NONLINEAR CONTROL OF AN UNDERWATER TOWED VEHICLE

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Abstract: This paper addresses the problem of pitch and depth control of an underwater towed vehicle. A nonlinear adaptive Lyapunov-based controller is designed and proven to regulate the pitch and depth tracking errors to zero. When in the presence of external disturbances and parameter uncertainties, the errors are shown to converge to a neighbourhood of the origin that can be made arbitrarily small. We show through computer simulations that the controlled system exhibits good performance about different operating conditions when subjected to sea-wave driven disturbances and in the presence of sensor noise.

Keywords: Nonlinear control, adaptive control, Lyapunov methods, trajectory tracking, underwater towed vehicle.

1. INTRODUCTION

Underwater towed vehicles (*towfish*) are commonly employed as sensor platforms for oceanographic data acquisition. Their widespread application in marine geophysics includes Side-Scan-Sonar systems, marine magnetometers, and gravimeters. See for example (Zumberge *et al.*, 1997; Parker, 1997). In physical oceanography towed vehicles are used, for example, to deploy current profilers for sampling of small-scale ocean turbulence (Gargett, 1994; Schuch, 2004). In these applications the attitude of the towfish may affect significantly the quality of the data acquired (Preston, 1992; Perrault *et al.*, 1997). To meet the requirements of a great number of oceanographic missions, two related control problems may be posed. The first one consists of tracking a depth profile where the desired depth depends on the horizontal position of the vehicle or is simply a function of time but does not depend on the bottom profile. The second problem arises in the context of bottom-following missions and consists of maintaining a desired altitude above the sea-bottom; the latter is a common requirement, in particular for marine geophysical data acquisition, see e.g. (Tivey and Schouten, 2003).

This paper addresses the simultaneous problem of pitch and depth control of a towed underwater vehicle. A towing arrangement is considered where the nose of the towfish is connected via a small umbilical (the pigtail section) to a depressor, which is in turn connected to and towed by a support ship using a long tow line (the main catenary), see Figure 1. When compared to a single-part towing, this arrangement is better suited to bottom-following surveys and has proven to yield good stability and significant rejection of external disturbances due to ship motion.

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1.1 Prior and related work

The dynamics of towing cables and towed vehicles have been studied by (Abkowitz, 1969). The effects of wave-driven motions of the towing vessel on a sonar platform are discussed in (Preston, 1992), which contains the results of sea-trials with different tow configurations employed to reduce external disturbances. In particular, the towfish pitch and heave obtained with a direct tow configuration is compared with the pitch obtained with a two-part tow with different pigtail lengths. The results show that a two stage towing arrangement can significantly attenuate towfish attitude disturbances due to the surface ship motion. Simultaneous 2-D modeling of the towing cable and towed vehicle is a problem addressed in (Perrault *et al.*, 1997), where the cable is modeled employing a lumped mass model and a PD closed-loop controller is developed to accomplish bottom following and pitch stabilization. Good altitude control is obtained at the expense of pitch control and vice-versa. A 3-D model of a two-part towing arrangement is studied in (Wu and Chwang, 2000) using a finite difference method. The results of the study show that a two-part towed system is an effective way of decoupling the towship motion from the towed vessel and emphasize the importance of the secondary cable length for the stability of the towfish. The concept of active stabilization of a towfish (using an internal servo-actuated mass which can trim the vehicle's centre of mass) is applied in (Woolsey and Gargett, 2002), resorting to LQR control techniques. A related problem of simultaneous control of yaw and depth of an autonomous underwater vehicle was posed and solved in (Aguiar, 1996) by resorting to sliding-mode control theory. A list of additional references reporting background work on the above and related problems may be found in (Schuch, 2004).

