Parameter Estimation for the Quad Rotor Dynamics [1]

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Abstract

The simulation of quadrotor dynamics is an essential tool for the development of autonomous behaviours for these platforms, since they prone to physical damage upon crashing. However, validation of these behaviours require the simulation to be faithful. Dynamical models for quadrotor can be formulated as a state space, continuous time, differential equations, depending on a set of physical parameters of the platform. These parameters are the mass, inertia tensor, and propeller thrust factor. This paper addresses the problem of estimating these parameters from flight data. A Least Squares (LS) approach is taken for the estimation. The estimation problem is formulated here, together with preliminary results. The approach is validated with simulated data. We also discuss how the presence of noise hinder the correct estimation of parameters.

1 Introduction

Developing simulators is very common in the field of robotics, regarding the quad rotors is a crucial step due to the risk of physical damaging in real flights. To have a reliable simulator is essential a good estimation of the parameters. In this paper, is presented a quad rotor dynamical model as well as a method to estimate his parameters, using specific flight data in order to simplify the LS estimations.

The LS is a widely used method and converges for the true value of the parameter [1], but as shown in [3], this is only true when the input data is noise free. In systems where the input and output are corrupted with noise, the estimation will be bias. Hence the method is tested using a simulator, in a first approach only the output data is corrupted. Secondly both output and input are corrupted with noise in order to verify the influence in the estimations.

2 Quad-rotor dynamic model

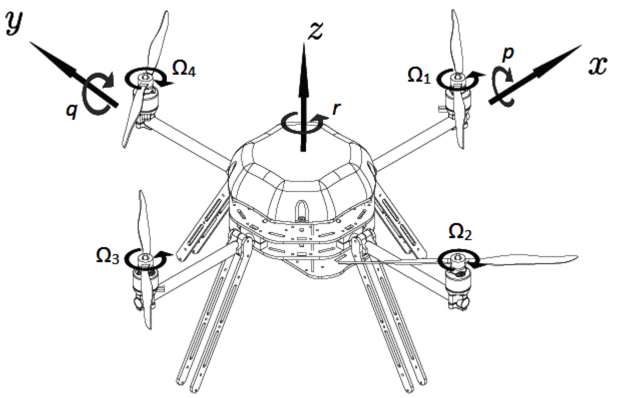

Figure 1: Quadrotor body-fixed frame

The quad-rotor model is fully studied in [2]. In this paper it is just presented the following set of equations

$$\begin{cases} \dot{p} = \dfrac{I_{yy}-I_{zz}}{I_{xx}}qr + \dfrac{U_2}{I_{xx}} & \text{(1a)} \\ \dot{q} = \dfrac{I_{zz}-I_{xx}}{I_{yy}}pr + \dfrac{U_3}{I_{yy}} & \text{(1b)} \\ \dot{r} = \dfrac{I_{xx}-I_{yy}}{I_{zz}}pq + \dfrac{U_4}{I_{zz}} & \text{(1c)} \end{cases}$$

$$\begin{cases} U_1 = b(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2) & \text{(2a)} \\ U_2 = bl(\Omega_4^2-\Omega_2^2) & \text{(2b)} \\ U_3 = bl(\Omega_3^2-\Omega_1^2) & \text{(2c)} \\ U_4 = d(\Omega_2^2+\Omega_4^2-\Omega_1^2-\Omega_3^2) & \text{(2d)} \end{cases}$$

Where $\dot{\omega} = [\dot{p},\dot{q},\dot{r}]^T$ is the angular accelerations and $\omega = [p,q,r]^T$ is the angular velocities with respect to x,y,z in the body-fixed frame. $\Omega = [\Omega_1,\Omega_2,\Omega_3,\Omega_4]$ is the propellers' speed and Throttle(U_1),Roll(U_2),Pitch(U_3) and Yaw(U_4) are the four possible movements of the quad-rotor.

The parameters that need to be estimated are the thrust (b), drag (d) and the inertia tensors I_{xx}, I_{yy} and I_{zz}.

