High resolution ECG analysis by an improved signal averaging method and comparison with a beat-to-beat approach

S. Jesus* and H. Rix

Laboratoire de Signaux et Systèmes, UA 814 du CNRS, Université de Nice, 41, Bd. Napoleon III, 06041 Nice Cedex, France

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ABSTRACT
The aim of this paper is to describe the analysis of a high resolution ECG recorded from the body surface. Standard signal averaging techniques are improved by using a new time delay estimation method which leads to a better alignment accuracy of P and T waves. A second method uses adaptive identification to achieve a beat by beat fine ECG estimation. Information provided by the two methods allows a better interpretation of low and very low level signals.

Keywords: Signal processing, ECG

INTRODUCTION
Since Puech¹ and Scherlag² revealed the interest in micropotentials in the study of heart disease considerable work has been carried out by many research groups. First results have been obtained by direct heart catheterization; signals are of a low level and a relatively high-frequency spectrum compared to those of the standard ECG; classical gain values (≈ 1000) and bandwidth (10–100 Hz) of current electrocardiographs do not allow micropotential extraction.

The main progress in ECG analysis has been the simultaneous proposal of Berburi et al.³, Flowers et al.⁴ and Stopczynck et al.⁵ in 1973, who established the basis of the high resolution ECG (HRECG). The ECG signal is recorded by a highly sensitive system from surface electrodes. Typical gain values are between 3 × 10⁴ and 5 × 10⁶. Frequency range is increased to 500 Hz. These two features are not enough to allow the analysis of low level signals, which are buried in muscular, motion and electrical noise.⁶ For some low level cardiac potentials (e.g. the His-Purkinje activity), a typical value of the signal to noise ratio (SNR) is −20 dB.

Signal averaging is the most popular and simplest method of improving the SNR of repetitive signals and the method has been used in HRECG with some success³–⁵. Its efficiency mainly depends on signal alignment accuracy and shape constancy. Furthermore, a signal estimate is given only after N summations. N is generally large, typically 100 < N < 300.

To avoid these disadvantages, recent researches have been carried out to obtain a possible beat to beat estimation of the fine ECG structure⁷–¹⁰. The idea is to carry out signal averaging in space rather than in time. Instead of summing N successive cardiac complexes (recorded with one electrode), the same cardiac complex recorded at N different locations on body surface (by N different electrodes) is summed. In this approach the noise is assumed to be isotropic. This method is called the low noise ECG (LNECG). Its weak points are the positioning of the N electrodes: only a limited number of electrodes can be used (up to 16). A low value of N limits the SNR gain of the method and enforces stringent constraints on acquisition systems¹⁰.

This paper deals with fine ECG analysis recorded from body surface by high resolution systems without any particular acquisition measures (e.g. Faraday cage, battery isolation, patient training, etc). The points of interest are the very low level signals and the fine structure of low SNR classical cardiac waves (P and T waves).

Two different approaches are investigated. First a signal averaging approach where an attempt is made to reduce jitter, caused by PR or ST interval variations, by direct P or T waves synchronization using a novel time delay estimation method. Second, a beat by beat estimation approach in order to avoid inherent problems of normal signal averaging and technical constraints of HR spatial ECG. The method uses smoothing, filtering and adaptive identification algorithms to estimate the ECG low level potentials on a beat by beat basis.

THEORY
Signal averaging
Signal averaging is highly dependent on the alignment accuracy of the signals to be averaged. ECG averaging is performed by detecting QRS complex, which has the highest SNR in the cardiac beat,
thereby ensuring a stable synchronization to extract low micropotentials occurring at a constant interval from the QRS. This dependence on stable synchronization and interval constancy enables the method to extract potentials other than those near QRS complex or physiologically linked to it.

The purpose is to improve synchronization possibilities for other ECG waves having lower SNR than the R waves (e.g. P and T waves). If synchronization is sufficiently precise, the jitter phenomenon influence is reduced when extracting low potentials near (and linked to) P or T waves. The new TDE method\(^1\) is recalled.