The problem of following a desired depth profile while the vehicle is being towed falls in the scope of trajectory-tracking control. The related problem of maintaining a constant altitude relative to the sea-bottom requires additional sensors to measure the altitude of the vehicle. Alternatively, the altitude of the towfish may be estimated online based on the position of the towing vessel, using a more complex cable model and an existent bathymetric map. The problem of accurate estimation of the position of a towed underwater body in the vertical plane during short periods of time has been addressed in (Damy *et al.*, 1994). The approach adopted in that study uses a numerical model of the towing cable, in order to transform GPS measurements of the towing vessel position into measurements of the towfish position and integrating this information with inertial sensor data acquired at the towfish. The position accuracy expected with the approach described is 10 cm.

Because depth control tends to affect the pitch angle of the vehicle being towed, which is also affected by disturbances transmitted via the towing cable, accurate pitch control becomes one of the key requirements of a good towing system. This motivated the problem addressed in the paper: develop a closed-loop control system to drive pitch to zero while tracking a desired depth reference. We assume that the towfish is stable in roll and controlled independently in yaw. We therefore restrict ourselves to controlling the vehicle in the vertical plane.

1.2 Proposed approach and main contribution of this work

The strategy adopted in the present paper for control system design borrows from nonlinear control theory. This choice was largely dictated by the requirement that the controller should yield good performance when the vehicle undergoes motions about different equilibrium conditions and exhibit robustness against vehicle parameter uncertainty. The equilibrium conditions are determined by, among other factors, the pigtail length and the towing speed. The towfish dynamic model that we adopted builds on the work reported in (Schuch, 2004). However, the problem that we tackle consists of controlling attitude and depth, and not just attitude.

The key contribution of the present work is the development of an adaptive controller that exhibits good performance about different equilibrium (operating) conditions and is robust against vehicle parameter uncertainty. The nonlinear control law derived is proven to stabilize the system even in the presence of bounded external disturbances and unmodeled dynamics.

The organization of the paper is as follows: Section 2 describes the dynamical model of the vehicle and formulates the corresponding problem of depth tracking and pitch regulation in the presence of parametric model uncertainty. In Section 3, a solution to this problem is proposed in terms of a nonlinear adaptive control law. Section 4 evaluates the performance of the control algorithms developed using computer simulations. Finally, Section 5 contains some concluding remarks and discusses problems that warrant further research.

2. VEHICLE MODEL AND CONTROL PROBLEM FORMULATION

2.1 Towfish characteristics and main assumptions

The towing system consists of the two-part towing arrangement described before and depicted in Figure 1. The vehicle is equipped with two control surfaces - bow and stern planes - equidistant from the centre of mass. The sensor suite

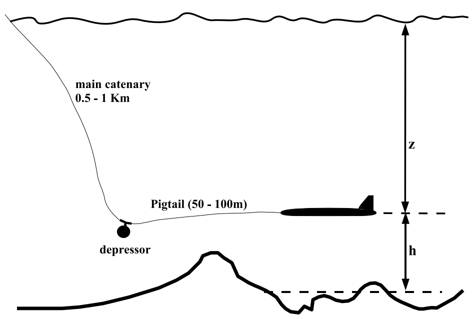

Fig. 1. Towing arrangement. h-altitude above the seabed; z-depth.

includes an Attitude and Heading Unit (AHU), a depth sensor, an (optional) sonar altimeter, and a Doppler Velocity Logger (DVL) that measures the velocity of the vehicle relative to the water. The towfish has a slightly positive buoyancy and the metacentric height is such that the vehicle is naturally stabilized in roll.

2.2 System modelling

In the present paper we adopt the approach proposed in (Schuch, 2004), leading to a simplified model where the perturbations induced by sea-waves are transmitted to the tow-fish by the depressor-pigtail subsystem, where the latter is modelled as a spring-damper system. The length of the pigtail can be used as a design parameter to filter the effect of the disturbances transmitted to the depressor by the towing vessel. To model the wave induced perturbations we apply the JONSWAP wave spectrum, see e.g. (Fossen, 2002).

