# Impact of Imperfect Channel Estimation on the Performance of *M*-QAM Hierarchical Constellations with Diversity

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***Abstract*— In this paper we derive general expressions for the performance of hierarchical multilevel quadrature amplitude modulation constellations (M-QAM), over flat Rayleigh fading environments with imperfect channel estimation. Cases of diversity reception with identical channels and with dissimilar channels, employing maximal ratio combining (MRC) are considered. Since hierarchical constellations are usually employed to achieve unequal bit error protection, the individual bit error rate (BER) of each bit stream is analyzed and it is shown that poor channel estimation has a more severe impact in the performance of the least protected bit streams.**

***Keywords- Channel estimation, diversity reception, quadrature amplitude modulation, Rayleigh fading.***

## I. INTRODUCTION

In the design of wireless communication networks, the limited spectrum available is one of the main restrictions for achieving high bit rate transmissions. The use of M-QAM is considered an attractive technique to achieve this objective due to its high spectral efficiency and has been studied for wireless systems by several authors [1]-[3].

A great deal of attention has been devoted to obtaining analytical expressions for the bit error rate (BER) performance of *M*-QAM. The symbol error rate (SER) of 16-QAM on *L* branch Rayleigh fading channels with Maximal Ratio Combining (MRC) was derived in [4]. [5] presents exact SER expression for *M*-QAM with *L* branch diversity reception in Rayleigh fading for the cases of MRC and selection combining (SC). These studies assume perfect channel state information (CSI). The CSI is required to rescale the received symbols so that they can correspond to symbols of the original constellation. Since in real systems the channel estimation is always imperfect it will have a significant impact in the performance of *M*-QAM constellations. Several authors have addressed the performance of QAM transmissions with imperfect channel estimation. A tight upper bound on the SER of 16-QAM with pilot symbol assisted modulation in Rayleigh fading channels was presented in [6]. The BER of 16-QAM and 64-QAM in flat Rayleigh fading with imperfect channel estimates was derived in [7]. In [8] this derivation was extended for the case of 16-QAM with diversity reception and MRC.

All the previous work referred so far refers to *M*-QAM uniform constellations. These constellations can be looked at as a subset of the more general case of hierarchical *M*-QAM constellations (which includes uniform and non uniformly spaced signal sets). These constellations can be used as a very simple method to provide unequal bit error protection and to improve the efficiency of a network. This idea is based on the work of Cover [9] who showed that in broadcast transmissions it is possible to exchange some of the capacity of the good communication links to the poor ones and this tradeoff can be worthwhile. In hierarchical constellations there are two or more classes of bits with different error protection, to which different streams of information can be mapped according to its importance. Depending on the propagation conditions, a given user can attempt to demodulate only the more protected bits or also the other bits that carry the additional information. Hierarchical 16-QAM and 64-QAM constellations have already been incorporated in the DVB-T standard [10]. In [11] a recursive algorithm for the exact BER computation of generalized *M*-QAM constellations in AWGN and fading channels was presented. Later closed-form expressions were obtained also for these channels [12]. At the moment the analytical BER performance for these constellations in Rayleigh channels with imperfect channel estimation has not been investigated yet.

Although the method used in [7] and [8] can be extended to general *M*-QAM constellations, the manipulations and development required for obtaining the expressions can become quite cumbersome. Thus, in this paper we adopt a different method and obtain general closed-form expressions for the BER performance in Rayleigh fading channels of generalized *M*-QAM constellations with diversity reception, employing MRC at the receiver. Cases of diversity reception with identical channels and also with dissimilar channels are considered. Recently, in [13], exact expressions were published

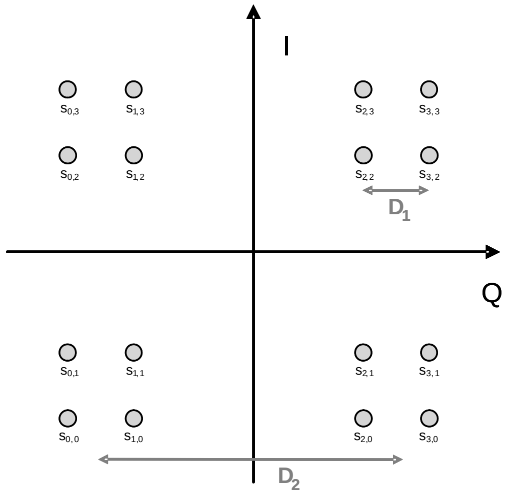


Figure 1. Hierarchical 16-QAM constellation.

for the performance of uniform *M*-QAM constellations but only for identical diversity channels.

