A Sensor Based Homing Strategy for Autonomous Underwater Vehicles

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***Abstract*— A new sensor based homing integrated guidance and control law is presented to drive an underactuated autonomous underwater vehicle (AUV) towards a fixed target, in 3D, using the information provided by an Ultra-Short Base Line (USBL) positioning system. The guidance and control law is firstly derived for the vehicle's kinematics expressed as the time differences of arrival (TDOAs) measured by the USBL sensor assuming the target at the infinity, and then extended for the dynamics of an underactuated AUV resorting to backstepping techniques. The proposed Lyapunov based control law yields almost global asymptotic stability (AGAS) in the absence of external disturbances and is further extended, keeping the same properties, to the case where known ocean currents affect the vehicle's dynamics. Simulations are presented and discussed that illustrate the performance and behavior of the overall closed loop system.**

I. Introduction

Advances in sensing devices, materials, and computational capabilities have provided the means to develop sophisticated underwater vehicles which nowadays display the capability to perform complex tasks in challenging and uncertain operation scenarios. In the last years several sophisticated Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) have been developed, endowing the scientific community with advanced research tools supported in onboard complex mission and vehicle control systems [1], [2], and [3].

The topic of guidance and control of underwater vehicles has been the subject of intense research in the past decades. The control of fully actuated robotic vehicles is nowadays fairly well understood, as evidenced by the large body of publications, see [4], [5], [6], and the references therein. However, the control of underactuated autonomous vehicles is still an active field of research. To tackle the problem of stabilization of an underactuated vehicle a variety of solutions have been proposed in the literature, see [7], [8], [9], and [10]. In [11], [12], and [13] three solutions are proposed to solve the trajectory tracking problem. In [14] a solution for the problem of following a straight line is presented and in [13] a waypoint tracking controller for an underactuated AUV is introduced. It turns out that all the aforementioned references share a common approach that is the vehicle position is computed in the inertial coordinate frame and the control laws are developed in body frame. Sensor based control has been a hot topic in the field of computer vision where the so-called visual servoing techniques have been subject of intensive research effort during the last years, see [15] and [16] for further information.

This paper addresses the design of an integrated guidance and control law to drive an underactuated AUV to a fixed target, in 3D. The solution for this problem, usually denominated as homing in the literature, is central to drive the vehicle to the vicinity of a base station or support vessel. It is assumed that an acoustic emitter is installed on a predefined fixed position in the mission scenario, denominated as target in the sequel, and an Ultra-Short Baseline (USBL) sensor, composed by an array of hydrophones, is rigidly mounted on the vehicle's nose. During the homing phase the target continuously emits acoustic waves that are received by the USBL hydrophone array and the time of arrival measured by each receiver, is synchronized, detected, and recorded. In the approach followed, it is assumed, for the sake of simplicity, that the target is placed at the infinity, where the planar wave approximation is valid. That is the distance between the source and the array is large when compared with both the wavelength and the distance between the USBL sensors. A Lyapunov based guidance and control law is firstly derived using the vehicle's kinematics directly expressed in terms of the time differences of arrival (TDOAs) obtained from the USBL data. The resulting control law is then extended for the dynamics of an underactuated AUV resorting to backstepping techniques. Afterwards, this strategy is further extended to the case where known ocean currents affect the vehicle's dynamics and almost global asymptotic stability (AGAS) is achieved in both cases. The implementation of the control laws also requires the vehicle's linear velocities, relative to the water and to the ground, as provided by a Doppler velocity log, and the vehicle attitude and angular velocities measured by an Attitude and Heading Reference System (AHRS).

The paper is organized as follows. In Section II the homing problem is introduced and the dynamics of the AUV are briefly described. Section III presents the USBL model, whereas in Section IV a solution for the control and guidance problem in the absence of external perturbations is proposed. This control law is further extended in Section V to the case where the vehicle dynamics are disturbed by constant known ocean currents. Simulation' results are presented and discussed in Section VI, and finally Section VII summarizes

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the main results of the paper.