Reference trajectory. The reference trajectory is defined to be a sufficiently smooth signal $z_d(t)$, which may be a function of the horizontal displacement $x(t)$. In computer simulations we approximate $z_d(x(t))$ by cubic-splines, thus ensuring that the reference trajectory is twice differentiable in time.

Nomenclature. We follow the standard convention of North-East-Down coordinate systems (Fossen, 2002). In the sequel, $\{I\}$ represents an inertial coordinate frame and $\{B\}$ denotes the body-fixed frame that moves with the vehicle. In a reference frame restricted to the vertical plane xz, vector $[x, z, \theta]'$ represents the pose of the vehicle expressed in $\{I\}$, with θ denoting the pitch angle. The vector $\nu = [u, w, q]'$ represents the surge, heave, and pitch rate components of the velocity of the vehicle expressed in $\{B\}$.

Kinematics. In our simplified kinematic model the pose and the velocity of the vehicle relative to $\{I\}$ are represented by vectors $\mathcal{X}_1 := [z, \theta]'$ and $\mathcal{X}_2 := \dot{\mathcal{X}}_1 = [\dot{z}, q]'$, respectively with

$$\begin{aligned} \dot{z} &= -u \ \sin(\theta) + \ \omega \ \cos(\theta), \\ \dot{\theta} &= q. \end{aligned} \tag{1}$$

Table 1. System parameters

Type of Parameter	Symbol	Value	Units
Added mass	$M_{\dot{u}}$	0	Kg
	$M_{\dot{w}}$	-78.14	Kg
	$Z_{\dot{u}}$	0	Kg
	$Z_{\dot{w}}$	-18.64	Kg
Added Inertia	$M_{\dot{q}}$	-260.55	$Kg\ m^2$
	$Z_{\dot{q}}$	170.90	$Kg\ m^2$
Mom. of Inertia	I_{yy}	174.49	$Kg\ m^2$
Hydrodyn.Damping	$M_{\|q\|q}$	0.1	$Kg\ m^2\ s^{-2}$
	$Z_{\|q\|q}$	-0.1	$Kg\ m\ s^{-2}$
Mass of the towfish	m	113.5	Kg
Towfish dry weight	W	1112	N
Length of the body	l_b	2.59	m
Buoyancy coef.	b_W	0.05	–
Body parameters	K_{fb}	0.351	–
	$K_{m\alpha}$	1.31	–
Elevators' param.	K_{fs}	2.62	–
	K_{md}	0	–

Dynamics. Neglecting the influence of the motions in sway and roll, the simplified body-fixed dynamic equations of interest are given by

$$\begin{aligned} &(m - Z_{\dot{w}})\dot{w} - Z_{\dot{u}}\dot{u} - Z_{\dot{q}}\dot{q} + (m - Z_{\dot{w}})uq \\ &+Z_{|q|q}|q|q + f_w(V, \theta) + g_w(\theta) = \\ &u_{\delta_c}(\nu, \delta, \theta) + \tau_w^e \end{aligned} \tag{2}$$

and

$$\begin{aligned} &(I_{yy} - M_{\dot{q}})\dot{q} - M_{\dot{u}}\dot{u} - M_{\dot{w}}\dot{w} \\ &+(Z_{\dot{w}} - X_{\dot{u}})uw + M_{|q|q}|q|q + f_\theta(V, \theta) \\ &+g_\theta(\theta) = u_{\delta_d}(\nu, \delta, \theta) + \tau_\theta^e, \end{aligned} \tag{3}$$

where $V = [u, w]'$, $f_w(.)$, and $f_\theta(.)$ represent hydrodynamic forces and moments associated with heave and pitch, respectively, $g_w(.)$ and $g_\theta(.)$ capture the restoring forces and moments, and τ_w^e and τ_θ^e denote unknown external forces and moments, including the towing force and the external disturbances. The variables u_{δ_c} and u_{δ_d} represent the forces and moments due to the common mode and the differential mode control actions, respectively. The constants $M_{\dot{q}}$, $M_{\dot{u}}$, $M_{\dot{w}}$, $Z_{\dot{q}}$, $Z_{\dot{u}}$, and $Z_{\dot{w}}$ represent added mass coefficients according to the notation of (SNAME, 1950). The symbols $M_{|q|q}$, and $Z_{|q|q}$ denote nonlinear damping coefficients of hydrodynamic origin. The parameter m is the mass of the vehicle including fluid inside the hull, and I_{yy} is the y-axis moment of inertia. The values used for these parameters are presented in Table 1.