The paper is organized as follows. Section II describes the model of the communication system, which includes the definition of hierarchical *M*-QAM constellations, the channel and the modeling of the channel estimation error. Section III derives the BER expressions and Section IV presents and analyzes some simulation and numerical results. The conclusions are given on Section V.

## II. SYSTEM AND CHANNEL MODEL

### A. *Hierarchical QAM Signal Constellations*

Hierarchical signal constellations (also called hierarchical constellations) are constellations where the distances along the I or Q axis between adjacent symbols can be different depending on their position. These constellations are thus able to provide unequal bit error protection. As an example, an hierarchical 16-QAM constellation can be constructed from a main QPSK constellation where each symbol is in fact another QPSK constellation, as shown in Figure 1. The basic idea is that the constellation can be viewed as a 16-QAM constellation if the channel conditions are good enough or as a QPSK constellation otherwise. In the latter situation, the received bit rate is reduced by half. These constellations can be characterized by the parameter $k=D_1/D_2$ ($0<k\leq 0.5$), as shown in Figure 1. If $k=0.5$, the resulting constellation corresponds to a uniform 16-QAM. This approach can be naturally extended to any QAM constellation size. The general expression for the definition of a symbol is

$$s = \sum_{l=1}^{\log_2\left(\sqrt{M}\right)} \left(\pm \frac{D_l}{2}\right) + \sum_{l=1}^{\log_2\left(\sqrt{M}\right)} \left(\pm \frac{D_l}{2}\right) j \, . \qquad (1)$$

The number of possible classes of bits with different error protection that can be obtained is $1/2 \cdot \log_2 M$ . In our analysis we consider that the parallel information streams are split in two, so that half of each stream goes for the in-phase branch and the other for the quadrature branch of the modulator. The resulting bit sequence in each branch is Gray encoded and mapped to the respective $\sqrt{M}$-PAM constellation symbols. The symbols from the in-phase and quadrature branches are then grouped together forming complex *M*-QAM symbols. The Gray coding for each $\sqrt{M}$-PAM constellation is performed according to the procedure described in [11]. First the constellation symbols are represented in an horizontal axis and are labeled from left to right with integers starting from 0 to $\sqrt{M}-1$. These labels are then converted to their binary representation so that each symbol $s_j$ can be represented by a $\log_2 M/2$-digit binary sequence: $b_j^1, b_j^2, \ldots, b_j^{\log_2 M/2}$. The corresponding Gray code is then computed using

$$\begin{aligned} g_j^1 &= b_j^1 \\ g_j^i &= b_j^i \oplus b_j^{i-1}, \qquad \text{i}=2,3,\ldots,\frac{\log_2 M}{2} \end{aligned} \qquad (2)$$

where $\oplus$ represents modulo-2 addition.

### B. *Received signal model*

Let us consider the case of a transmission over an $L$ diversity branch flat Rayleigh fading channel where all the branches can have different fading powers. Assuming perfect carrier and symbol synchronization, each received signal sample can be modelled as

$$r_k = \alpha_k \cdot s + n_k, \quad k = 1 \ldots L \qquad (3)$$

where $\alpha_k$ is the channel coefficient for diversity path $k$, $s$ is the transmitted symbol and $n_k$ represents additive white thermal noise. Both $\alpha_k$ and $n_k$ are modelled as complex gaussian random variables with $E[\alpha_k]=0$, $E\left[|\alpha_k|^2\right]=2\sigma_{\alpha_k}^2$ (average fading power for path $k$), $E[n_k]=0$ and $E\left[|n_k|^2\right]=2\sigma_n^2=N_0$ ($N_0/2$ is the noise power spectral density). Since $\alpha_k$ and $n_k$ are complex gaussian variables, the probability density function of the received signal sample, $r_k$, conditioned on the transmitted symbol, $s$, is also gaussian, and its mean is 0. The channel's corresponding autocorrelation and cross correlation functions can be expressed as [14]