II. Problem statement

Let $\{I\}$ be an inertial coordinate frame, and $\{B\}$ the body-fixed coordinate frame, whose origin is located at the center of mass of the vehicle. Consider $\mathbf{p} = [x,\ y,\ z]^T$ as the position of the origin of $\{B\}$, described in $\{I\}$, $\mathbf{v} = [u,\ v,\ w]^T$ the linear velocity of the vehicle relative to $\{I\}$, expressed in body-fixed coordinates, and $\boldsymbol{\omega} = [p,\ q,\ r]^T$ the angular velocity, also expressed in body-fixed coordinates. The vehicle kinematics can be written as

$$\dot{\mathbf{p}} = {}^{I}_{B}\mathbf{R}\,(\boldsymbol{\lambda})\mathbf{v} \quad \dot{\boldsymbol{\lambda}} = \mathbf{Q}(\boldsymbol{\lambda})\boldsymbol{\omega} \tag{1}$$

where $\mathbf{R} = {}^{I}_{B}\mathbf{R} = ({}^{B}_{I}\mathbf{R})^T$ is the rotation matrix from $\{B\}$ to $\{I\}$, verifying $\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}(\boldsymbol{\omega})$, and $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x})\mathbf{y} = \mathbf{x}\times\mathbf{y}$, with $\times$ denoting the cross product.

The vehicle's dynamic equations of motion, can be written in a compact form as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} = -\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} - \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v} - \mathbf{g}_{\mathbf{v}}(\mathbf{R}) + \mathbf{b}_{\mathbf{v}}u_{\mathbf{v}} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} - \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} - \mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R}) + \mathbf{B}_{\boldsymbol{\omega}}\mathbf{u}_{\boldsymbol{\omega}} \end{cases} \tag{2}$$

where

- $\mathbf{M} = \text{diag}\{m_u, m_v, m_w\}$ is a positive definite diagonal mass matrix;
- $\mathbf{J} = \text{diag}\{J_{xx}, J_{yy}, J_{zz}\}$ is a positive definite inertia matrix;
- $u_{\mathbf{v}} = \tau_u$ is the force control input that acts along the x_B axis;
- $\mathbf{u}_{\boldsymbol{\omega}} = [\tau_q,\ \tau_r]^T$ are the torque control inputs that affect the rotation of the vehicle about the y_B and z_B axes, respectively;
- $\mathbf{D}_{\mathbf{v}}(\mathbf{v}) = \text{diag}\{X_u + X_{|u|u}|u|, Y_v + Y_{|v|v}|v|, Z_w + Z_{|w|w}|w|\}$ is the matrix of the linear motion drag coefficients;
- $\mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega}) = \text{diag}\{K_p + K_{|p|p}|p|, M_q + M_{|q|q}|q|, N_r + N_{|r|r}|r|\}$ is the matrix of the rotational motion drag coefficients;
- $\mathbf{b}_{\mathbf{v}} = [1,\ 0,\ 0]^T$ and $\mathbf{B}_{\boldsymbol{\omega}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$;
- $\mathbf{g}_{\mathbf{v}}(\mathbf{R}) = \mathbf{R}^T[0,\ 0,\ W - B]^T$ represents the gravitational and buoyancy effects, W and B respectively, on the vehicle's linear motion;
- $\mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R}) = \mathbf{S}(\mathbf{r}_B)\mathbf{R}^T[0,\ 0,\ B]^T$ accounts for the effect of the center of buoyancy displacement relatively to the center of mass on the vehicle rotational motion.

Assume that the vehicle is neutrally buoyant, i.e., $W = B$ and therefore $\mathbf{g}_{\mathbf{v}}(\mathbf{R}) = \mathbf{0}$. Further consider that the vehicle's added masses associated with the sway and heave motions are similar, that is $m_v \simeq m_w$, which constitutes a reasonable assumption for most ROV like underwater vehicles.

The homing problem considered in this paper can be stated as follows:

Problem Statement: *Consider an underactuated AUV with kinematics and dynamics given by* (1) *and* (2)*, respectively. Assume that there is a target placed in a fixed position, in 3D, that emits continuously a known acoustic wave. Design a sensor based integrated guidance and control law to drive the vehicle towards the target using the time differences of arrival of the acoustic signal as measured by an USBL sensor installed on the AUV.*

III. USBL model

During the homing approach phase the vehicle is far away from the acoustic emitter, that is, the distance from the vehicle to the target is much larger than the distance between any pair of receivers. Therefore, the plane-wave assumption is valid. Let $\mathbf{r}_i = [x_i,\ y_i,\ z_i]^T \in \mathbb{R}^3,\ \ i = 1,\ 2,\ \ldots,\ N$, denote the positions of the N acoustic receivers installed on the USBL sensor and consider a plane-wave traveling along the opposite direction of the unit vector $\mathbf{d} = [d_x,\ d_y,\ d_z]^T$, as shown in Figure 1. Notice both $\mathbf{r}_i$ and $\mathbf{d}$ are expressed in the body frame.