2.3 Problem Formulation

Let $z_d : [0, \infty) \rightarrow \mathbb{R}$ be a sufficiently smooth time-varying reference trajectory with a uniformly bounded time-derivative and let the desired pitch angle be $\theta_d = 0$. The problem that we address can be formally described as follows:

Consider the vehicle model represented by equations (1),(2), and (3). *Design a state-feedback control law such that all closed-loop signals are bounded and the tracking error norm* $\|[z - z_d,\ \theta]\|$ *converges exponentially fast to a neighbourhood of the origin that can be made arbitrarily small in the presence of parameter model uncertainty.*

3. CONTROLLER DESIGN

We propose a Lyapunov-based adaptive control law to solve the trajectory tracking problem formulated in section 2.

3.1 Nonlinear controller design

Let $\mathcal{X}_{1d}(t) := [z_d(t), 0]'$ represent the desired trajectory and define $e := \mathcal{X}_1 - \mathcal{X}_{1d}$.

Step1. Convergence of e. Consider the Lyapunov control function $V_1 := \frac{1}{2}e'e$ whose time-derivative is

$$\dot{V}_1 = (\mathcal{X}_1 - \mathcal{X}_{1d})'(\mathcal{X}_2 - \dot{\mathcal{X}}_{1d}). \qquad (4)$$

To make $\dot{V}_1$ negative definite (n.d.) we can regard $\mathcal{X}_2$ as a virtual control and set $\mathcal{X}_2 = \dot{\mathcal{X}}_{1d} - K_a e$, for some positive definite (p.d.) diagonal matrix K_a. Introducing the error variable $\bar{z} = \mathcal{X}_2 - (\dot{\mathcal{X}}_{1d} - K_a e)$, where $\mathcal{X}_{1d}$ is required to be bounded and twice time-differentiable, we rewrite (4) as

$$\dot{V}_1 = -e'K_a e + e'\bar{z}. \qquad (5)$$

Step2. Backstepping for z. Define the following model parameters calculated from the vehicle's parameters (see Table 1 for the definitions of the vehicle's parameters used in the forthcoming expressions):

$$a_1 := -\frac{1}{m - Z_{\dot{w}}}; \quad a_2 := -\frac{1}{(I_{yy} - M_{\dot{q}})}$$

$$\Gamma_1 := \frac{1}{(m - Z_{\dot{w}})}[K_{fb}, K_{fs}, W_z, (m - X_{\dot{u}})]'$$

$$\Gamma_2 := \frac{1}{(I_{yy} - M_{\dot{q}})}[M_{|q|q}, X_{\dot{u}}, K_{md}, K_{m\alpha}, W_\theta]'$$

and take $\Gamma := diag(\Gamma_1', \Gamma_2')$, $A := diag(a_1, a_2) > 0$, with $W_z := b_W W$ and $W_\theta := (1 + b_W)W \sin\theta$. Consider also the following functions of the measured variables:

$$g_1 := -\dot{u}\sin\theta - uq\cos\theta - wq\sin\theta; \quad g_2 := 0$$

$$\beta_1 := \cos\theta[-f_v(V)\theta, -2f_v(V)\theta, \cos\theta, uq]'$$

$$\beta_2 := [-|q|q, uw, f_v(V), f_v(V)\theta, \sin\theta]'$$

with $f_v(V) = \frac{1}{2}\rho|V|^2$. Define also $\beta := [\beta_1', \beta_2']'$, $f := \Gamma\beta$, and $g := [g_1, g_2]'$. Using the above definitions,simple manipulations of equations (1),(2) and (3) yield

