A 2D SENSOR BASED CONTROL LAW FOR HOMING OF AUVS IN THE HORIZONTAL PLANE

Pedro Batista [*,1] **Carlos Silvestre** [*] **Paulo Oliveira** [*]

[*] *Instituto Superior Técnico, Institute for Systems and Robotics Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

Abstract: A new sensor based homing integrated guidance and control law is presented to drive an underactuated autonomous underwater vehicle (AUV) towards a fixed target, in the horizontal plane, using the information provided by an Ultra-Short Base Line (USBL) positioning system. Almost global asymptotic stability (AGAS) is achieved, both in the absence of external disturbances and in the presence of constant known ocean currents. Simulations are presented to illustrate the performance and behavior of the overall closed loop system.

Keywords: Marine systems, autonomous vehicles, nonlinear control.

1. INTRODUCTION

In the last years the scientific community has witnessed the development of several sophisticated Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) which provide advanced research tools supported in onboard advanced mission and vehicle control systems (Sarradin *et al.*, 2002; Silvestre and Pascoal, 2004; Silvestre *et al.*, 1998). The control of these vehicles has naturally been subject of intense work, and while the control of fully actuated vehicles is nowadays fairly well understood, as evidenced by the large body of publications, e.g. (Isidori, 1995; Nijmeijer and van der Schaft, 1990; Sastry, 1999) and the references therein, the control of underactuated vehicles, which are quite appellative as they allow for cost and weight reduction, as well as an increase of autonomy, is still an active field of research. To tackle the problems of stabilization and trajectory tracking of an underactuated vehicle several solutions have been proposed in the literature, see (Wichlund *et al.*, 1995; Reyhanoglu, 1996; Pettersen and Nijmeijer, 1998; Mazenc *et al.*, 2002) and (Aguiar and Hespanha, 2003; Aguiar *et al.*, 2003), respectively. In (Indiveri *et al.*, 2000) a solution for the problem of following a straight line is presented and in (Aguiar and Pascoal, 2002) a way point tracking controller for an underactuated AUV is introduced. It turns out that in all the aforementioned references the vehicle position is computed in the inertial coordinate frame and the control laws are developed in the body frame, disregarding onboard sensors. Sensor-based control has been a hot topic in the field of computer vision where the so-called visual servoing techniques have been subject of an intensive research effort during the last years, see (Cowan *et al.*, 2002; Malis and Chaumette, 2002) for further information.

This paper presents a sensor-based integrated guidance and control law to drive an underactuated AUV to a fixed target, in 2D. The solution for this problem, usually denominated as homing

[1] The work of P. Batista was supported by a PhD Student Scholarship from the POCTI Programme of FCT, SFRH/BD/24862/2005.
[2] This work was supported by the Portuguese FCT POSI programme under framework QCA III and by the project MAYA of the AdI.

in the literature, is central to drive the vehicle to the vicinity of a base station or support vessel. It is assumed that an acoustic emitter is installed on a predefined fixed position in the mission scenario, denominated as target in the sequel, and an Ultra-Short Baseline (USBL) sensor, composed by an array of hydrophones, is rigidly mounted on the vehicle's nose. During the homing phase the target continuously emits acoustic waves that are received by the USBL hydrophone array and the time of arrival measured by each receiver, is synchronized, detected, and recorded. In the approach followed, it is assumed, for the sake of simplicity, that the target is placed at the infinity, where the planar wave approximation is valid. That is the distance between the source and the array is large when compared with both the wavelength and the distance between the USBL sensors. The implementation of the control laws also requires the vehicle's linear velocities, relative to the water and to the ground, as provided by a Doppler velocity log, and the vehicle attitude and angular velocities measured by an Attitude and Heading Reference System (AHRS).