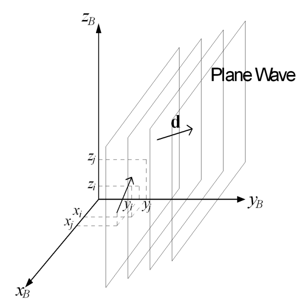

Fig. 1. Plane Wave and the USBL system

Let t_i be the instant of time of arrival of the plane-wave at i^{th} receiver and V_p the velocity of propagation of the sound in water. Then, assuming that the medium is homogeneous and neglecting the velocity of the vehicle, which is a reasonable assumption since $\|\mathbf{v}\| << V_p$, the time difference of arrival between receivers i and j satisfies

$$V_p\,(t_i - t_j) = -\left[d_x\,(x_i - x_j) + d_y\,(y_i - y_j) + d_z\,(z_i - z_j)\right] \tag{3}$$

Denote by $\Delta_1 = t_1 - t_2,\ \Delta_2 = t_1 - t_3,\ \ldots,\ \Delta_M = t_{N-1} - t_N$ all the possible combinations of TDOAs, and let $\boldsymbol{\Delta} = [\Delta_1,\ \Delta_2,\ \cdots,\ \Delta_M]^T$. Define also

$$\begin{aligned} \mathbf{r}_x &= [x_1 - x_2,\ x_1 - x_3,\ \cdots,\ x_{N-1} - x_N]^T \\ \mathbf{r}_y &= [y_1 - y_2,\ y_1 - y_3,\ \cdots,\ y_{N-1} - y_N]^T \\ \mathbf{r}_z &= [z_1 - z_2,\ z_1 - z_3,\ \cdots,\ z_{N-1} - z_N]^T \end{aligned}$$

and $\mathbf{H}_R \in \mathbb{R}^{M\times 3}$ as $\mathbf{H}_R = [\mathbf{r}_x,\ \ \mathbf{r}_y,\ \ \mathbf{r}_z]$. Then, the generalization of (3) for all TDOAs yields

$$\boldsymbol{\Delta} = -\frac{1}{V_p}\mathbf{H}_R\mathbf{d} \tag{4}$$

Due to the plane-wave assumption, that assumes the target at the infinity, the direction of propagation expressed in the inertial frame is constant. Therefore, the time derivative of (4) can be written as

$$\dot{\boldsymbol{\Delta}} = \frac{1}{V_p}\mathbf{H}_R\mathbf{S}(\boldsymbol{\omega})\mathbf{d}$$

To write the time derivative of the TDOA vector $\boldsymbol{\Delta}$ in closed form, define $\mathbf{H}_Q \in \mathbb{R}^{3\times 3}$ as

$$\mathbf{H}_Q = \frac{1}{V_p}\mathbf{H}_R^T\mathbf{H}_R$$

which is assumed to be non-singular. This turns out to be a weak hypothesis as it is always true if, at least, 4 receivers are mounted in noncoplanar positions. In those conditions $\mathbf{H}_R$ has maximum rank and so does $\mathbf{H}_Q$. Then,

$$\mathbf{d} = -\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

and

$$\dot{\boldsymbol{\Delta}} = -\frac{1}{V_p}\mathbf{H}_R\mathbf{S}(\boldsymbol{\omega})\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

which corresponds to a closed form for the dynamics of $\boldsymbol{\Delta}$, assuming the target at the infinity.

IV. Controller design

In this section an integrated nonlinear closed loop guidance and control law is derived for the homing problem stated earlier in Section II. Assuming there are no ocean currents the idea behind the control strategy proposed here is to steer the vehicle directly towards the emitter. The synthesis of the guidance and control law resorts extensively to the Lyapunov's direct method and backstepping techniques.

To steer the vehicle towards the target, consider first the error variable

$$\mathbf{z}_1 := \boldsymbol{\Delta} + \frac{1}{V_p}\mathbf{r}_x$$

As $\mathbf{z}_1$ converges to zero, the vehicle aligns itself in the direction of the target. However, this condition is not sufficient to ensure the desired behavior of the vehicle during the homing phase as it can still move away from the target. In order to avoid that, define a second scalar error variable

$$z_2 := [1,\ 0,\ 0]\mathbf{v} - V_d$$

where V_d is a positive constant that corresponds to the desired velocity during the homing stage. When z_2 converges to zero, the surge velocity u converges to the desired velocity V_d. Since the vehicle is correctly aligned if $\mathbf{z}_1$ is driven to zero, one could think that ensuring that both $\mathbf{z}_1$ and z_2 converge to zero, the vehicle would always approach the target. However, this is not true as the sway and heave velocities are left free. Despite that, it will be shown that, with the control law based upon these two error variables, these velocities converge to zero, which completes a set of sufficient conditions that solves the problem at hand.