The distance between the center of the quad-rotor and the center of the propellers, l, is a known parameter.

3 Parameters Estimation

In this section a method is proposed which aims to estimate the parameters individually, using flight data with specific movements in order to simplify the equations presented in section 2.

3.1 Thrust estimation

To estimate b, only equation (2a) is used. Assuming that the quad-rotor is hovering, $U_1 = mg$, where m is the quad-rotor mass and g the acceleration due to gravity. Using this assumption

$$\overbrace{mg}^{\mathrm{Y}} = b\overbrace{(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2)}^{\mathrm{X}} \tag{3}$$

and b LS estimation [1] is

$$\hat{b} = (X'X)^{-1}(X'Y) \tag{4}$$

In actual flights it could be tricky to keep the quad-rotor hovering, therefore another solution is proposed. Using Newton's second law over z axe

$$F = m\ddot{z} = b(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2) - mg \tag{5}$$

Doing a double integrating on equation (5) and considering that the final and initial positions are the same, b is deducted as follows

$$\begin{aligned} \int\int_{t_1}^{t_2}(m\ddot{z}) &= \int\int_{t_1}^{t_2}(b(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2)-mg) \\ m\overbrace{[z(t_2)-z(t_1)]}^{0} &= b\int\int_{t_1}^{t_2}(b(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2-mg) \qquad (6) \\ \hat{b} &= \frac{mg\int\int_{t_1}^{t_2}(1)}{\int\int_{t_1}^{t_2}(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2)} \end{aligned}$$

3.2 I_{xx} Estimation

To estimate I_{xx}, flight data will only contain roll movements, which implies that $\omega = [p,0,0]^T$. Applying this,equation (1a) becomes

$$\dot{p} = \frac{bl(\Omega_4^2-\Omega_2^2)}{I_{xx}} \tag{7}$$

In order to perform the LS regression it is necessary to turn the equation into a discrete domain. To approximate the derivative, a forward finite difference is used

[1] This work was supported by the FCT project [PEst-OE/EEI/LA0009/2013]

$$\frac{p(t+h)-p(t))}{h} = \frac{bl(\Omega_4^2-\Omega_2^2)}{I_{xx}} \tag{8}$$

using b value estimated previously, I_{xx} LS estimation is given by

$$\begin{aligned} \overbrace{(\Omega_4^2-\Omega_2^2)}^{Y} &= \overbrace{\frac{p(t+h)-p(t))}{h\hat{b}l}}^{X} I_{xx} \\ \hat{I_{xx}} &= (X'X)^{-1}(X'Y) \end{aligned} \tag{9}$$

3.3 I_{yy} Estimation

This case is similar to the previous one, but instead of roll movements, pitch movements are used, hence, $\omega = [0,q,0]^T$. Applying as before, to equation (1b) and doing a forward finite difference to approximate the derivative

$$\frac{q(t+h)-q(t))}{h} = \frac{\hat{b}l(\Omega_3^2-\Omega_1^2)}{I_{yy}} \tag{10}$$

the I_{yy} LS estimation is

$$\begin{aligned} \overbrace{(\Omega_3^2-\Omega_1^2)}^{Y} &= \overbrace{\frac{q(t+h)-q(t))}{h\hat{b}l}}^{X} I_{yy} \\ \hat{I_{yy}} &= (X'X)^{-1}(X'Y) \end{aligned} \tag{11}$$

3.4 I_{zz} Estimation

To estimate I_{zz} the equations(1a) and (1b) are used, in this case the flight data will contain the three movements Roll,Pitch and Yaw. Using the forward finite diference and the previous estimations

$$\overbrace{-\frac{p(t+h)-p(t)}{h}\hat{I_{xx}} + qr\hat{I_{yy}} + \hat{b}l(\Omega_4^2-\Omega_2^2)}^{Y_1} = \overbrace{qr}^{X_1} I_{zz} \tag{12}$$

$$\underbrace{\frac{q(t+h)-q(t)}{h}\hat{I_{yy}} + pr\hat{I_{xx}} - \hat{b}l(\Omega_3^2-\Omega_1^2)}_{Y_2} = \underbrace{pr}_{X_2} I_{zz} \tag{13}$$

the I_{zz} LS estimation is

$$\hat{I_{zz}} = \left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}' \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}' \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right) \tag{14}$$