The paper is organized as follows. In Section 2 the homing problem is introduced and the dynamics of the horizontal plane of the AUV are briefly described, whereas the USBL model is presented in Section 3. In Section 4 the vehicle's kinematics, directly expressed in terms of the time differences of arrival (TDOAs) obtained from the USBL data, are used to derive a Lyapunov-based guidance and control law. This control control law is then extended to the dynamics of an underactuated AUV resorting to backstepping techniques. Afterwards, this strategy is further extended, in Section 5, to the case where known ocean currents affect the vehicle's dynamics and almost global asymptotic stability (AGAS) is achieved in both cases. Simulation' results are presented and discussed in Section 6, and finally Section 7 summarizes the main results of the paper.

2. PROBLEM STATEMENT

Let $\{I\}$ be an inertial coordinate frame, and $\{B\}$ a body-fixed coordinate frame, whose origin is located at the center of mass of the vehicle. Consider $\mathbf{p} = [x,\ y]^T$ as the position of the origin of $\{B\}$, described in $\{I\}$, ψ the orientation of the vehicle relative to $\{I\}$, $\mathbf{v} = [u,\ v]^T$ the linear velocity of the vehicle relative to $\{I\}$, expressed in body-fixed coordinates, and ω the angular velocity. The vehicle kinematics can be written as

$$\dot{\mathbf{p}} = {}^{I}_{B}\mathbf{R}\,(\psi)\mathbf{v} \quad \dot{\psi} = \omega \tag{1}$$

where $\mathbf{R} = {}^{I}_{B}\mathbf{R} = ({}^{B}_{I}\mathbf{R})^T$ is the rotation matrix from $\{B\}$ to $\{I\}$, verifying $\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}(\omega)$, and $\mathbf{S}(x)$ is the skew-symmetric matrix

$$\mathbf{S}(x) = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$$

The vehicle's dynamic equations of motion can be written in a compact form as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} = -\mathbf{S}(\omega)\mathbf{M}\mathbf{v} - \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v} + \mathbf{g}u_{\mathbf{v}} \\ J\dot{\omega} = -d_{\omega}(\omega)\omega + u_{\omega} \end{cases} \tag{2}$$

where $\mathbf{M} = \mathrm{diag}\{m_u, m_v\}$ is the positive definite diagonal mass matrix, $\mathbf{D}_{\mathbf{v}}(\mathbf{v}) = \mathrm{diag}\{d_u + d_{|u|u}|u|, d_v + d_{|v|v}|v|\}$ captures the hydrodynamic damping effects on the linear velocity, $d_{\omega}(\omega) = d_{\omega} + d_{|\omega|\omega}\omega$ captures the hydrodynamic damping effects on the angular velocity, and $\mathbf{g} = [1,\ 0]^T$. The control inputs $u_{\mathbf{v}} = \tau_u$ and $u_{\omega} = \tau_{\omega}$ are the surge force and the yaw torque, respectively.

The homing problem considered in this paper can be stated as follows:

Problem Statement. *Consider an underactuated AUV with kinematics and dynamics given by* (1) *and* (2)*, respectively. Assume that, in the plane where the vehicle is moving, there is a target placed in a fixed position that emits continuously a well known acoustic wave. Design a sensor based integrated guidance and control law to drive the vehicle towards the target using the time differences of arrival of the acoustic signal as measured by an USBL sensor installed on the AUV.*

3. USBL MODEL

During the homing approach phase the vehicle is far away from the acoustic emitter, that is, the distance from the vehicle to the target is much larger than the distance between any pair of receivers. Therefore, the plane-wave assumption is valid. Let $\mathbf{r}_i = [x_i,\ y_i]^T \in \mathbb{R}^2,\ \ i = 1,\ 2,\ \ldots,\ N$, denote the positions of the N acoustic receivers installed on the USBL sensor and consider a plane-wave traveling along the opposite direction of the unit vector $\mathbf{d} = [d_x,\ d_y]^T$, as shown in Figure 1. Notice both $\mathbf{r}_i$ and $\mathbf{d}$ are expressed in the body frame.