To synthesize the control law start by defining the Lyapunov functions

$$V_1 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1$$

where

$$\mathbf{H}_L = \left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\right)^T\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\right)$$

and

$$V_2 = \frac{1}{2}z_2^2 \tag{5}$$

After a few computations, the time derivative $\dot{V}_2$ can be written as

$$\begin{aligned}\dot{V}_2 \;=\; & z_2[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_\mathbf{v}u_\mathbf{v}\\ & -z_2\left([1,\ 0,\ 0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} + \mathbf{D}_\mathbf{v}(\mathbf{v})\mathbf{v}\right]\right)\end{aligned}$$

Choosing

$$\begin{aligned}u_\mathbf{v} \;=\; & \frac{[1,\ 0,\ 0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v} + \mathbf{D}_\mathbf{v}(\mathbf{v})\mathbf{v} + \mathbf{g}_\mathbf{v}(\mathbf{R})\right]}{[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_\mathbf{v}}\\ & -\frac{k_2 z_2}{[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_\mathbf{v}}\end{aligned} \tag{6}$$

where $k_2 > 0$ is a control gain, the time derivative of (5) becomes $\dot{V}_2 = -k_2 z_2^2$ which yields global asymptotic stability of z_2. Furthermore the convergence is exponentially fast.

The time derivative $\dot{V}_1$, after some algebraic manipulations, can be written as

$$\dot{V}_1 = \boldsymbol{\omega}^T\mathbf{S}([1,\ 0,\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

Following the standard backstepping technique it is possible to regard $\boldsymbol{\omega}$ as a virtual control input that can be used to make $\dot{V}_1 \leq 0$. This is achieved by setting $\mathbf{B}_{\boldsymbol{\omega}}{}^T\boldsymbol{\omega}$ equal to $\mathbf{B}_{\boldsymbol{\omega}}{}^T\boldsymbol{\omega}_d$, where

$$\boldsymbol{\omega}_d := -\mathbf{K}_1\mathbf{S}([1,\ 0,\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

and $\mathbf{K}_1 = \text{diag}\{0, k_{12}, k_{13}\}$, $k_{12} > 0$, $k_{13} > 0$, is a control gain matrix. To accomplish this define a third error variable

$$\mathbf{z}_3 = \mathbf{B}_{\boldsymbol{\omega}}{}^T(\boldsymbol{\omega} - \boldsymbol{\omega}_d)$$

and the augmented Lyapunov function

$$V_3 = V_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3$$

The time derivative of V_3 can be written as

$$\begin{aligned}\dot{V}_3 \;=\; & -\left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right]^T\mathbf{K}_1\left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right]\\ & -\mathbf{z}_3{}^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\mathbf{J}^{-1}\left[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} + \mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R})\right]\\ & -\mathbf{z}_3^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\left(\dot{\boldsymbol{\omega}}_d - \mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right)\\ & +\mathbf{z}_3{}^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\mathbf{J}^{-1}\mathbf{B}_{\boldsymbol{\omega}}\mathbf{u}_{\boldsymbol{\omega}}\end{aligned}$$

Setting

$$\begin{aligned}\mathbf{u}_{\boldsymbol{\omega}} \;=\; & \left(\mathbf{B}_{\boldsymbol{\omega}}{}^T\mathbf{J}^{-1}\mathbf{B}_{\boldsymbol{\omega}}\right)^{-1}\left[\mathbf{B}_{\boldsymbol{\omega}}{}^T\left(\mathbf{J}^{-1}\left[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega}\right.\right.\right.\\ & \left.\left.+\mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R})\right] + \dot{\boldsymbol{\omega}}_d - \mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right)\\ & \left.-\mathbf{K}_3\mathbf{z}_3\right]\end{aligned} \tag{7}$$

where $\mathbf{K}_3$ is a positive definite control gain matrix, one obtains $\dot{V}_3 \leq 0$, with $\dot{\boldsymbol{\omega}}_d$ given by

$$\dot{\boldsymbol{\omega}}_d = \mathbf{K}_1\mathbf{S}([1\ 0\ 0]^T)\mathbf{S}(\boldsymbol{\omega})\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

The following theorem states the main result of this section.

Theorem 1: Consider a vehicle with kinematics and dynamics given by equations (1) and (2), respectively, moving without ocean currents. Then, with the control law (6) and (7), the error variable z_2 converges globally asymptotically to zero and almost global asymptotic stability is warranted for the error variables $\mathbf{z}_1$ and $\mathbf{z}_3$. Furthermore, the sway, heave and roll velocities converge to zero, solving the homing problem stated in Section II.