$$\dot{\mathcal{X}}_2 = f + A\bar{u} + g + d$$

where $d := P(\theta)A\tau^e$, with $\tau^e := [\tau_w^e, \tau_\theta^e]'$, $P(\theta) = diag(\cos\theta, 1)$, and $\bar{u} := [u_{\delta_c}, u_{\delta_d}]'$ is the control action to be determined. For simplicity of presentation, at this stage we consider that $d = 0$. This is appropriate to do when $\theta \approx 0$, the towing force has no vertical component, and there exist no external disturbances. These assumptions will be lifted later. Neglecting the term $-\dot{u}\sin\theta$ in g_1, the dynamic equation of the error $\bar{z}$ can be written as

$$\dot{\bar{z}} := f + A\bar{u} + g + K_a\dot{e} - \ddot{\mathcal{X}}_{1d} \qquad (6)$$

where $\ddot{\mathcal{X}}_{1d}$ is the second time-derivative of the reference trajectory.

We introduce the augmented Lyapunov function

$$V_2 := V_1 + \frac{1}{2}\bar{z}'\bar{z} = \frac{1}{2}e'e + \frac{1}{2}\bar{z}'\bar{z} \qquad (7)$$

and write its time-derivative, applying (6), as

$$\dot{V}_2 = -e'K_a e + \bar{z}'[e + f + A\bar{u} + g + K_a\dot{e} - \ddot{\mathcal{X}}_{1d}]. \quad (8)$$

At this stage, one could use the control $\bar{u}$ to drive $\bar{z}$ to zero if we knew exactly the remaining terms inside the brackets in (8). This is not practical to do, because some of the parameters of the vehicle are not known with good accuracy . Hence, we define the variables $\hat{a}_1$, $\hat{a}_2$, $\hat{\Gamma}_1$, and $\hat{\Gamma}_2$ to represent estimates of a_1, a_2, Γ_1, and Γ_2, respectively, and set the control using the estimated model parameters as follows:

$$\bar{u} = -\hat{A}^{-1}[e + \hat{f} + g + K_a\dot{e} - \ddot{\mathcal{X}}_{1d} + K_b\bar{z}], \quad (9)$$

where K_b is a p.d. diagonal matrix, $\hat{A} := diag(\hat{a}_1, \hat{a}_2)$, and $\hat{f} := \hat{\Gamma}\beta = diag(\hat{\Gamma}_1', \hat{\Gamma}_2')\beta$. We require that $\ddot{\mathcal{X}}_{1d}$ be bounded in order to guarantee boundedness of the control signal.

Define the estimation errors $\tilde{A} := A - \hat{A}$, $\tilde{\Gamma} := \Gamma - \hat{\Gamma}$, and $\tilde{\Pi} := \Pi - \hat{\Pi}$ with $\Pi := [\Gamma, A]'$, $\hat{\Pi} := [\hat{\Gamma}, \hat{A}]'$, and take $\Phi := [\beta', -(A^{-1}(\hat{\Gamma}\beta + e + g + K_a\dot{e} - \ddot{\mathcal{X}}_{1d} + K_b\bar{z}))']'$. Using the control law (9), straightforward algebraic manipulations yield the time-derivative of V_2 as

$$\begin{aligned}\dot{V}_2 &= -e'K_a e - \bar{z}'K_b\bar{z} \\ &+\bar{z}'[\tilde{\Gamma}\beta - \hat{A}^{-1}\tilde{A}(\hat{\Gamma}\beta + e + g + K_a\dot{e} - \ddot{\mathcal{X}}_{1d} + K_b\bar{z})] \\ &= -e'K_a e - \bar{z}'K_b\bar{z} + \bar{z}'\tilde{\Pi}'\Phi\end{aligned} \qquad (10)$$

Step3. Adaptive control. We introduce a third Lyapunov function

$$V_3 := V_2 + \frac{1}{2r}\left\|\tilde{\Pi}\right\|_F^2 = \frac{1}{2}e'e + \frac{1}{2}\bar{z}'\bar{z} + \frac{1}{2r}\left\|\tilde{\Pi}\right\|_F^2 \qquad (11)$$

for some scalar $r > 0$, where $\|.\|_F$ stands for the Frobenius norm.