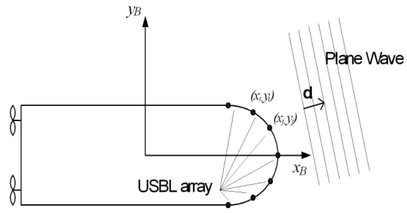

Fig. 1. Plane Wave and the USBL system

Let t_i be the instant of time of arrival of the plane-wave at i^{th} receiver and V_S the velocity of propagation of the sound in water. Then, assuming that the medium is homogeneous and neglecting the velocity of the vehicle, which is a reasonable assumption since $\|\mathbf{v}\| << V_S$, the time difference of arrival between receivers i and j satisfies

$$V_S\,(t_i - t_j) = -\left[d_x\,(x_i - x_j) + d_y\,(y_i - y_j)\right] \tag{3}$$

Denote by $\Delta_1 = t_1 - t_2$, $\Delta_2 = t_1 - t_3$, $\ldots$, $\Delta_M = t_{N-1} - t_N$, where $M = N\,(N-1)/2$, all the possible combinations of TDOA, and let $\boldsymbol{\Delta} = [\Delta_1,\ \Delta_2,\ \cdots,\ \Delta_M]^T$. Define also

$$\mathbf{r}_x = [x_1 - x_2,\ x_1 - x_3,\ \cdots,\ x_{N-1} - x_N]^T$$
$$\mathbf{r}_y = [y_1 - y_2,\ y_1 - y_3,\ \cdots,\ y_{N-1} - y_N]^T$$

and $\mathbf{H}_R \in \mathbb{R}^{M\times 2}$ as $\mathbf{H}_R = [\mathbf{r}_x,\ \mathbf{r}_y]$. Then, the generalization of (3) for all TDOAs yields

$$\boldsymbol{\Delta} = -\frac{1}{V_S}\mathbf{H}_R\mathbf{d} \tag{4}$$

Due to the plane-wave assumption, that assumes the target at the infinity, the direction of propagation expressed in the inertial frame is constant. Therefore, the time derivative of (4) can be written as

$$\dot{\boldsymbol{\Delta}} = \frac{1}{V_S}\mathbf{H}_R\mathbf{S}(\omega)\mathbf{d} \tag{5}$$

To write the time derivative of the TDOA vector $\boldsymbol{\Delta}$ in closed form, define $\mathbf{H}_Q \in \mathbb{R}^{2\times 2}$ as

$$\mathbf{H}_Q = \frac{1}{V_S}\mathbf{H}_R^T\mathbf{H}_R$$

which is assumed to be non-singular. This turns out to be a weak hypothesis as it is always true if, at least, 3 receivers are mounted in noncolinear positions. In those conditions $\mathbf{H}_R$ has maximum rank and so does $\mathbf{H}_Q$. Then,

$$\mathbf{d} = -\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta} \tag{6}$$

Substituting (6) in (5) gives

$$\dot{\boldsymbol{\Delta}} = -\frac{1}{V_S}\mathbf{H}_R\mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta} \tag{7}$$

which corresponds to a closed form for the dynamics of $\boldsymbol{\Delta}$, assuming the target at the infinity.

4. CONTROLLER DESIGN

In this section an integrated nonlinear closed loop guidance and control law is derived for the homing problem stated earlier in Section 2. Assuming there are no ocean currents the idea behind the control strategy proposed here is to steer the vehicle directly towards the emitter. The synthesis of the guidance and control law resorts extensively to the Lyapunov's direct method and backstepping techniques.

To steer the vehicle towards the target, consider first the error variable

$$\mathbf{z}_1 := \boldsymbol{\Delta} + \frac{1}{V_S}\mathbf{r}_x$$

As $\mathbf{z}_1$ converges to zero, the vehicle aligns itself with the direction of the target. However, this condition is not sufficient to ensure the desired behavior of the vehicle during the homing phase as it can still move away from the target. In order to avoid that, define a second scalar error variable

$$z_2 := [1,\ 0]\mathbf{v} - V_d$$

where V_d is a positive constant that corresponds to the desired velocity during the homing stage. When z_2 converges to zero, the surge velocity u converges to the desired velocity V_d. Since the vehicle is correctly aligned if $\mathbf{z}_1$ is driven to zero, one could think that ensuring that both $\mathbf{z}_1$ and z_2 converge to zero, the vehicle would always approach the target. However, this is not true as the sway velocity was left free. Despite this fact, it will be shown that, with the control law based upon these two error variables, the sway velocity converges to zero, which completes a set of sufficient conditions that solves the problem at hand.

