aligned}$$

Now, setting

$$\begin{aligned}\mathbf{u}_\omega \;=\; & \mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} + \mathbf{D}_\omega(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_\omega(\mathbf{R})\\ & +\mathbf{J}\left(\dot{\boldsymbol{\omega}}_d + \mathbf{z}_3 - k_3\mathbf{z}_4\right),\end{aligned} \tag{8}$$

where k_3 is a third scalar positive control gain, finally yields $\dot{V}_2 = -k_1z_1^2 - k_2\mathbf{z}_3^T\mathbf{z}_3 - k_3\mathbf{z}_4^T\mathbf{z}_4$. The time derivative $\dot{\boldsymbol{\omega}}_d$ is not presented here for the sake of simplicity.

The following theorem is the main result of this section.

Theorem 1: Consider a vehicle with kinematics and dynamics given by equations (1) and (2), respectively, moving in the absence of ocean currents and suppose the homing problem stated in Section II defined outside a ball of radius R_{min} and centered at the target's position. Further assume that

$$R_{min} > \frac{m_u}{\min\{Y_v, Z_w\}}V_d. \tag{9}$$

Then, with the control law (7)-(8), the equilibrium point $\mathbf{z} = \left[z_1, \mathbf{z}_3^T, \mathbf{z}_4^T\right]^T = \mathbf{0}$ is globally asymptotically stable and the sway and heave velocities converge to zero, thus solving globally the homing problem stated in Section II.

Proof: The Lyapunov function V_2 is, by construction, continuous, radially unbounded and positive definite. With the control law (7)-(8), the time derivative $\dot{V}_2$ results in

$$\dot{V}_2 = -k_1z_1^2 - k_2\mathbf{z}_3^T\mathbf{z}_3 - k_3\mathbf{z}_4^T\mathbf{z}_4, \tag{10}$$

which is negative semi-definite. Therefore, V_2 is nonincreasing along all state trajectories, which remain bounded for all time. Moreover, V_2 approaches its own limit. Resorting to LaSalle' Theorem, it follows from (10) that z_1, $\mathbf{z}_3$ and $\mathbf{z}_4$ converge to zero. Because $\overline{q}$ is a unit quaternion, $q_0^2 + \mathbf{q}_v^T\mathbf{q}_v = 1$. Thus, when $\mathbf{z}_3$ converges to zero, $\overline{q}$ approaches $\pm(1,0,0,0)$ which means that $\mathbf{R}_e \rightarrow \mathbf{I}$ [17]. Therefore, the vehicle's x axis aligns itself with the desired direction.

To complete the stability analysis all that is left to do is to show that the sway and heave velocities also converge to zero. Expanding the dynamics of the sway and heave velocities as in (2) yields

$$\begin{cases} \dot{v} = -\frac{Y_v + Y_{|v|v}|v|}{m_v}v + \frac{m_w}{m_v}pw - \frac{m_u}{m_v}ur, \\ \dot{w} = -\frac{Z_w + Z_{|w|w}|w|}{m_w}w - \frac{m_v}{m_w}pv + \frac{m_u}{m_w}uq. \end{cases} \tag{11}$$

Now, after a few straightforward computations it is possible to conclude that, when $\mathbf{z}$ converges to zero, the angular velocity converges to

$$\lim_{\mathbf{z}\to\mathbf{0}} \boldsymbol{\omega} = \frac{1}{\rho}\left[0, w, -v\right]^T .$$

On the other hand, when z_1 converges to zero, u converges to V_d. Thus, when $\mathbf{z}$ converges to zero, the dynamics of the sway and heave velocities can be written as the Linear Time Varying System (LTVS) driven by a vanishing disturbance $\mathbf{d}(t)$

$$\begin{bmatrix} \dot{v} \\ \dot{w} \end{bmatrix} = \mathbf{A}(t) \begin{bmatrix} v \\ w \end{bmatrix} + \mathbf{d}(t), \tag{12}$$

where

$$\mathbf{A}(t) = \begin{bmatrix} -\frac{Y_v + Y_{|v|v}|v| - \frac{m_u}{\rho}V_d}{m_v} & \frac{m_w}{m_v}p \\ -\frac{m_v}{m_w}p & -\frac{Z_w + Z_{|w|w}|w| - \frac{m_u}{\rho}V_d}{m_w} \end{bmatrix} .$$