$$\begin{cases} R_{\alpha_k^I}(\tau) = R_{\alpha_k^Q}(\tau) = E\left\{\alpha_k^I(t)\alpha_k^I(t+\tau)\right\} = \sigma_\alpha^2 J_0(2\pi f_D \tau) \\ R_{\alpha_k^I\alpha_k^Q}(\tau) = E\left\{\alpha_k^I(t)\alpha_k^Q(t+\tau)\right\} = 0 \end{cases} \qquad (4)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $f_D$ is the Doppler frequency and $\alpha_k^I$ and $\alpha_k^Q$ are the in-phase and quadrature components of the fading coefficient $\alpha_k$.

The receiver performs Maximal Ratio Combining (MRC) of the received signals. Since the mapping of the bits into the constellation symbols is performed independently to the I and Q branches, the decision variable in the receiver is either the real or the imaginary part of the result of the MRC. So, for the

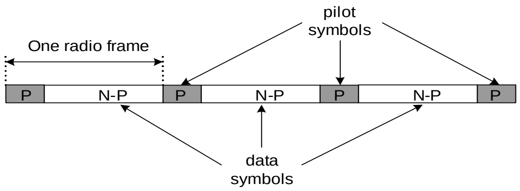


Figure 2. Frame structure for a PSAM system.

detection of the in-phase bits the decision variable can be expressed as

$$z_{re} = \mathrm{Real}\left\{\sum_{k=1}^{L} r_k \cdot \hat{\alpha}_k^*\right\}. \tag{5}$$

### C. Channel Estimation

In this analysis we consider that the communication system employs a pilot symbol assisted modulation (PSAM) philosophy [6], where the transmitted symbols are formatted in frames of length $N$ with $P$ pilot symbols periodically inserted into the data sequence, as shown in Figure 2. The pilot symbols are known by the receiver and are used for channel estimation purposes. The channel estimate for each pilot symbol, $S_p$, is obtained with

$$\hat{\alpha}_k(i) = \frac{S_p^*}{|S_p|^2} r_k(i), \qquad i = 1,...,P \tag{6}$$

To obtain the BER expressions it is necessary to model the channel estimate which depends on the type of estimation algorithm employed. Next it will be shown two examples for modeling the channel estimate.

#### 1) Basic channel estimator

If the channel fading rate is sufficiently low so that it can be considered approximately constant during the duration of the frame then it is possible to use the same channel estimate for all the data symbols. This channel estimate can be computed simply as the average of the channel estimates of the $P$ pilot symbols:

$$\hat{\alpha}_k(t) = \frac{1}{P}\sum_{i=1}^{P}\hat{\alpha}_k(i), \qquad t = P+1,...,N \tag{7}$$

From (7), we see that the channel estimate, $\hat{\alpha}_k$, is a sum of zero mean complex gaussian variables and thus it is also a zero mean complex gaussian variable. The variance of $\hat{\alpha}_k$ depends on the position $t$ in the frame and can be written as

$$E\left[\left|\hat{\alpha}_k(t)\right|^2\right] = E\left[\hat{\alpha}_k(t)\hat{\alpha}_k^*(t)\right]$$

$$= \frac{1}{P^2}\left[\sum_{i=1}^{P}\sum_{j=1}^{P} 2\sigma_{\alpha_k}^2 J_0(2\pi f_D |i-j| T_s) + P\frac{1}{|S_p|^2}N_0\right], \tag{8}$$

where $T_s$ is the symbol period. The second order moment of $r_k$ and $\hat{\alpha}_k$, is given by

$$E\left[r_k(t)\hat{\alpha}_k^*(t)\right] = \frac{s}{P}\sum_{i=1}^{P} 2\sigma_{\alpha_k}^2 J_0(2\pi f_D |i-t| T_s) \tag{9}$$