To synthesize the control law, define the Lyapunov function

$$V_1 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}z_2^2$$

where $\mathbf{H}_L = \left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\right)^T\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\right)$. After a few computations, the time derivative $\dot{V}_1$ can be written as

$$\dot{V}_1 = z_2[1,\ 0]\mathbf{M}^{-1}\left(\mathbf{g}u_{\mathbf{v}} - [\mathbf{S}(\omega)\mathbf{M}\mathbf{v} + \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v}]\right)$$
$$-\omega[1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

Setting $u_{\mathbf{v}}$ equal to

$$u_{\mathbf{v}} = \frac{[1,\ 0]\mathbf{M}^{-1}\left[\mathbf{S}(\omega)\mathbf{M}\mathbf{v} + \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v}\right] - k_2 z_2}{[1,\ 0]\mathbf{M}^{-1}\mathbf{g}} \tag{8}$$

where $k_2 > 0$ is a control gain, and ω equal to ω_d,

$$\omega_d := k_1[1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$$

where k_1 is a second positive scalar control gain, the time derivative $\dot{V}_1$ becomes strictly non-positive. Although $u_{\mathbf{v}}$ is a real control input, the same cannot be said about ω, which was regarded here as a virtual control input. Following the standard backstepping technique, define a third error variable

$$z_3 = \omega - \omega_d$$

and the augmented Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_3^2 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$$

The time derivative $\dot{V}_2$ can be written as

$$\dot{V}_2 = -k_1\left([1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right)^2 - k_2 z_2^2 + z_3\left(-\dot{\omega_d}\right.$$
$$\left. + \frac{1}{J}\left[-d_\omega(\omega)\omega + u_\omega\right] - [1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}\right)$$

Now, setting

$$u_\omega = d_\omega(\omega)\omega + J\left(\dot{\omega_d} + [1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta} - k_3 z_3\right) \tag{9}$$

where $k_3 > 0$ is a control gain, one obtains $\dot{V}_2 \leq 0$, with $\dot{\omega}_{=}k_1\omega[1,\ 0]\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta}$.

The following theorem states the main result of this section.

Theorem 1. Consider a vehicle with kinematics and dynamics given by equations (1) and (2), respectively, moving without ocean currents. Then, with the control law (8) and (9), the origin $\mathbf{z} = [\mathbf{z}_1^T, z_2, z_3]^T = \mathbf{0}$ is an almost global asymptotic stable equilibrium point. Moreover, the sway velocity converges to zero, thus solving almost globally the homing problem stated in Section 2.

PROOF. Before going into the details a sketch of the proof is first offered. The convergence of the error variable $\mathbf{z}$ is a straightforward application of the Lyapunov's second method. The analysis of the vehicle's sway equation of motion, when $\mathbf{z}$ converges to zero, allows to conclude the convergence to zero of the sway velocity.

The function V_2 is, by construction, continuous, radially unbounded, and positive definite for feasible values of $\mathbf{z}_1$. This can be easily shown expanding V_2

$$V_2 = \frac{1}{2}\left(\mathbf{d} - [1,\ 0]^T\right)^T\left(\mathbf{d} - [1,\ 0]^T\right) + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 > 0\ \forall_{\mathbf{d}\neq[1,\ 0]^T \wedge z_2\neq 0 \wedge z_3 \neq 0 \Leftrightarrow \mathbf{z}\neq\mathbf{0}}$$

Moreover, with the control law (9), the time derivative $\dot{V}_2$ results in

$$\dot{V}_2 = -k_1\left([1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{\Delta}\right)^2 - k_2 z_2^2 - k_3 z_3^2$$

which is negative semi-definite and it is also straightforward to show that

$$\dot{V}_2 = 0 \Leftrightarrow (\mathbf{z} = \mathbf{0}) \vee \left(\mathbf{z}_1 = \frac{2}{V_S}\mathbf{r}_x, z_2 = 0, z_3 = 0\right)$$