Now, due to the fact that p also converges to zero and using (9), there exists t_0 such that for all $t > t_0$ the eigenvalues of the symmetric matrix $\mathbf{E}(t) = \frac{1}{2}\left[\mathbf{A}(t) + \mathbf{A}^T(t)\right]$, which are all real, remain strictly in the left-half complex plane. Thus, the LTVS (12) is asymptotically stable, which concludes this proof. ∎

V. Control in the presence of ocean currents

In this section the results from the previous section are generalized for the case where constant ocean currents are present. Firstly, the integrated guidance and control law synthesized in the previous section is modified assuming that the ocean current is known. Afterwards, a globally asymptotically stable observer that relies on the information provided by the USBL sensor is proposed. Finally, the stability of the complete closed loop system is addressed.

A. Controller Design

Consider that the vehicle is moving with water relative velocity $\mathbf{v}_r$ in the presence of an ocean current $\mathbf{v}_c$, both expressed in body-fixed coordinates. It is further assumed that the current velocity is constant in the inertial frame. The dynamics of the vehicle can then be rewritten as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}}_r = -\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v}_r - \mathbf{D}_{v_r}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{b}_{\mathbf{v}}u_{\mathbf{v}} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\mathbf{S}(\mathbf{v}_r)\mathbf{M}\mathbf{v}_r - \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} - \mathbf{D}_{\omega}(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_{\omega}(\mathbf{R}) + \mathbf{u}_{\omega} \end{cases} \tag{13}$$

and the vehicle's velocity relative to the inertial frame, expressed in body-fixed coordinates, is $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_c$.

In this new mission scenario the control strategy synthesized in Section IV cannot be directly used, as the new control objective is to align the velocity of the vehicle relative to the inertial frame with the target's direction instead of the x axis of the vehicle. However, if the attitude error could be expressed as in Section IV, a similar control law could perhaps be used.

Consider the vehicle reference relative velocity $\mathbf{v}_R = [V_d, 0, 0]^T$, expressed in $\{B\}$. The error variable z_1, which accounts for the surge speed, is naturally modified to $z_1 := [1,\ 0,\ 0]\mathbf{v}_r - V_d$. Redefining the quaternion error $\overline{q}$ to correctly express the new attitude error, the error variables z_2 and $\mathbf{z}_3$ may remain unchanged. In order to do so, define a new coordinate system $\{E\}$ based on the direction of the emitter as follows: let the x axis of $\{E\}$ have direction $\mathbf{d}$, the y axis the direction of $\mathbf{i}_x \times \mathbf{d}$, where $\mathbf{i}_x = [1, 0, 0]^T$, and the z axis have the direction of $\mathbf{d} \times (\mathbf{i}_x \times \mathbf{d})$, all expressed in the body-fixed frame. The rotation matrix from $\{E\}$ to $\{B\}$ is given by

$${}^{B}_{E}\mathbf{R} = \left[\ \mathbf{d} \left| \frac{\mathbf{i}_x \times \mathbf{d}}{\|\mathbf{i}_x \times \mathbf{d}\|} \right| \frac{\mathbf{d} \times (\mathbf{i}_x \times \mathbf{d})}{\|\mathbf{d} \times (\mathbf{i}_x \times \mathbf{d})\|}\ \right], \tag{14}$$

where $\mathbf{d}$, using (4), is directly obtained from the TDOA provided by the USBL sensor. When $\mathbf{d}$ is parallel to $\mathbf{i}_x$, (14) does not define a rotation matrix. In this particular case, the rotation can be defined, e.g., by ${}^{B}_{E}\mathbf{R} = \mathbf{I}$, $\mathbf{d} = \mathbf{i}_x$ and ${}^{B}_{E}\mathbf{R} = \mathrm{diag}\{-1, 1, -1\}$, $\mathbf{d} = -\mathbf{i}_x$. Notice that, in the coordinate system $\{E\}$, the target's direction $\mathbf{d}$ is, by construction,

$${}^{E}(\mathbf{d}) = [1,\ 0,\ 0]^T . \tag{15}$$

Denote by ${}^{E}\left(\mathbf{v}_r^O\right)$ the velocity of the vehicle relative to the water, expressed in $\{E\}$, when the vehicle is moving directly towards the target with speed V_d and no lateral velocity. Then, the relationship

$$\frac{{}^{E}\left(\mathbf{v}_r^O\right) + {}^{E}(\mathbf{v}_c)}{\|\, {}^{E}\left(\mathbf{v}_r^O\right) + {}^{E}(\mathbf{v}_c)\|} = {}^{E}(\mathbf{d})$$

is satisfied. Using (15), it is straightforward to conclude that

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{E}\left(\mathbf{v}_r^O\right) = -\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{E}(\mathbf{v}_c) .$$