#### 2) Channel estimation using a FIR

Instead of using only the pilot symbols of the current frame it is possible to transmit only one pilot symbol per frame ($P$=1) and then perform the channel estimation in the receiver taking into account also the pilot symbols of the neighbor frames. To obtain the channel estimates for the data symbols, an interpolation is performed using a finite impulse response (FIR) filter that uses the pilot symbols estimates of the $\lfloor (W-1)/2 \rfloor$ previous and $\lfloor W/2 \rfloor$ following frames, according to

$$\hat{\alpha}_k((u-1)\cdot N + t) = \sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} h_t^{j-u+1}\hat{\alpha}_k(j\cdot N+1), \tag{10}$$

where $u$ is the current frame index, $t=2,\ldots,N$ is the symbol position in the frame, $h_t^{j-u+1}$ are the interpolation coefficients of the estimation filter and $\hat{\alpha}_k(j\cdot N+1)$ are the channel estimates of the pilot symbols. These coefficients depend on the interpolation algorithm employed. There are several proposed algorithms in the literature like the optimal Wiener filter interpolator [6], the low pass sinc interpolator [15] or the low-order Gaussian interpolator [16].

Looking at expression (10), we see that, similarly to the previous case, the channel estimate, $\hat{\alpha}_k$, is also a zero mean complex gaussian variable. The variance of the channel estimate for symbol $t$ in frame $u$ can be written as

$$E\left[\left|\hat{\alpha}_k((u-1)\cdot N+t)\right|^2\right] =$$

$$= 2\sigma_{\alpha_k}^2 \sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1}\ \sum_{i=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} h_t^{j-u+1}h_t^{i-u+1} J_0(2\pi f_D |i-j| N\cdot T_s)$$

$$+\frac{1}{|S_p|^2}N_0\sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1}\left(h_t^{j-u+1}\right)^2 \tag{11}$$

The second order moment of $r_k$ and $\hat{\alpha}_k$, is given by

$$E\left[r_k((u-1)\cdot N+t)\cdot\hat{\alpha}_k^*((u-1)\cdot N+t)\right] =$$

$$= 2\sigma_{\alpha_k}^2 s \sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} h_t^{j-u+1} J_0\left(2\pi f_D \left|(u-1-j)\cdot N+t-1\right| T_s\right) \tag{12}$$

## III. BER Performance Analysis

Our target is to obtain the error probability of each different bit type $i_m$ ($m=1,\ldots,log_2(M)/2$) in a constellation. As shown in the previous section the parameters that define the channel estimate model depend on the position in the transmitted frame. This means that the BER will also be dependent on this position. So, it is necessary to average the individual BER's over all the possible locations in a frame.

Due to the mapping considered, the I and Q branches are symmetric and thus we can develop our study using the decision variable for only one of the branches. In the following we will consider only the decision variable for the I branch, i.e., using (5). Although the BER performance of a *M*-QAM constellation in a Rayleigh channel with perfect channel estimation can be obtained reducing the constellation simply to a $\sqrt{M}$-PAM constellation, in the presence of imperfect channel estimation this simplification is not possible. This happens due to the existence of a phase error even after channel compensation, which adds interference from the quadrature components to the in-phase components and vice-versa. Nevertheless, the existing symmetries in the constellations still allows a simplification since it is only required to perform the computation of the error probability for each bit $i_m$ by averaging the conditional BER's over all existing constellation symbols in only one of the quadrants.

In [12] an explicit closed-form expression for the bit error probability of generalized hierarchical QAM constellations in AWGN and Rayleigh channels was derived. It is possible to adapt this expression for the situation of imperfect channel estimation which is a more general case where the constellation can not be simply analyzed as a PAM constellation. In this situation, and admitting that the transmitted symbols are equiprobable, i.e., $P\left(s_{j,f}\right)=1/M$, the BER expression can be written as

$$P_b\left(i_m\right)=\frac{1}{\left(N-P\right)}\frac{2}{\sqrt{M}}\sum_{t=1}^{N-P}\sum_{j=0}^{\sqrt{M}/2-1}\left[1-g_j^k+\left(-1\right)^{g_j^k}\sum_{l=1}^{2^{(k-1)}}\left[\left(-1\right)^{l+1}\right.\right.$$

$$\left.\left.\times\frac{2}{\sqrt{M}}\sum_{f=0}^{\sqrt{M}/2-1}\text{Prob}\left\{z_{re}<\mathbf{b}_m\left(l\right)\sum_{k=1}^{L}\left|\hat{\alpha}_k\right|^2\left|s_{j,f},t\right.\right\}\right]\right],\quad(13)$$