It is now important to prove that the equilibrium point not coincident with the origin, which corresponds to the situation where the vehicle is aligned towards the opposite direction of the target, is an unstable equilibrium point. To show that consider the function

$$V_{uns} = \frac{1}{2}\mathbf{z}_{uns}^T\mathbf{H}_L\mathbf{z}_{uns} - \frac{1}{2}z_2^2 - \frac{1}{2}z_3^2 \tag{10}$$

where

$$\mathbf{z}_{uns} = \mathbf{\Delta} - \frac{1}{V_S}\mathbf{r}_x$$

The time derivative of (10) can be written as

$$\dot{V}_{uns} = k_1\left[[1,\ 0]\mathbf{S}(1)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{\Delta}\right]^2 + k_2 z_2^2 + k_3 z_3^2$$

Since $V_{uns}(\mathbf{0}) = 0$, $V_{uns}(\mathbf{z}_i, z_2, z_3)$ can assume strictly positive values arbitrarily close to the origin and $\dot{V}_{uns}$ is positive definite in a neighborhood of the origin, then the origin of V_{uns} is unstable ((Slotine and Li, 1991), Theorem 4.4). Therefore, the only stable equilibrium point of V_2 is the origin $(\mathbf{0}, 0, 0)$. Thus, almost global asymptotic convergence of the error variables $(\mathbf{z}_1, z_2, z_3)$ to the origin is achieved.

To complete the stability analysis all that is left to do is to show that the sway velocity converges to zero. The dynamics of the sway velocity can be written as

$$\dot{v} = -\frac{m_u}{m_v}u\omega - \frac{d_v + d_{|v|v}|v|}{m_v}v$$

Taking the limit of angular velocity when $\mathbf{z}$ converges to zero yields $\lim_{\mathbf{z}\to\mathbf{0}}\omega = 0$, since $\omega \to \omega_d$ and $\omega_d \to 0$. On the other hand, u converges to the desired velocity V_d. Therefore, the sway velocity converges to zero, completing the proof.

5. CONTROL IN THE PRESENCE OF OCEAN CURRENTS

Consider that the vehicle is moving with water relative velocity $\mathbf{v}_r$ in the presence of a known ocean current $\mathbf{v}_c$, both expressed in body-fixed coordinates. It is further assumed that the current velocity is constant in the inertial frame. The dynamics of the vehicle can then be rewritten as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}}_r = -\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r - \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{g}u_{\mathbf{v}} \\ J\dot{\omega} = -d_\omega(\omega)\omega + u_\omega \end{cases} \tag{11}$$

and the vehicle's velocity relative to the inertial frame, expressed in body-fixed coordinates, is $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_c$.

Under these conditions it is possible to conclude that the guidance and control strategy synthesized in Section 4 does not solve the homing problem at hand, as the new control objective is to align the velocity of the vehicle relatively to the inertial frame towards the target instead of the x axis of the vehicle. Consider the vehicle reference relative velocity $\mathbf{v}_R := [V_d,\ 0]^T$ that corresponds to a desired velocity relative to $\{I\}$, expressed in $\{B\}$, of $\mathbf{v}_d = \mathbf{v}_R + \mathbf{v}_c$. The vehicle is moving towards the target when the velocity vector $\mathbf{v}$ is aligned with the direction of the target, which corresponds to a TDOA vector given by

$$\mathbf{\Delta}_d = -\frac{1}{V_S}\mathbf{H}_R\mathbf{d}_d$$

where $\mathbf{d}_d = \frac{\mathbf{v}_R + \mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|}$. Obviously the previous statement is only valid if $V_d + V_c\cos(\theta_c) > 0$, where $V_c\cos(\theta_c)$ represents the projection of the current on the vehicle's x axis. Otherwise, it would be impossible for the vehicle to approach the target.