**Principle**
Consider a real signal \(z_i(t)\). Let
\[
\int_{\infty}^{+\infty} z_i(t) \, dt = A \tag{1}
\]
be the surface of \(z_i(t)\) and assume \(A \neq 0\); let
\[
Z_i(t) = (1/A) \int_{-\infty}^{t} z_i(x) \, dx \tag{2}
\]
be its normalized integral. The normalized integral of the signal
\[
z_i(t) = k_j z_i(t - D_y) \tag{3}
\]
is related to \(Z_i(t)\) by
\[
Z_j(t) = Z_i(t - D_y) \tag{4}
\]
where \(k_j\) and \(D_y\) are two constants. The method is based on the following result: the delay \(D_y\) between \(z_i(t)\) and \(z_j(t)\) can be obtained by
\[
D_y = \int_{-\infty}^{+\infty} [Z_i(t) - Z_j(t)] \, dt \tag{5}
\]

**Application**
With noise \(y_i(t)\) and \(y_j(t)\) are dealt with instead of \(z_i(t)\) and \(z_j(t)\)
\[
y_i(t) = z_i(t) + v_i(t) \tag{6}
\]
\[
y_j(t) = z_j(t) + v_j(t)
\]
where \(v_i(t)\) and \(v_j(t)\) are two additive noise processes. Generally the noisy signals are replaced by a positive function to avoid surface \(A\) becoming zero. Furthermore, considering stationary noises, the signals are assumed to be band limited so that they are still integrable, an assumption that is not very restrictive for the application considered here. Dealing with repetitive signals which are positive in a given interval, without losing any information one can work with
\[
y^+(t) = \max(y(t), 0)
\]
In this case one can compute an estimator \(\hat{D}_y\) of \(D_y\) by applying relation (5). The uncertainty of \(\hat{D}_y\) be reduced by noting that shifting \(y_j(t)\) by a known quantity \(\tau\)
\[
y_j(t) = y_j(t - \tau) = y_i(t - D_y - \tau)
\]
one can calculate
\[
Q(\tau) = \int [Y_i(t) - Y_j(t - \tau)] \, dt
\]
for a series of \(\tau\) values \(\{\tau_l: l = 1, L\}\); in (8) \([a, b]\) the observation interval and \(Y_i(t)\) and \(Y_j(t - \tau_l)\) the normalized integrals of \(y_i(t)\) and \(y_j(t - \tau_l)\) respectively. The zero crossing of the regression line, of the form \(y = \alpha \tau + \beta\), of the \(Q(\tau_l)\) versus \(\tau_l\) gives an estimator of \(D_y\)
\[
\hat{D}_y = \beta/\alpha
\]

**Beat-to-beat estimation**
In order to avoid signal averaging constraints and obtain rapid information for immediate diagnostic beat-to-beat ECG estimation is highly desirable. Until now, researches in this field have suffered recording instrumentation capabilities and increased procedure complexity.

Here a totally different approach is tried, applying a particular family of adaptive identification smoothing algorithms.

The problem is assumed to be the estimation of a deterministic unknown signal corrupted by additive noise with a poor SNR (< 0 dB). In this case standard identification techniques give poor results\(^2\). Furthermore, in this method an ARM representation associated with a stationarity assumption is generally used and the generation process is excited by a continuous input.

In this approach these two assumptions are not absolutely necessary. The deterministic signal to be extracted is modelled by the impulse response of a process represented in a state variable form starting from unknown initial conditions. The algorithm generates a smooth adaptive joint estimate of the model parameters and the desired signal; only the model order is assumed to be known. The same algorithm produces a smooth estimate of the initial state at each iteration.

After convergence the signal reconstruction obtained by a Kalman filter conditioned on the estimated parameters and the observation set.

**Signal modelling**
The deterministic signal is assumed to be represented by an ARMA model defined by
\[
z(k) = -\sum_{i=1}^{L} a_i z(k - i) + \sum_{i=1}^{M} b_i u(k - i) \tag{10}
\]
where \(u(\bullet)\) is the impulse input and the coefficients \{\(a_i\)\} and \{\(b_i\)\} characterize the system to be identified. Assuming that \(u(\bullet)\) is an impulse arriving just before starting the observation one can consider that all the information due to \(u(\bullet)\) is contained in an initial state \(x_0 \neq 0\) and rewritten equation (10), i
an equivalent state variable form\(^{13,14}\)
\[
\begin{align*}
\begin{cases}
  x(k+1) = J x(k) + \theta(k) z(k) & x(0) = x_0 \neq 0 \\
  z(k) = C x(k)
\end{cases}
\end{align*}
\]
where
\[
J = \begin{pmatrix}
  0 & \ldots & 0 \\
  1 & \ldots & \ldots \\
  \vdots & \ddots & \ddots \\
  0 & \ldots & 1 \\
\end{pmatrix}
\]
\[
C = (0, \ldots, 0, 1)
\]
\[
\theta(k) = (-a_0(k), \ldots, -a_1(k))
\]