where

$$\mathbf{b}_m\left(l\right)=\frac{\mathbf{d}_\mathbf{s}\left(\left(2l-1\right)2^{1/2\cdot\log_2 M-m}\right)+\mathbf{d}_\mathbf{s}\left(\left(2l-1\right)2^{1/2\cdot\log_2 M-m}+1\right)}{2}\quad(14)$$

and

$$\mathbf{d}_\mathbf{s}\left(j\right)=\sum_{i=1}^{1/2\cdot\log_2 M}\left(2b_j^i-1\right)D_{1/2\cdot\log_2 M-i+1}\;.\qquad(15)$$

Thus, to compute the analytical BER it is necessary to obtain an expression for $\text{Prob}\left\{z_{re}<\mathbf{b}_m\left(l\right)\sum_{k=1}^{L}\left|\hat{\alpha}_k\right|^2\left|s_{j,f},k\right.\right\}$. In the following derivations we will drop the indexes *t* (symbol position in the frame), *i* (in-phase branch label of the constellation symbol) and *f* (quadrature branch label of the constellation symbol) and replace $\mathbf{b}_m\left(l\right)$ for *w* for simplicity of notation. To avoid that the decision borders depend explicitly on the channel estimate, the probability expression can be rewritten as

$$\text{Prob}\left(z_{re}<w\sum_{k=1}^{L}\left|\hat{\alpha}_k\right|^2\right)=\text{Prob}\left(\text{Real}\left\{\sum_{k=1}^{L}\left(r_k-w\cdot\hat{\alpha}_k\right)\hat{\alpha}_k^*\right\}<0\right)$$
$$=\text{Prob}\left(z'_{re}<0\right),\qquad(16)$$

where the modified variables

$$z'_{re}=\sum_{k=1}^{L}z'_{re_k}=\sum_{k=1}^{L}\text{Real}\left\{r'_k\cdot\hat{\alpha}_k^*\right\}\qquad(17)$$

and

$$r'_k=r_k-w\cdot\hat{\alpha}_k\qquad(18)$$

are defined. Since $r_k$ and $\hat{\alpha}_k$ are complex random gaussian variables and *w* is a constant then $r'_k$ also has a gaussian distribution. The second moment of $r'_k$ is given by

$$E\left[\left|r'_k\right|^2\mid s\right]=2\left|s\right|^2\sigma_{\alpha_k}^2+N_0-2w\text{Re}\left\{E\left[r_k\cdot\hat{\alpha}_k^*\mid s\right]\right\}+w^2E\left[\left|\hat{\alpha}_k\right|^2\right]\quad(19)$$

the cross moment of $r'_k$ and $\hat{\alpha}_k$ is

$$E\left[r'_k\hat{\alpha}_k^*\mid s\right]=E\left[r_k\cdot\hat{\alpha}_k^*\mid s\right]-wE\left[\left|\hat{\alpha}_k\right|^2\right]\qquad(20)$$

and the cross-correlation coefficient between $r'_k$ and $\hat{\alpha}_k$ is defined as

$$\mu'_k=\frac{E\left[r'_k\hat{\alpha}_k^*\mid s\right]}{\sqrt{E\left[\left|r'_k\right|^2\mid s\right]E\left[\left|\hat{\alpha}_k\right|^2\mid s\right]}}=\left|\mu'_k\right|e^{-\varepsilon'_k\cdot j}\,.\qquad(21)$$

We will first compute the PDF of $z'_{re}$ conditioned on *s* for each individual reception path. We start by writing the PDF of the decision variable $z'_{re_k}$ ($z'_{re_k}=\text{Real}\left(r'_k\cdot\hat{\alpha}_k^*\right)$), conditioned on *s* [17], which is

$$p\left(z'_{re_k}\mid s\right)=\frac{1}{F_k}\exp\left[G_k\cdot z'_{re_k}\right]\exp\left[-H_k\cdot\left|z'_{re_k}\right|\right]\qquad(22)$$

with

$$F_k=2\sigma_{r'_k}\sigma_{\hat{\alpha}_k}\sqrt{1-\left|\mu'_k\right|^2\left(\sin\varepsilon'_k\right)^2}$$
$$G_k=\frac{\left|\mu'_k\right|\cos\varepsilon'_k}{\sigma_{r'_k}\sigma_{\hat{\alpha}_k}\left(1-\left|\mu'_k\right|^2\right)}\qquad(23)$$
$$H_k=\frac{\sqrt{1-\left|\mu'_k\right|^2\left(\sin\varepsilon'_k\right)^2}}{\sigma_{r'_k}\sigma_{\hat{\alpha}_k}\left(1-\left|\mu'_k\right|^2\right)}$$

where $\sigma_{r'_k}^2=E\left[\left|r'_k\right|^2\mid s\right]/2$.