To solve the homing problem in the presence of currents, the error variables defined in Section 4 are now naturally redefined as

$$\mathbf{z}_1 := \mathbf{\Delta} + \frac{1}{V_S}\mathbf{H}_R\mathbf{d}_d$$

and

$$z_2 := [1,\ 0]\mathbf{v}_r - V_d$$

Consider the same Lyapunov function as in Section 4

$$V_1 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}z_2^2$$

To compute the time derivative $\dot{V}_1$, the time derivative of $\mathbf{d}_d$ must first be determined. Keeping in mind that the velocity of the fluid expressed in the inertial coordinate frame is constant, and after a few straightforward algebraic manipulations, one obtains

$$\dot{\mathbf{z}}_1 = -\frac{1}{V_p}\mathbf{H}_R\mathbf{S}(\omega)\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1 + \frac{1}{V_p}\mathbf{H}_R\left[\frac{\mathbf{v}_R^T\mathbf{S}(\omega)\mathbf{v}_c}{\|\mathbf{v}_R + \mathbf{v}_c\|^2}\mathbf{d}_d + \frac{1}{\|\mathbf{v}_R + \mathbf{v}_c\|}\mathbf{S}(\omega)\mathbf{v}_R\right] \tag{12}$$

Using (2), (7), and (12), and after some more algebraic manipulations, the time derivative $\dot{V}_1$ becomes

$$\dot{V}_1 = z_2[1,\ 0]\mathbf{M}^{-1}\left(\mathbf{g}u_{\mathbf{v}} - [\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r + \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)]\right) + \omega\frac{\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T\left[\frac{\mathbf{v}_R^T\mathbf{S}(1)\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{d}_d + \mathbf{S}(1)\mathbf{v}_R\right]}{\|\mathbf{v}_R + \mathbf{v}_c\|}$$

Now, setting

$$u_{\mathbf{v}} = \frac{[1,\ 0]\mathbf{M}^{-1}[\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r + \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)] - k_2 z_2}{[1,\ 0]\mathbf{M}^{-1}\mathbf{g}} \tag{13}$$

where k_2 is a positive control gain, and ω equal to ω_d,

$$\omega_d := -k_1\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T\left[\frac{\mathbf{v}_R^T\mathbf{S}(1)\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{d}_d + \mathbf{S}(1)\mathbf{v}_R\right]$$

where k_1 is another positive control gain, $\dot{V}_1$ becomes negative semi-definite. Since ω is not a real control variable, and using the same technique as in Section 4, consider a third error variable defined as

$$z_3 = \omega - \omega_d$$

and the augmented Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_3^2 = \frac{1}{2}\mathbf{z}_1^T\mathbf{H}_L\mathbf{z}_1 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$$

The time derivative of V_2 can now be written as

$$\dot{V}_2 = -k_1\frac{\left(\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T\left[\frac{\mathbf{v}_R^T\mathbf{S}(1)\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{d}_d + \mathbf{S}(1)\mathbf{v}_R\right]\right)^2}{\|\mathbf{v}_R + \mathbf{v}_c\|} - k_2 z_2^2 + z_3\left(\frac{1}{J}[-d_\omega(\omega)\omega + u_2] - \dot{\omega}_d\right) + z_3\frac{\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T\left[\frac{\mathbf{v}_R^T\mathbf{S}(1)\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{d}_d + \mathbf{S}(1)\mathbf{v}_R\right]}{\|\mathbf{v}_R + \mathbf{v}_c\|}$$

For the sake of simplicity, the derivative $\dot{\omega}_d$ is not presented here. Now, setting

$$u_\omega = -J\frac{\left(\mathbf{H}_Q^{-1}\mathbf{H}_R^T\mathbf{z}_1\right)^T}{\|\mathbf{v}_R + \mathbf{v}_c\|}\left[\frac{\mathbf{v}_R^T\mathbf{S}(1)\mathbf{v}_c}{\|\mathbf{v}_R+\mathbf{v}_c\|}\mathbf{d}_d + \mathbf{S}(1)\mathbf{v}_R\right] + d_\omega(\omega)\omega + J(\dot{\omega}_d - k_3 z_3) \tag{14}$$

where $k_3 > 0$ is a control gain, $\dot{V}_2$ is made negative semi-definite.