Proof: Before going into the details a sketch of the proof is first offered. The convergence of the error variables in $\mathbf{z}_1$, z_2 and $\mathbf{z}_3$ is a straightforward application of the Lyapunov's second method. The analysis of the vehicle's equations of motion, when $\mathbf{z}_1$, z_2 and $\mathbf{z}_3$ converge to zero, allows to conclude the convergence to zero of the sway, heave, and roll velocities.

The Lyapunov function V_2 is, by construction, continuous, radially unbounded and positive definite. With the control law (6), the time derivative $\dot{V}_2$ results negative definite. Therefore, the origin $z_2 = 0$ is a global asymptotic stable equilibrium point. Furthermore, since

$$\dot{V}_2 = -k_2 z_2^2 = -2k_2 V_2$$

z_2 converges exponentially fast to zero.

The function V_3 is, also by construction, continuous, radially unbounded and positive definite for feasible values of $\mathbf{z}_1$. This can be easily shown as if V_3 is expanded one obtains

$$\begin{aligned} V_3 &= \frac{1}{2}\left(\mathbf{d} - [1,\ 0,\ 0]^T\right)^T \left(\mathbf{d} - [1,\ 0,\ 0]^T\right) + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 \\ &> 0\ \forall_{\mathbf{d}\neq[1,\ 0,\ 0]^T \wedge \mathbf{z}_3 \neq 0 \Leftrightarrow \mathbf{z}_1 \neq \mathbf{0} \wedge \mathbf{z}_3 \neq 0} \end{aligned}$$

Moreover, with the control law (7), the time derivative $\dot{V}_3$ results in

$$\begin{aligned} \dot{V}_3 &= -\left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right]^T \mathbf{K}_1 \left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right] \\ &\quad -\mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3 \end{aligned}$$

which is negative semi-definite and it is also straightforward to show that

$$\dot{V}_3 = 0 \Leftrightarrow (\mathbf{z}_1 = \mathbf{0}, \mathbf{z}_3 = \mathbf{0}) \vee \left(\mathbf{z}_1 = \frac{2}{V_p}\mathbf{r}_x, \mathbf{z}_3 = \mathbf{0}\right)$$

It is now important to prove that the equilibrium point not coincident with the origin where $\dot{V}_3 = 0$, that corresponds to the situation where the vehicle is aligned towards the opposite direction of the target, is an unstable equilibrium point. To show that consider the function

$$V_i = \frac{1}{2}\mathbf{z}_i^T\mathbf{H}_L\mathbf{z}_i - \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 \tag{8}$$

where

$$\mathbf{z}_i = \boldsymbol{\Delta} - \frac{1}{V_p}\mathbf{r}_x$$

The time derivative of (8) can be written as

$$\begin{aligned} \dot{V}_i &= \left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right]\mathbf{K}_1\left[\mathbf{S}([1\ 0\ 0]^T)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right] \\ &\quad +\mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3 \end{aligned}$$

Since $V_i(\mathbf{0}) = 0$, $V_i(\mathbf{z}_i, \mathbf{z}_3)$ can assume strictly positive values arbitrarily close to the origin and $\dot{V}_i$ is positive definite in a neighborhood of the origin, then the origin of V_i is unstable ([17], Theorem 4.4). Therefore, the only stable equilibrium point of V_3 is the origin $(\mathbf{0}, \mathbf{0})$. Thus, almost global asymptotic convergence of the error variables $(\mathbf{z}_1, \mathbf{z}_3)$ to the origin is achieved.

To complete the stability analysis all that is left to do is to show that the sway, heave and roll velocities converge to zero. The dynamics of the sway and heave velocities can be written as

$$\begin{bmatrix} \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -\frac{Y_v + Y_{|v|v}|v|}{m_v} & \frac{m_w}{m_v}p \\ -\frac{m_v}{m_w}p & -\frac{Z_w + Z_{|w|w}|w|}{m_w} \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix} + \begin{bmatrix} -\frac{m_u}{m_v}ur \\ \frac{m_u}{m_w}uq \end{bmatrix} \tag{9}$$

Taking the limit of the pitch and yaw velocities when $\mathbf{z} = (\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3)$ converges to zero yields

$$\lim_{\mathbf{z}\to\mathbf{0}} [q,\ r]^T = \mathbf{0}^T$$

On the other hand, u converges to the desired velocity V_d. Therefore, the dynamics of the sway and heave velocities can be regarded as a linear time varying system with an external disturbance that converges to zero. It can be shown that, using the assumption $m_v = m_w$, the linear time varying system (9) is asymptotically stable, for arbitrary values of p. Therefore, since the external input, here regarded as a disturbance, converges to zero, so do the sway and heave velocities.