From (17), the decision variable corresponds to the sum of *L* variables with PDF's similar to (22). According to [18], the

PDF of the sum of independent random variables can be computed as the inverse Fourier transform of the product of the individual characteristic functions. The characteristic function of the PDF defined in (22) is obtained applying the Fourier transform, which results in

$$\Psi_k(\upsilon j) = -\frac{2H_k}{F_k(\upsilon j - G_k - H_k)(\upsilon j - G_k + H_k)} \quad (24)$$

Let us divide the $L$ diversity branches in $L'$ different sets of diversity paths, where in each set there are $\rho_k$ ($k = 1,...,L'$) received paths with equal powers, satisfying $L = \sum_{k=1}^{L'} \rho_k$ .Then the total characteristic function (given by the product of the individual characteristic functions) can be written as

$$\Psi(\upsilon j) = \prod_{k=1}^{L'} \left(\frac{-2H_k}{F_k}\right)^{\rho_k} \frac{1}{(\upsilon j - G_k - H_k)^{\rho_k} (\upsilon j - G_k + H_k)^{\rho_k}} . \quad (25)$$

Decomposing (25) as a sum of simple fractions according to the method proposed in [19], and then applying the inverse Fourier transform, we obtain the PDF of $z'_{re}$ conditioned on $s$

$$\begin{aligned} p(z'_{re}) &= \mathrm{F}^{-1}\{\Psi(j\upsilon)\} \\ &= \prod_{k=1}^{L'} \left(\frac{-2H_k}{F_k}\right)^{\rho_k} \left[ -\sum_{k=1}^{L'} \sum_{i=1}^{\rho_k} \frac{A_{k,i}^1}{(i-1)!} z_{re}'^{i-1} e^{(G_k+H_k)z'_{re}} u(-z'_{re}) \right. \\ &\left. + \sum_{k=1}^{L'} \sum_{i=1}^{\rho_k} \frac{A_{k,i}^2}{(i-1)!} z_{re}'^{i-1} e^{(G_k-H_k)z'_{re}} u(z'_{re}) \right], \quad (26) \end{aligned}$$

where

$$A_{k,i}^1 = \frac{1}{(\rho_k - i)!} \left[ \frac{\partial^{\rho_k - i}}{\partial s^{\rho_k - i}} \left( \prod_{\substack{j=1 \\ j \neq k}}^{L'} \frac{1}{(s - G_j - H_j)^{\rho_j}} \times \prod_{j=1}^{L'} \frac{1}{(s - G_j + H_j)^{\rho_j}} \right) \right]_{s = G_k + H_k} \quad (27)$$

To compute (16) it is necessary to integrate this PDF from $-\infty$ to $0$:

$$\begin{aligned} \mathrm{Prob}(z'_{re} < 0) &= \int_{-\infty}^{0} p(z'_{re})\, dz'_{re} \\ &= \prod_{k=1}^{L'} \left(\frac{-2H_k}{F_k}\right)^{\rho_k} \sum_{k=1}^{L'} \sum_{i=1}^{\rho_k} (-1)^i A_{k,i}^1 (G_k + H_k)^{-i} \quad (28) \end{aligned}$$

If the diversity branches have all equal powers, i.e., $\rho_k = L$ , $L' = 1$ and $F_k$, $G_k$ and $H_k$ are all equal (the $k$ index can be dropped) results

$$\mathrm{Prob}(z'_{re} < 0) = \frac{1}{(2F \cdot H)^L} \sum_{k=1}^{L} \binom{2L-k-1}{L-k} \left(\frac{2H}{G+H}\right)^k . \quad (29)$$