Since $\left\| {}^{E}\left(\mathbf{v}_r^O\right) \right\| = V_d$, there are only two possible values left for the first component of ${}^{E}\left(\mathbf{v}_r^O\right)$. However, this component can be shown to be always positive under the assumption that $V_d > V_c$, which is a reasonable assumption. In fact, if this assumption is not satisfied, it can be impossible for the vehicle to approach the target as its relative velocity may be insufficient to counteract the ocean current. Thus, the signal ambiguity is solved and ${}^{E}\left(\mathbf{v}_r^O\right)$ uniquely defined. Now, an error definition equivalent to (5) can be written as

$$\mathbf{R}_e[V_d, 0, 0]^T := \mathbf{v}_r^O . \tag{16}$$

The same control strategy as in Section IV can be applied with minor changes in the control law: the relative velocity is now used to feed the control law $u_{\mathbf{v}}$ and the quaternion attitude error is obtained from (16). The induced rotation $\boldsymbol{\omega}_l$ also changes but is here omitted for the sake of simplicity [18]. The control law is now given by

$$u_{\mathbf{v}} = \frac{[1,\ 0,\ 0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v}_r + \mathbf{D}_{v_r}(\mathbf{v}_r)\right] - k_1 z_1}{[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_{\mathbf{v}}} \tag{17}$$

and

$$\begin{aligned} \mathbf{u}_{\omega} = {} & \mathbf{S}(\mathbf{v}_r)\mathbf{M}\mathbf{v}_r + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} + \mathbf{D}_{\omega}(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_{\omega}(\mathbf{R}) \\ & + \mathbf{J}\left(\dot{\boldsymbol{\omega}}_d + \mathbf{z}_3 - k_3\mathbf{z}_4\right) . \end{aligned} \tag{18}$$

Global asymptotic stability is achieved, as in Section IV, for

$$R_{min} > \frac{2m_u}{\min\{Y_v, Z_w\}} V_d. \tag{19}$$

B. A Globally Exponentially Stable Ocean Current Observer

In the previous section it was assumed that the velocity of the ocean current was known, which is perfectly feasible using an extra sensor, e.g., a Doppler velocity log when the vehicle is close to the seabed. However, when the vehicle is far from the sea bottom its inertial velocity is no longer available onboard and therefore an alternative solution must be adopted. In this section, a nonlinear observer that makes use of the TDOA and target range measurements provided by the USBL sensor, and the water relative velocity supplied by a Doppler velocity log is proposed, and its stability analyzed.

The position of the target expressed in the body frame can be obtained directly from the USBL data. Using (4), it can be written

$$\mathbf{e} = -\rho \mathbf{H}_Q^{-1} \mathbf{H}_R^T \boldsymbol{\Delta}. \tag{20}$$

As the emitter is fixed in the universal frame, the time derivative of its position expressed in the body frame is given by $\dot{\mathbf{e}} = -\mathbf{v}_r - \mathbf{v}_c - \mathbf{S}(\boldsymbol{\omega})\mathbf{e}$. Because the current is assumed to be constant (in the inertial frame), the time derivative of the current expressed in the body frame simply results in $\dot{\mathbf{v}}_c = -\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_c$. A globally exponentially stable observer for the water velocity expressed in the body frame is presented in the following theorem.

Theorem 2: Consider the observer in the body coordinate frame given by

$$\dot{\hat{\mathbf{e}}} = -\mathbf{v}_r - \hat{\mathbf{v}}_c - \mathbf{S}(\boldsymbol{\omega})\mathbf{e} + [\mathbf{S}(\boldsymbol{\omega}) + k_{obs}\mathbf{I}](\mathbf{e} - \hat{\mathbf{e}}) \tag{21a}$$
$$\dot{\hat{\mathbf{v}}}_c = -\mathbf{S}(\boldsymbol{\omega})\hat{\mathbf{v}}_c - (\mathbf{e} - \hat{\mathbf{e}}), \tag{21b}$$

where $\hat{\mathbf{e}}$ is the estimate of the emitter's position, $\mathbf{e}$ is the observed variable, given by (20), $\hat{\mathbf{v}}_c$ is the estimate of the velocity of the current, all expressed in the body-fixed frame, and $k_{obs} > 0$ is an observer gain. Then, the estimation errors

$$\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}} \tag{22a}$$
$$\tilde{\mathbf{v}}_c = \mathbf{v}_c - \hat{\mathbf{v}}_c \tag{22b}$$

converge globally exponentially fast to zero.