The roll velocity motion equation is similar to the equation of a stable pendulum affected by a disturbance that converges to zero. From [18] one can conclude that this angular velocity also converges to zero, therefore completing the proof. ∎

V. Control in the presence of ocean currents

In this section the results from the previous sections are generalized for the case where known ocean currents are present. Consider that the vehicle is moving with water relative velocity $\mathbf{v}_r$, expressed in the body-fixed coordinate frame, and that the water is also moving with constant velocity $\mathbf{v}_c$ relatively to the inertial frame, expressed in body-fixed coordinates. Then, the dynamics of the vehicle can be rewritten as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}}_r = -\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v}_r - \mathbf{D}_{v_r}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{b}_{\mathbf{v}}u_{\mathbf{v}} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\mathbf{S}(\mathbf{v}_r)\mathbf{M}\mathbf{v}_r - \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} - \mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} - \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R}) + \mathbf{B}_{\boldsymbol{\omega}}\mathbf{u}_{\boldsymbol{\omega}} \end{cases} \tag{10}$$

and the vehicle's velocity relative to the inertial frame, expressed in body-fixed coordinates, is $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_c$.

Under these conditions it is possible to conclude that the guidance and control strategy synthesized in Section IV does not solve the current homing problem, as the new control objective is to align the velocity of the vehicle relatively to the inertial frame towards the target instead of the x axis of the vehicle. Consider the vehicle reference relative velocity $\mathbf{v}_R := [V_d,\ 0,\ 0]^T$ that corresponds to a desired velocity relative to $\{I\}$ and expressed in $\{B\}$ of $\mathbf{v}_d = \mathbf{v}_R + \mathbf{v}_c$. The vehicle is moving towards the target when the velocity vector $\mathbf{v}$ is aligned with the direction of the target, which corresponds to a TDOA vector given by

$$\boldsymbol{\Delta}_d = -\frac{1}{V_p}\mathbf{H}_R\mathbf{d}_d$$

where

$$\mathbf{d}_d = \frac{\mathbf{v}_R + \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|}$$

Obviously the previous statement is only valid if $V_d + V_c\cos(\theta_c) > 0$, where $V_c\cos(\theta_c)$ represents the projection of the current on the vehicle's x axis. Otherwise, it would be impossible for the vehicle to approach the target.

To solve the homing problem in the presence of currents consider the error variables

$$\mathbf{z}_1 := \mathbf{\Delta} + \frac{1}{V_p}\mathbf{H}_R\mathbf{d}_d$$

and

$$z_2 := [1,\ 0,\ 0]\mathbf{v}_r - V_d$$

The convergence of the error variable z_2 to zero can be achieved following a similar procedure as the one presented in Section IV. Defining the Lyapunov function

$$V_2 = \frac{1}{2}z_2^2$$

it is straightforward to show that setting

$$u_{\mathbf{v}} = \frac{[1,\ 0,\ 0]\mathbf{M}^{-1}\left[\mathbf{S}(\boldsymbol{\omega})\mathbf{M}\mathbf{v}_r + \mathbf{D}_{v_r}(\mathbf{v}_r)\mathbf{v}_r\right]}{[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_{\mathbf{v}}} - \frac{k_2 z_2}{[1,\ 0,\ 0]\mathbf{M}^{-1}\mathbf{b}_{\mathbf{v}}} \quad (11)$$

the time derivative of V_2 becomes $\dot{V}_2 = -k_2 z_2^2$. Therefore, z_2 converges exponentially to zero.

To drive $\mathbf{z}_1$ to zero, consider the same Lyapunov function as in Section IV,

$$V_1 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1$$

Using the fact that $\mathbf{H}_L$ is symmetric the time derivative of V_1 is given by

$$\dot{V}_1 = \mathbf{z}_1^T\mathbf{H}_L\dot{\mathbf{z}_1}$$

Since the velocity of the fluid expressed in the inertial coordinate frame is constant, the time derivative of $\mathbf{v}_c$ is $\dot{\mathbf{v}}_c = -\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_c$, and the time derivative of $\mathbf{d}_d$ results in

$$\dot{\mathbf{d}}_d = -\mathbf{S}(\boldsymbol{\omega})\mathbf{d}_d + \frac{\mathbf{v}_R^T\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|^2}\mathbf{d}_d + \frac{1}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_R$$