The time-derivative of V_3 is

$$\dot{V}_3 = -e'K_a e - \bar{z}'K_b\bar{z} + \mathbf{tr}(\tilde{\Pi}'(\Phi\bar{z}' - \frac{1}{r}\dot{\hat{\Pi}})). \quad (12)$$

Let Π_0 represent an initial estimate of Π. Setting

$$\dot{\hat{\Pi}} = r[\Phi\bar{z}' - s(\hat{\Pi} - \Pi_0)] \qquad (13)$$

for some scalar $s > 0$ yields

$$\dot{V}_3 = -e'K_a e - \bar{z}'K_b\bar{z} + \mathbf{tr}(s\tilde{\Pi}'(\hat{\Pi} - \Pi_0)).$$

Making $\tilde{\Pi}_0 := \Pi - \Pi_0$ and applying the equality $\mathbf{tr}(s\tilde{\Pi}'(\hat{\Pi} - \Pi_0)) = -\frac{1}{2}s\left\|\tilde{\Pi}\right\|_F^2 - \frac{1}{2}s\left\|\hat{\Pi} - \Pi_0\right\|_F^2 + \frac{1}{2}s\left\|\hat{\Pi} - \Pi_0\right\|_F^2$ it follows that

$$\dot{V}_3 \leq -e'K_a e - \bar{z}'K_b\bar{z} - \frac{1}{2}s\left\|\tilde{\Pi}\right\|_F^2 + \frac{1}{2}s\left\|\tilde{\Pi}_0\right\|_F^2 \qquad (14)$$

Although we cannot ensure that $\dot{V}_3$ is always negative, it is possible to show that this is sufficient to ensure practical stability. Note also that the dynamics of $\hat{\Pi}$ established in (13) ensure that the values of the parameters estimated by the adaptive controller do not diverge from the corresponding initial values.

External forces. Let $f_{td} := [f_{tdx}, f_{tdz}]'$ be the vector representing the sum of the towing force and the external disturbances, expressed in $\{I\}$. The corresponding terms in the dynamics equations (2) and (3) are represented by $\tau^e := [\tau^e_w, \tau^e_\theta]' = Q(\theta) f_{td}$, with

$$Q(\theta) := \begin{bmatrix} \sin\theta & \cos\theta \\ \dfrac{-l_b}{2}\sin\theta & \dfrac{-l_b}{2}\cos\theta \end{bmatrix}$$

where l_b is the length of the towfish (see Table 1). These unknown terms cannot be taken into account in the control law (9) yielding, instead of (10),

$$\dot{V}_2 = -e'K_a e - \bar{z}'K_b\bar{z} + \bar{z}'\tilde{\Pi}'\Phi + \bar{z}'d \tag{15}$$

and

$$\dot{V}_3 \le -e'K_a e - \bar{z}'K_b\bar{z} - \frac{1}{2}s\left\|\tilde{\Pi}\right\|_F^2 + \frac{1}{2}s\left\|\tilde{\Pi}_0\right\|_F^2 + \bar{z}'d. \tag{16}$$

3.2 Stability analysis

We now apply a reasoning similar to the one in the proof of Theorem 1 in (Aguiar and Hespanha, 2003) to prove the following theorem:

Theorem I Consider the closed-loop system Σ consisting of the vehicle model (1),(2),(3) and the adaptive feedback controller (6), (9), and (13). Given a bounded, sufficiently smooth time-varying reference trajectory $\mathcal{X}_d : [0,\infty) \to \mathbb{R} \times \{0\}$, the following holds:

i) For any initial condition, the solution to Σ exists globally, all the closed-loop signals are bounded, and the tracking error e satisfies

$$\|e\| \le e^{-\lambda t}c_0 + \epsilon \tag{17}$$

where λ, c_0, and ϵ are positive constants, and c_0 depends on the initial conditions.

ii) By appropriate choice of the controller parameter K_b, the rate of convergence λ and the radius ϵ can be chosen at will.