The following theorem is the main result of this section.

Theorem 2. Consider a vehicle with kinematics and dynamics given by equations (1) and (11), respectively, moving in the presence of constant ocean currents. Then, with the control law (13) and (14), the origin $\mathbf{z} = [\mathbf{z}_1^T, z_2, z_3]^T = \mathbf{0}$ is an almost global asymptotic equilibrium point. Moreover, the sway velocity converges to zero, thus solving the homing problem stated in Section 2 in the presence of constant known ocean currents.

PROOF. The proof of the theorem follows the same steps of the proof of Theorem 1. The Lyapunov function V_2 is continuous, radially unbounded, positive definite for feasible values of $\mathbf{z}_1$, and its time derivative, with the control law (13) and (14), is made negative semi-definite. The only point not coincident with the origin, where $\dot{V}_2 = 0$, that corresponds to the situation where the vehicle is moving in the opposite direction of the desired one, is an unstable equilibrium point, as in the proof of Theorem 1. Therefore, the origin $\mathbf{z} = \mathbf{0}$ is an almost globally asymptotically stable equilibrium point. Following the same steps as in Theorem 1, the convergence of the sway velocity is guaranteed, thus completing this proof.

6. SIMULATION RESULTS

To illustrate the performance of the proposed integrated guidance and control laws a computer simulation is presented in this section. In the simulation a simplified model of the horizontal plane of the SIRENE vehicle was used, assuming the vehicle is directly actuated in force and torque (Silvestre *et al.*, 1998).

Assume the vehicle has to counteract an ocean current with velocity $[-0.5,\ -0.5]^T$m/s, expressed in the inertial frame. The vehicle starts at position $[0,\ 500]^T$ m and the acoustic pinger is located at position $[500,\ 500]^T$ m. The control parameters were chosen as follows: $k_1 = 0.025$, $k_2 = 0.04$ and $k_3 = 20$. The desired velocity was set to $V_d = 2$ m/s, and a semi-spherical symmetric USBL sensor with seven receivers was placed on the vehicle's nose. Figure 2 shows the trajectory described by the vehicle, whereas Figures 3 and 4 display the evolution of the vehicle's velocities and control inputs, respectively. From the figures

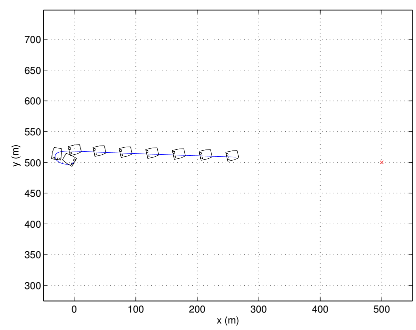

Fig. 2. Trajectory described by the vehicle in the presence of currents

it can be concluded that the vehicle is driven towards the target describing a smooth trajectory. The control inputs are smooth and the angular and sway velocities converge to zero, as expected.

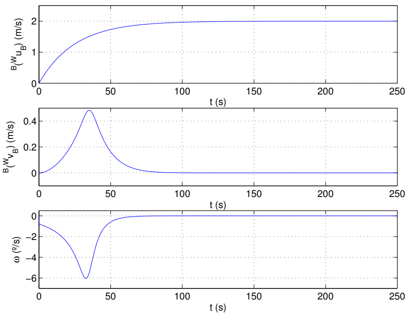

Fig. 3. Time evolution of body-fixed velocities of the vehicle in the presence of currents

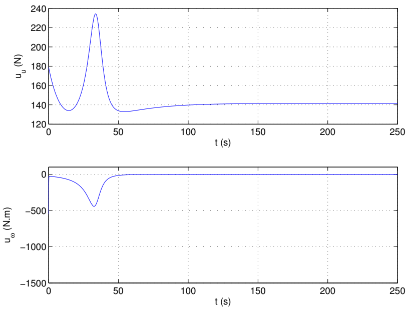

Fig. 4. Time evolution of control inputs in the presence of currents

7. CONCLUSIONS

The paper presented a new homing sensor based integrated guidance and control law to drive an underactuated AUV to a fixed target in 2D using the information provided by an USBL positioning system. The guidance and control laws were firstly derived for the vehicle's kinematics expressed as TDOAs measured by the USBL sensor and then extended to the dynamics of an AUV resorting to backstepping techniques. Almost global asymptotic stability was achieved for the guidance and control law in the presence (and absence) of known ocean currents. Simulations were presented and discussed to illustrate the performance of the proposed solutions.