Proof: The time derivatives of the errors $\tilde{\mathbf{e}}$ and $\tilde{\mathbf{v}}_c$ can be written, after some computations, as

$$\dot{\tilde{\mathbf{e}}} = -\tilde{\mathbf{v}}_c - [\mathbf{S}(\boldsymbol{\omega}) + k_{obs}\mathbf{I}]\tilde{\mathbf{e}} \tag{23a}$$
$$\dot{\tilde{\mathbf{v}}}_c = -\mathbf{S}(\boldsymbol{\omega})\tilde{\mathbf{v}}_c + \tilde{\mathbf{e}}. \tag{23b}$$

Considering the global diffeomorphic coordinate transformation $\mathbf{z}_{obs} = \mathbf{T}(\mathbf{R})[\tilde{\mathbf{e}}^T, \tilde{\mathbf{v}}_c^T]^T$, where

$$\mathbf{T}(\mathbf{R}) = \left[\begin{array}{c|c} \mathbf{R} & \mathbf{0}_{3\times3} \\ \hline \mathbf{0}_{3\times3} & \mathbf{R} \end{array}\right],$$

a linear time invariant exponentially stable system is obtained for the new variable $\mathbf{z}_{obs}$, from which follows that the origin of (23) is globally exponentially stable. ■

C. Closed-loop stability analysis

The presence of an observer to estimate the velocity of the ocean current introduces an error $\tilde{\mathbf{u}}_{\boldsymbol{\omega}} = \hat{\mathbf{u}}_{\boldsymbol{\omega}} - \mathbf{u}_{\boldsymbol{\omega}}$ in the control input $\mathbf{u}_{\boldsymbol{\omega}}$. Indeed, due to the error $\tilde{\mathbf{v}}_c$ in the velocity of the ocean current, the control law (18) is now replaced by

$$\begin{aligned}\hat{\mathbf{u}}_{\boldsymbol{\omega}} = {} & \mathbf{S}(\mathbf{v}_r)\mathbf{M}\mathbf{v}_r + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} + \mathbf{D}_\omega(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_\omega(\mathbf{R}) \\ & + \mathbf{J}\left(\hat{\dot{\boldsymbol{\omega}}}_d + \hat{\mathbf{z}}_3 - k_3\hat{\mathbf{z}}_4\right),\end{aligned} \tag{24}$$

where $\hat{\dot{\boldsymbol{\omega}}}_d$, $\hat{\mathbf{z}}_3$ and $\hat{\mathbf{z}}_4$ are the estimates of $\dot{\boldsymbol{\omega}}_d$, $\mathbf{z}_3$, and $\mathbf{z}_4$, respectively. Notice that the error of the velocity of the current appears both directly and indirectly, as some of the variables depend implicitly on the velocity of the current, namely $\mathbf{v}_r^O$ and the quaternion $\mathbf{q}$. However, the maps from $\tilde{\mathbf{v}}_c$ to $\tilde{\mathbf{v}}_r^O$ and $\tilde{\mathbf{q}}$ are smooth and the origin of $\tilde{\mathbf{v}}_c$ is mapped onto the origin in both cases.

The stability of the overall closed loop system is addressed in the following theorem. An additional assumption on the boundedness of the velocity and acceleration of the vehicle is needed. This assumption, although strong from the theoretical point of view, has a clear physical interpretation as the propulsion system of the AUV limits the available force and torque which implies upper bounds for the velocities and accelerations.

Theorem 3: Consider the nonlinear system consisting of a vehicle with kinematics and dynamics given by equations (1) and (13), respectively, the current observer (21) and the control law given by (17) and (18), where the necessary variables are replaced by their estimates obtained from the observer. Consider the homing problem as stated in Theorem 1. Assume (19) and that the velocity and acceleration of the vehicle are bounded. Then, the equilibrium point $\mathbf{z} = [z_1, \mathbf{z}_3^T, \mathbf{z}_4^T]^T = \mathbf{0}$ is locally uniformly asymptotically stable and the sway and heave velocities converge to zero, thus solving locally the aforementioned problem in the presence of constant unknown ocean currents.