Proof Starting with (16) we apply Young's inequality to show that for any scalar constant $\gamma > 0$

$$\dot{V}_3 \le -e'K_a e - \bar{z}'(K_b - \frac{\gamma}{2}I)\bar{z} - \frac{1}{2}s\left\|\tilde{\Pi}\right\|_F^2 + \frac{1}{2}\Delta^2, \tag{18}$$

where $\Delta = \frac{\|d\|}{\sqrt{\gamma}} + \sqrt{s}\left\|\tilde{\Pi}_0\right\|_F$. From (11) and (18), and assuming that K_b satisfies $K_b > \frac{\gamma}{2}I$ we conclude that there exists a sufficiently small constant $\lambda > 0$ such that the following inequality holds:

$$\dot{V}_3 \le -\lambda V_3 + \frac{1}{2}\Delta^2. \tag{19}$$

We prove (i) by applying the Comparison Lemma (Khalil, 2002) and showing that along the solutions of Σ

$$V_3(t) \le \mathbf{e}^{-\lambda t}V_3(0) + \frac{1}{2\lambda}\Delta^2. \tag{20}$$

This shows that all the control signals remain bounded and the solutions of the system exist globally and are ultimately bounded with ultimate bound $\frac{1}{2\lambda}\Delta^2$. Considering the definition of V_3, $\|e\|$ converges to a ball of radius $\frac{\Delta}{\sqrt{\lambda}}$. From (20) we can also conclude that the close-loop system is input-to-state stable (ISS) with respect to bounded parametric uncertainties and bounded external disturbances.

To prove (ii) we show that it is possible to make the radius $\frac{\Delta}{\sqrt{\lambda}}$ arbitrarily small by an appropriate choice of the controller parameters. For a given limiting radius ϵ and a given convergence rate λ it can be shown that

$$\frac{\Delta}{\sqrt{\lambda}} \le \epsilon = \frac{d_b}{\sqrt{\lambda\gamma}} + \sqrt{\frac{s}{\lambda}}\left\|\tilde{\Pi}_0\right\|_F$$

with $d_b := \sup_{t\ge 0}\|d\|$. Thus, we can select

$$\gamma := d_b^2/\lambda\left(\epsilon - \sqrt{\frac{s}{\lambda}}\left\|\tilde{\Pi}_0\right\|_F\right)^2$$

provided that we make

$$K_b - \frac{\gamma}{2}I = K_b - \frac{d_b^2}{2\lambda\left(\epsilon - \sqrt{\frac{s}{\lambda}}\left\|\tilde{\Pi}_0\right\|_F\right)^2}I \ge \frac{1}{2}I > 0.$$

Thus it is shown that (19) is verified and therefore (20) holds.

4. SIMULATION RESULTS

In this section we illustrate the performance of the controll algorithms developed, using computer simulations. We simulated the system at the approximate towing speeds of 4 $knts$, 6 $knts$, and 8 $knts$ using pigtail lengths of 50 m and 100 m. The results presented below correspond to the towing speed of $6knts$ which do not differ considerably from those obtained at the other speeds. The pigtail length is 100 m. The control planes have a maximum deflection of ± 30 deg and the actuators' dynamics are approximated by a first order system with a time constant equal to 0.1 s.