In this representation we assume that there is no dynamic noise and that all the uncertainty is modelled by an additive white Gaussian noise process on the observation equation
\[
y(k) = z(k) + v(k)
\]
Note that the estimation of the desired signal is
\[
\hat{z}(k) = \hat{x}_x(k)
\]
The problem is resumed to the estimation of \(\theta(k)\) and the state \(x(k)\). This problem is achieved in a joint representation of equation (11) by forming the following augmented state space model:
\[
\begin{align*}
\begin{cases}
  x_a(k+1) &= \begin{pmatrix} J & \frac{x_p(k)I}{O} \end{pmatrix} x_a(k) + \begin{pmatrix} \frac{x_p(k)I}{O} \end{pmatrix} x_p(k) \\
  y(k) &= \begin{pmatrix} I & O \end{pmatrix} x_a(k) + v(k)
\end{cases}
\end{align*}
\]
with
\[
x_a(k) = \begin{pmatrix} x(k) \\ \theta(k) \end{pmatrix}
\]
The non linear problem of estimating \(x_a(k)\) is generally solved by the extended Kalman filter, but in order to avoid the above mentioned problems we use a different family of algorithms: the partitioned algorithms originally due to Lainiotis\(^{13}\).

**Generalized partitioned algorithms (GPA)\(^{13}\)**

The main idea is to make a partition of the initial augmented state in a nominal state and a remainder:
\[
x_a(0) = x_a s(0) + x_a r(0)
\]
the two separate vectors are assumed to be independent and Gaussian. This partitioning allows one to include the \textit{a priori} information that the user may have about the process, in the nominal part, and then iteratively estimate the remainder.

A particularly interesting partitioning of equation (13) is
\[
\begin{align*}
\hat{x}_a s(0) &= \begin{pmatrix} \hat{x}(0) \\ 0 \end{pmatrix} & \hat{x}_a r(0) &= \begin{pmatrix} 0 \\ \hat{\theta}(0) \end{pmatrix}
\end{align*}
\]
In order to produce an adaptive algorithm a known reference model is introduced and the algorithm estimates only the deviation from this reference model. The reference is generated by a nominal Kalman filter, conditioned by a parameter vector \(\hat{\theta}_o\) updated at each iteration.

The particular choice, equation (14), gives separate smoothing equations to the initial state and the parameters. This choice leads to a reduction of computational complexity. Only a \(p \times p\) matrix inversion is necessary at each iteration.

Convergence is reached when
\[
\|\hat{\theta}_{j+1} - \hat{\theta}_j\| < \epsilon
\]
is fulfilled.

**ECG DATA PROCESSING**

ECG signals shown in this section have been recorded at the Laboratoire de Cardiologie, Hôpital Pasteur, and presented in a short form in reference\(^7\). A high amplification recording device (Pancardiographe, Biosignal) has been used. A gain of about \(10^5\) and a bandpass filter of 0.5–300 Hz were chosen. Data acquisition was made with a sampling rate of 1 kHz and a 12 bits quantization for an input level of ± 10 V. Real ECG signals have been recorded with a two-track magnetic tape recorder; the two synchronous tracks contain a low level ECG for QRS detection and synchronization and a high level ECG for HR-ECG processing (averaging or beat by beat estimation).

In order to reduce QRS detection error, the low level ECG track was preprocessed with a digital moving average bandpass filter as proposed by\(^{16}\).

The filter structure has been changed to avoid introducing a time delay and loss of synchronization between the two tracks. The filtered signal \(y(n)\) is computed by the convolution relationship
\[
y(n) = (h_T * e) (n)
\]
i.e.
\[
y(n) = \sum_{i=-\infty}^{+\infty} h_T(i) e(n - i)
\]
where \(e(n)\) is the input signal and \(h_T(n)\) is the filter impulse response given by
\[
h_T(n) = \begin{cases}
  (n + K + 1)/(K + 1)^2 - K \leq n \leq 0 \\
  (K + 1 - n)/(K + 1)^2 - 1 \leq n \leq K + 1 \\
  0 & \text{otherwise}
\end{cases}
\]
According to equation (18) the summation in equation (17) is reduced to a finite number of terms. The frequency response (which is the Fourier transform of the impulse response) is then
\[
H_T(f) = \frac{1}{(K + 1)^2} \frac{\sin^2 \pi f}{\sin^2 \pi f}
\]
Figure 1 P wave processing at rest by signal averaging five successive P waves, mean signals after 20, 40, 60 and summations with, b, R and, c, P wave synchronization.