Proof: Consider the system

$$\dot{\mathbf{z}} = \mathbf{f}_1(\mathbf{z}, \tilde{\mathbf{v}}_c), \tag{25}$$

where $u_{\mathbf{v}}$ and $\mathbf{u}_{\boldsymbol{\omega}}$ are replaced by (17) and (24), respectively, and $\tilde{\mathbf{v}}_c$ is here regarded as the system input. Following the same steps as in Theorem 1 it is straightforward to conclude that the autonomous system $\dot{\mathbf{z}} = \mathbf{f}_1(\mathbf{z}, \mathbf{0})$ has a uniformly asymptotically stable equilibrium point at the origin $\mathbf{z} = \mathbf{0}$. Moreover, $\mathbf{f}_1(\mathbf{z}, \tilde{\mathbf{q}})$ is continuously differentiable and the Jacobian matrices $[\partial\mathbf{f}_1/\partial\mathbf{z}]$ and $[\partial\mathbf{f}_1/\partial\tilde{\mathbf{v}}_c]$, using the assumption on the boundedness of the velocity and acceleration of the vehicle, are bounded in some neighborhood of $(\mathbf{z} = \mathbf{0}, \tilde{\mathbf{v}}_c = \mathbf{0})$. Thus, the system (25) is locally ISS (Lemma 5.4, [19]). As the observer error was shown to be 0-GES, it follows from Lemma 5.6 [19] that the origin of the cascaded system (23) and (25) is uniformly asymptotically stable. Following the same steps as in Theorem 1, it is possible to prove that $\mathbf{R}_e \to \mathbf{I}$ and that the sway and heave velocities converge to zero, which concludes this proof. ■

VI. SIMULATION RESULTS

To illustrate the performance of the proposed integrated guidance and control law a computer simulation is presented in this section. In the simulation a simplified model of the SIRENE vehicle was used, assuming the vehicle is directly actuated in force and torque [3].

In this simulation the vehicle has to counteract a constant unknown ocean current with velocity $[0,\ -1,\ 0]^T$ m/s, expressed in the inertial frame. An observer with gain $k_{obs} = 5$ estimates this current to feed the control law, as described in Section V. The vehicle starts at position $[0,\ 0,\ 50]^T$ m and the acoustic transponder is located at position $[500,\ 500,\ 500]^T$ m. A semi-spherical symmetric USBL sensor with seventeen receivers is assumed to be placed on the vehicle's nose. The control parameters were set to $k_1 = 0.025$, $k_2 = 0.0005$, and $k_3 = 10$ and the desired velocity set to $V_d = 2$ m/s. Fig. 2 shows the trajectory described by the vehicle, whereas Fig. 3 displays the evolution of the vehicle's velocities and control inputs.

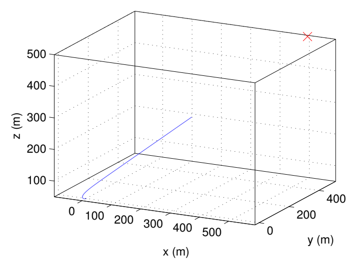

Fig. 2. Trajectory described by the vehicle in the presence of currents

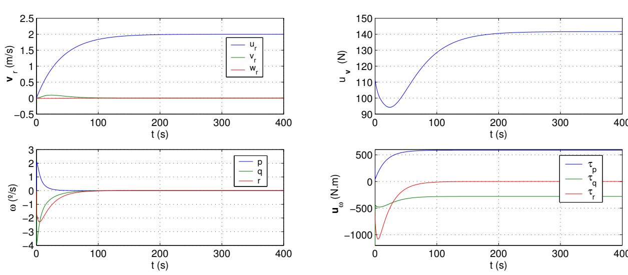

Fig. 3. Time evolution of the vehicle velocities and control inputs in the presence of ocean currents

As expected, the trajectory and control inputs are smooth and the angular, sway, and heave velocities converge to zero.

VII. CONCLUSIONS

The paper presented new homing sensor based integrated guidance and control laws to drive an underactuated AUV to a fixed target in 3D using the information provided by an USBL positioning system. Under the presence (and absence) of constant known ocean currents global asymptotic stability was achieved with the proposed laws. To estimate unknown constant ocean currents a globally exponentially stable observer that also resorts to the USBL data was presented and local asymptotic stability for the overall closed loop system was achieved. Simulation results were presented illustrating the performance of the proposed solutions.

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