REFERENCES

Aguiar, A. P. and A. M. Pascoal (2002). Dynamic positioning and way-point tracking of underactuated AUVs in the presence of ocean currents. *Proc. 41st IEEE Conference on Decision and Control* **2**, 2105 – 2110.

Aguiar, A. P. and J. P. Hespanha (2003). Position Tracking of Underactuated Vehicles. *Proc. of the 2003 American Control Conference* **3**, 1988 – 1993.

Aguiar, A. P., L. Cremean and J. P. Hespanha (2003). Position Tracking for a Nonlinear Underactuated Hovercraft: Controller Design and Experimental Results. *Proc. 42nd IEEE Conference on Decision and Control, 2003* **4**, 3858 – 3863.

Cowan, N. J., J.D. Weingarten and D.E. Koditschek (2002). Visual servoing via navigation functions. *IEEE Transactions on Robotics and Automation* **18**(4), 521–533.

Indiveri, G., M. Aicardi and G. Casalino (2000). Nonlinear Time-Invariant Feedback Control of an Underactuated Marine Vehicle Along a Straight Course. *Proc. 5th IFAC Conference on Manoeuvring and Control of Marine Craft,* pp. 221–226.

Isidori, A. (1995). *Applied Nonlinear Control.* 3rd ed.. Springer-Verlag.

Malis, E. and F. Chaumette (2002). Theoretical improvements in the stability analysis of a new class of model-free visual servoing methods. *IEEE Transactions on Robotics and Automation* **18**(2), 176–186.

Mazenc, F., K. Y. Pettersen and H. Nijmeijer (2002). Global Uniform Asymptotic Stabilization of an Underactuated Surface Vessel. *Proc. 41st IEEE Conference on Decision and Control* **1**, 510–515.

Nijmeijer, H. and A. J. van der Schaft (1990). *Nonlinear Dynamical Control Systems.* Springer-Verlag.

Pettersen, K. Y. and H. Nijmeijer (1998). Global Practical Stabilization and Tracking for an Underactuated Ship - a Combined Averaging and Backstepping Approach. *Proc. IFAC Conf. Systems Structure Control* pp. 59–64.

Reyhanoglu, M. (1996). Control and Stabilization of an underactuated surface vessel. *Proc. 35th IEEE Conf. Decision and Control* **3**, 2371–2376.

Sarradin, P. M., K.O. Leroy, H. Ondreas, M. Sibuet, M. Klages, Y. Fouquet, B. Savoye, J. F. Drogou and J. L. Michel (2002). Evaluation of the first year of scientific use of the French ROV Victor 6000. *Proc. of the 2002 International Symposium on Underwater Technology* pp. 11–16.

Sastry, S. (1999). *Nonlinear Systems: Analysis, Stability and Control.* Springer-Verlag.

Silvestre, C., A. Aguiar, P. Oliveira and A. Pascoal (1998). Control of the SIRENE Underwater Shuttle: System Design and Tests at Sea. *Proc. 17th International Conference on Offshore Mechanics and Artic Engineering (OMAE'98- Conference), Lisboa, Portugal.*

Silvestre, C. and A. Pascoal (2004). Control of the INFANTE AUV using gain scheduled static output feedback. *Control Engineering Practice* **12**, 1501–1509.

Slotine, J.J. E and W Li (1991). *Applied Nonlinear Control.* Prentice-Hall, Inc.

Wichlund, K.Y., O. J. Sordalen and O. Egeland (1995). Control of vehicles with second-order nonholonomic constraints: Underactuated vehicles. *Proc. 3rd Eur. Control Conf.* pp. 3086–3091.