which shows a pure real transfer function with a phase characteristic equal to zero for all frequencies to ensure no time delay. The cut off frequencies

\[ f_{c1} = \frac{1}{T_i(K_1 + 1)} \quad f_{c2} = \frac{1}{T_i(K_2 + 1)} \]

where \( K_1 \) and \( K_2 \) are the number of samples averager one and two respectively, \( T_i \) is the sampling interval. To remove base line variations and high frequency noise we choose \( K_1 = 300 \) and \( K_2 = 6 \) which gives a band width \( BP \)

\[ BP = f_{c2} - f_{c1} = 142 - 3.3 \text{ Hz} \]

Records were made from normal, healthy young people at rest and in the recovery phase following light exercise (cardiac frequency varying from 90 to 60 beats min\(^{-1}\)).

**Signal averaging**

In each case comparison was made between the different synchronizations:

1. R wave detection using standard level detection threshold technique.
2. P (or T) wave triggering using our proposed method.

*Figure 1* shows an example of P wave (and T segment) processing recorded at rest. *Figure 2* shows another example of P wave processing recorded on the same person, but in the recovery phase of a light exercise.
Figure 2. P wave processing in the recovery phase of a light exercise: a, five successive P waves, mean signals after 20, 40, 60, 80 and 100 summations with, b, R and, c, P wave synchronization.

Exercise. Figure 3 is an example of TP interval averaging.

In Figure 1a there is a series of five noisy P wave and their P-R interval. Figure 1b and c shows the mean signals of Figure 1a after 20, 40, 60 and 80 summations in b with R wave synchronization and in c with P wave synchronization using our method. Comparing these two figures, one can see that the P wave mean shape is better conserved in c than in b.

Figure 2a shows a series of five noisy P waves recorded on the recovering phase after a light exercise. These P waves are averaged with R wave synchronization (Figure 2b) and with P wave synchronization (Figure 2c). Again, one can note a better P wave mean shape conservation in c than in b. Moreover, low potentials before and after P can be seen in b and are erased in a by the jitter phenomenon due to PR variations.

Figure 3, shows the averaged TP interval obtained with three different synchronizations: R wave before TP (a), P wave after TP (b) and T wave before TP (c). This example shows how recurrent low potentials (arrows on Figure 3c) on TP interval, linked to T wave can be extracted only by triggering from T wave itself (c) and not from R (a) or P wave (b).
Beat to beat estimation

This section illustrates the results of the beat by beat estimation algorithm (GPA) processing the signals shown in Figure 1(a) and Figure 2(a). Results given in Figure 4 and 5 respectively.

Initialization was \( \theta(0) = (0) \) and \( \alpha(0) = 0 \). The order was chosen equal to two (\( p = 2 \)), convergence was reached after 5 or 6 iterations. Reconstruction was made by a Kalman filter, initialized \( \alpha(0/N)_{\text{conv}}, P(0/N)_{\text{conv}} \) and \( \theta_{\text{conv}} \), values of smoothed initial state estimate, its error covariance matrix and the parameter vector after convergence.

A good reconstruction of the signals has been accomplished.

DISCUSSION

The authors have presented two different approaches to the processing of high resolution \( ECG \) signals; signal averaging and beat by beat estimation.

To improve signal averaging capabilities a synchronization method was proposed that allows stable triggering on low SNR signals. The rest of the paper will be devoted to the extraction of low potentials physiology linked to \( P \) or \( T \) waves, and (3) a sorting between \( L \) potential by successive synchronizing in \( R \), \( P \) or waves and (4) a good estimate of the probabilistic distribution for \( PR \) or \( ST \) interval.

The second approach allows shape variabil...
analysis and avoids the main problems attached to signal averaging. The fine P wave structure could be obtained on a beat by beat basis and shape changes from one beat to another are more obvious. Nevertheless, low potentials cannot be distinguished from noise and the beat by beat signal interpretation is better achieved by comparison with the mean signal obtained by averaging. In effect, comparing Figure 2(c) with Figure 5 one can note that some low potentials extracted in the PR interval are present in some but not all beats.

CONCLUSIONS

The results suggest that signal averaging is an objective method with the useful property of extracting only signals lying at a constant time interval from a known triggering point. Time interval constancy generally implies a physiological link, which makes medical interpretation easier. This property makes signal averaging essential even if a beat by beat estimation is practicable. Nevertheless, these two approaches use different amounts of information and give different amounts of information; in fact, they are complementary. Finally, it should be noted that beat by beat estimation is a new tool which may take several years to be fully understood and used.

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