After straightforward algebraic manipulations the time derivative of $\mathbf{z}_1$ can be written as

$$\dot{\mathbf{z}_1} = -\frac{1}{V_p}\mathbf{H}_R\Big[\mathbf{S}(\boldsymbol{\omega})\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{\Delta} + \mathbf{S}(\boldsymbol{\omega})\mathbf{d}_d - \frac{\mathbf{v}_R^T\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|^2}\mathbf{d}_d - \frac{1}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{S}(\boldsymbol{\omega})\mathbf{v}_R\Big] \quad (12)$$

Using (12), and after some more algebraic manipulations, the time derivative of V_1 becomes

$$\dot{V}_1 = \frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\boldsymbol{\omega}^T\mathbf{S}([1\ 0\ 0]^T)\boldsymbol{\omega}_c$$

where

$$\boldsymbol{\omega}_c = \mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1 - \frac{1}{\|\mathbf{v}_R+\mathbf{v}_c\|}\left[\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T\mathbf{d}_d\right]\mathbf{v}_c$$

Just like in Section IV, it is now possible to regard $\boldsymbol{\omega}$ as a virtual control input that one can use to make $\dot{V}_1 \leq 0$. This is achieved by setting $\mathbf{B}_{\boldsymbol{\omega}}{}^T\boldsymbol{\omega}$ equal to $\mathbf{B}_{\boldsymbol{\omega}}{}^T\boldsymbol{\omega}_d$,

$$\boldsymbol{\omega}_d := -\mathbf{K}_1\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c$$

where $\mathbf{K}_1 = \operatorname{diag}\{0, k_{12}, k_{13}\},\ k_{12} > 0,\ k_{13} > 0$, is a control gain matrix. To accomplish this, consider a third error variable defined as

$$\mathbf{z}_3 = \mathbf{B}_{\boldsymbol{\omega}}{}^T(\boldsymbol{\omega} - \boldsymbol{\omega}_d)$$

and the augmented Lyapunov function

$$V_3 = V_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3$$

The time derivative of V_3 can be written as

$$\begin{aligned}\dot{V}_3 = &-\frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\left[\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c\right]^T\mathbf{K}_1\mathbf{S}([1,\ 0,\ 0])^T\boldsymbol{\omega}_c \\ &-\mathbf{z}_3^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\left(\mathbf{J}^{-1}\left[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega} + \mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R})\right]\right) \\ &+\mathbf{z}_3^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\left(-\dot{\boldsymbol{\omega}}_d + \frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c\right) \\ &+\mathbf{z}_3^T\mathbf{B}_{\boldsymbol{\omega}}{}^T\mathbf{J}^{-1}\mathbf{B}_{\boldsymbol{\omega}}\mathbf{u}_{\boldsymbol{\omega}}\end{aligned}$$

For the sake of simplicity the derivative of $\boldsymbol{\omega}$ is not presented here. Now, setting

$$\begin{aligned}\mathbf{u}_{\boldsymbol{\omega}} = &\left(\mathbf{B}_{\boldsymbol{\omega}}{}^T\mathbf{J}^{-1}\mathbf{B}_{\boldsymbol{\omega}}\right)^{-1}\Big[\mathbf{B}_{\boldsymbol{\omega}}{}^T\big(\mathbf{J}^{-1}\left[\mathbf{S}(\mathbf{v})\mathbf{M}\mathbf{v} + \mathbf{S}(\boldsymbol{\omega})\mathbf{J}\boldsymbol{\omega}\right. \\ &\left.+\mathbf{D}_{\boldsymbol{\omega}}(\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{g}_{\boldsymbol{\omega}}(\mathbf{R})\right] + \dot{\boldsymbol{\omega}}_d \\ &-\frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c\big) - \mathbf{K}_3\mathbf{z}_3\Big]\end{aligned} \quad (13)$$

where $\mathbf{K}_3 \in \mathbb{R}^{2\times 2}$ is a positive definite control gain matrix, $\dot{V}_3$ becomes

$$\begin{aligned}\dot{V}_3 = &-\frac{V_d}{\|\mathbf{v}_R+\mathbf{v}_c\|}\left[\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c\right]^T\mathbf{K}_1\left[\mathbf{S}([1,\ 0,\ 0]^T)\boldsymbol{\omega}_c\right] \\ &-\mathbf{z}_3^T\mathbf{K}_3\mathbf{z}_3\end{aligned}$$

which is negative semi-definite.

The following theorem is the main result of this section.