3.5 Drag Estimation

The last parameter to estimate is d, where the data will only contain Yaw movements and the equation (1c) is used. This results in $\omega = [0,0,r]^T$ and approximating the derivative by the forward finite difference and using the I_{zz} estimation

$$\overbrace{(\Omega_2^2+\Omega_4^2-\Omega_1^2-\Omega_3^2)}^{Y} = \overbrace{\left(\frac{r(t+h)-r(t)}{h}\hat{I_{zz}}\right)}^{X} \left(\frac{1}{d}\right) \tag{15}$$

the d LS estimation is

$$\hat{d} = \frac{1}{(X'X)^{-1}(X'Y)} \tag{16}$$

4 Results

To test the method, a *simulink* simulator developed in [2] is used. A block is included to add white Gaussian noise to the variables, in order to examine the efficiency of the method. The simulator assumes the following

	Estimation without noise in X		Estimation with noise in X	
Flight	LS	Double Int.	LS	Double Int.
1	$5.415e-5$	$5.427e-5$	$5.4195e-5$	$5.4378e-5$
2	$5.4148e-5$	$5.427e-5$	$5.4180e-5$	$5.437e-5$
3	$5.418e-5$	$5.4268e-5$	$5.4186e-5$	$5.4379e-5$

Table 1: Estimation values of parameter b with and without noise in X

	Least-Squares Estimation without noise in X			
Flight	I_{xx}	I_{yy}	I_{zz}	d
1	$8.1e-3$	$8.0e-3$	$15.1e-3$	$1.0985e-6$
2	$7.9e-3$	$8.0e-3$	$13.9e-3$	$1.0968e-5$
3	$8.1e-3$	$8.1e-3$	$13.6e-3$	$1.101e-6$
	Least-Squares Estimation with noise in X			
Flight	I_{xx}	I_{yy}	I_{zz}	d
1	$5.6e-3$	$5.5e-3$	$6.7e-3$	$1.2056e-6$
2	$5.5e-3$	$5.5e-3$	$3.5e-3$	$1.1175e-6$
3	$5.5e-3$	$5.6e-3$	$7.5e-3$	$1.1825e-6$

Table 2: Estimation values of parameters I_{xx}, I_{yy}, I_{zz} and d with and without noise in X

values for the parameters

$$\begin{aligned} b &= 5.42e-5 \\ d &= 1.1e-6 \\ I_{xx} = I_{yy} &= 8.1e-3 \\ I_{zz} &= 14.2e-3 \end{aligned}$$

The measurements were taken with flights of 100 seconds and with a sampling period of 0.05 seconds. For each parameter were done six flights, where three of them were only corrupted with noised in the output data and the other three had noise in both output and input data.

Table (1) shows the estimations values of b, both method presented in section 3.1 were tested. The two methods proved to be effective, even in the presence of noise in the input data.

Table (2) presents the estimations values for the remaining parameters, it can be observed that if the input data is error free the estimations are close to the true value, but in the scenario where both output and input are noisy, the estimation of I_{xx} and I_{yy} are bias [3] and the error propagates to the remaining estimations.

5 Conclusions

This paper presented a method to estimate the parameters of a quad rotor dynamic model,the preliminary results shows lack of precision of the method when both input and output data are corrupted with noise.

It is noteworthy that all inertia tensors can be estimated at the same time using equations (12) and (13) from section (3.4), but in that scenario the movements Roll,Pitch and Yaw must be executed simultaneously, which is a tricky flight to perform. Therefore it was chosen to estimate them separately for a better estimation of I_{xx} and I_{yy} .

Future work consist in estimating the parameters with real data taking into account that a noise free input is needed, which can be difficult to achieve. Another approach is to make a non-parametric estimation of the system.

References

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