In the simulations, the vehicle is made to track the desired depth profile represented in Figure 2(a) while regulating the pitch angle to zero. The initial conditions for the towfish are $(z, \theta, \dot{z}, q) = (24.7, 0, 0, 0)$. The simulation starts with velocities $u = 3m/s$, $w = 0$, and $q = 0$. The controller parameters have been set as follows: $K_a = 0.05I_2$, $K_b = 1.1I_2$, $r = 10^{-6}$, $s = 10^{-6}$. The initial estimates of the model parameters represented by Π_0 are set to $\pm 10\%$ of the corresponding true values.

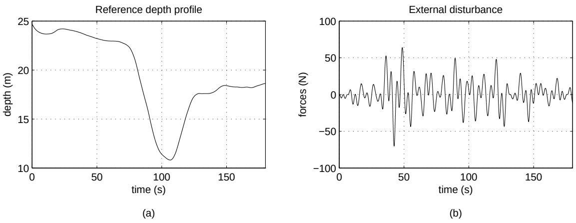

Fig. 2. (a)Depth reference; (b) Force due to external disturbances

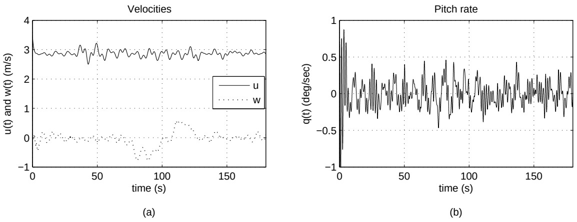

Fig. 3. Towfish velocities: (a)surge and heave; (b) pitch rate

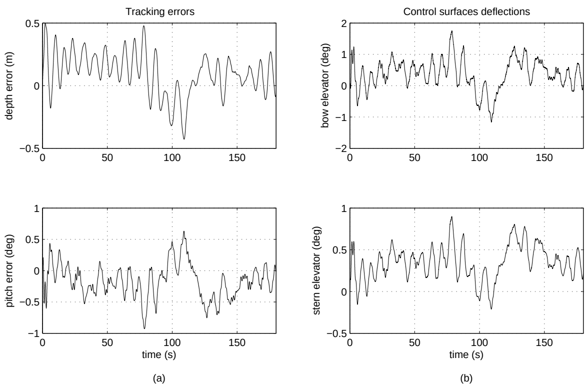

Fig. 4. (a) Tracking errors; (b) Deflection of control surfaces

Measurement noise. To test the robustness of the proposed adaptive controller with respect to sensor noise, the measurements of u, w, q, z, and θ used in the simulations are affected by mutually independent additive gaussian white noise whose intensities are the following, respectively: $\sigma_u^2 = (0.05m/s)^2$, $\sigma_w^2 = (0.1m/s)^2$, $\sigma_q^2 = (0.5deg/s)^2$, $\sigma_z^2 = (0.1m)^2$, and $\sigma_\theta^2 = (0.1deg)^2$.

Results. The intensity of the force caused by sea waves is plotted in Figure 2(b). The velocities of the towfish are shown in Figure 3. The tracking errors achieved by the system, and the actuation of the control surfaces are depicted in Figure 4. We observe that in general the depth and pitch errors verify $|e_z| \leq 50\ cm$ and $|e_\theta| \leq 1$ deg, respectively.

5. CONCLUSIONS AND FUTURE WORK

The paper addressed the problem of pitch and depth control of an underwater towed vehicle. A nonlinear adaptive Lyapunov-based controller was designed and tested in simulation. The results obtained show that the nonlinear Lyapunov-based controller proposed is adequate for depth or bottom following missions and precise control of the attitude of a towed vehicle. The controller proved to be robust against vehicle parameter uncertainty and bounded external disturbances. The controlled system exhibits good performance at different equilibrium conditions, in the presence of sensor noise and external disturbances, and meets the requirements of the envisioned applications of a towfish.

Future work on this subject will start by extending the application of these developments to the control in both yaw and pitch. We expect to apply these techniques to the control of an actual towfish used in marine geophysical surveying.

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