Theorem 2: Consider a vehicle with kinematics and dynamics given by equations (1) and (10), respectively, moving in the presence of ocean currents. Then, with the control law (11) and (13), the error variable z_2 converges globally asymptotically to zero and almost global asymptotic stability is warranted for the error variables in $\mathbf{z}_1$ and $\mathbf{z}_3$. Furthermore, the sway, heave and roll velocities converge to zero, solving the homing problem stated in Section II.

Proof: The proof of convergence of the error variables $\mathbf{z}_1$, z_2 and $\mathbf{z}_3$ is similar to the one presented in Theorem 1. When $\mathbf{z}_1$, z_2 and $\mathbf{z}_3$ converge to zero so do the heave and sway velocities. Using similar arguments as in Theorem 1 the resulting roll motion converges to zero which completes a set of sufficient conditions that ensures that the proposed control law solves the homing problem. ■

VI. Simulation Results

To illustrate the performance of the proposed integrated guidance and control laws a computer simulation is presented in this section. A simplified model of the SIRENE vehicle was used, assuming the vehicle is directly actuated in force and torque [3].

Assume the vehicle has to counteract an ocean current with velocity $[0,\ -1,\ 0]^T$ m/s, expressed in the inertial frame. The vehicle starts at position $[0,\ 0,\ 50]^T$ m and the acoustic pinger is located at position $[500,\ 500,\ 500]^T$ m. The control parameters were chosen as follows: $\mathbf{K}_1 = \text{diag}(0, 10^{-4}, 10^{-4})$, $k_2 = 0.025$ and $\mathbf{K}_3 = \text{diag}(40, 40)$. The desired velocity was set to $V_d = 2$ m/s, and a semi-spherical symmetry USBL sensor with seventeen receivers was placed on the vehicle's nose. Figure 2 shows the trajectory described by the vehicle, whereas Figures 3 and 4 display the evolution of the vehicle's velocities, and control inputs and Euler angles, respectively. From the figures it

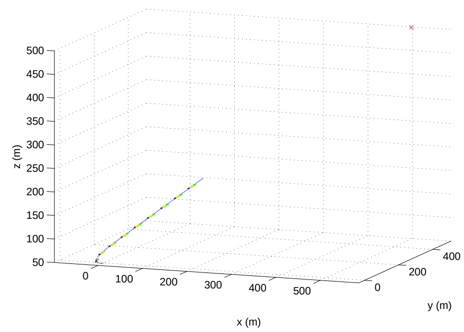

Fig. 2. Trajectory described by the vehicle in the presence of currents

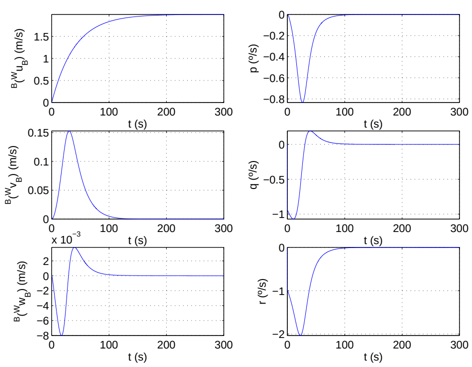

Fig. 3. Time evolution of body-fixed velocities of the vehicle in the presence of currents

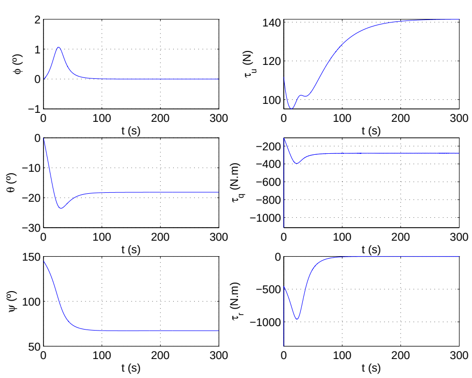

Fig. 4. Time evolution of Euler angles and control inputs in the presence of currents

can be concluded that the vehicle is driven towards the target describing a smooth trajectory. The control inputs are smooth and the resulting angular and lateral velocities converge to zero, as expected.

VII. Conclusions

The paper presented a new homing sensor based integrated guidance and control law to drive an underactuated AUV to a fixed target in 3D using the information provided by an USBL positioning system. The guidance and control laws were firstly derived for the vehicle's kinematics expressed as TDOAs measured by the USBL sensor and then extended to the dynamics of an AUV resorting to backstepping techniques. Almost global asymptotic stability was achieved for the guidance and control law in the presence (and absence) of known ocean currents. Simulation results are presented and discussed that illustrate the performance of the closed loop system.

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