If all the received diversity branches are different, i.e., $\rho_k = 1$ for any $k$ and $L' = L$ , the $A_{k,i}^1$ coefficients are computed as

$$A_{k,1}^1 = \prod_{\substack{j=1 \\ j \neq k}}^{L} \frac{1}{(G_k + H_k - G_j - H_j)^{\rho_j}} \times \prod_{j=1}^{L} \frac{1}{(G_k + H_k - G_j + H_j)^{\rho_j}} , \quad (30)$$

resulting

$$\mathrm{Prob}(z'_{re} < 0) = \prod_{k=1}^{L} \left(\frac{-2H_k}{F_k}\right) \sum_{k=1}^{L} -\frac{A_{k,1}^1}{(G_k + H_k)} \quad (31)$$

## IV. Numerical Results

To verify the validity of the obtained expressions, some simulations were run using the Monte Carlo method. The results obtained are plotted as a function of $E_S/N_0$ ($E_S$ - symbol energy). Figure 3 shows the simulated BER performance of hierarchical 64-QAM ($k_1$=0.4 and $k_2$=0.4), employing basic estimation and considering four diversity branches with Rayleigh fading ($f_d T_s$=1x$10^{-3}$) and different powers ([0dB -3dB -6dB -9dB]). The frame format is composed by 1 pilot and 14 data symbols. The analytical results, computed using expressions (13) and (31), are also plotted. The curves corresponding to perfect channel estimation are drawn in both figures. It is clear that the analytical results accurately match the simulated ones. We also see that the non uniformity of the constellation used clearly result in differentiated performances for the different bit classes.

Figure 4 compares the performances of an hierarchical 256-QAM ($k_1$=0.4, $k_2$=0.4 and $k_3$=0.4) constellation for a two equal branch diversity reception, in a slow Rayleigh fading environment ($f_d T_s$=1x$10^{-4}$) using two different channel estimation algorithms. Results for perfect estimation, a basic channel estimator (P=4, N=18) and a low pass sinc channel estimation filter (K=20, N=18) are presented. The performance of the basic channel estimator is very close to the perfect channel estimation case and is better than the performance of the sinc filter for low $E_s/N_0$. This is due to the fact that because the channel evolution is slow, for low signal to noise ratios the estimation error is basically caused by the thermal noise. Since the basic channel estimator performs an averaging operation using 4 pilot symbols the effect of the thermal noise is reduced. Note that the better performance achieved with the basic estimator for low $E_s/N_0$ values requires the transmission of four times more pilot symbols. For high $E_s/N_0$ values, the error caused by the channel evolution prevails over the thermal noise becoming the main source for the channel estimation error. Since the basic channel estimator does not perform any interpolation between consecutive frames it clearly exhibits a higher irreducible BER floor than the sinc filter. It is important to note that these BER floors are more problematic for the least protected bits since in these cases they have much higher magnitude and may compromise the reception of these streams.

## V. Conclusions

In this paper we have derived analytical expressions that allow the computation of the exact BER performance of the individual bit classes for any hierarchical square $M$-QAM constellation, in the presence of imperfect channel estimation. Rayleigh fading environments with equal or unequal receiving diversity branches, employing MRC, are considered. It was

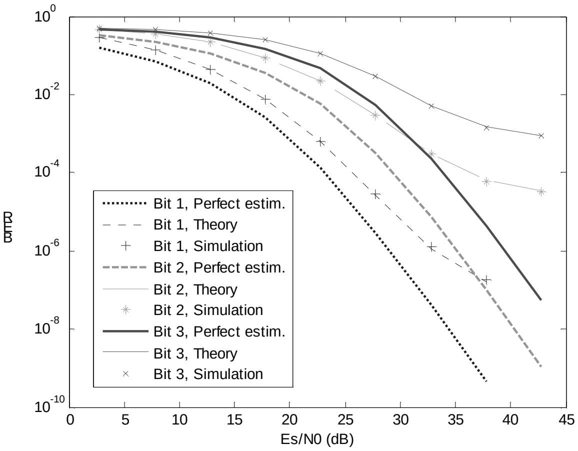


Figure 3 – Theoric and simulated BER performances of hierarchical 64-QAM ($k_1$=0.4 and $k_2$=0.4), with 4 diversity branches with different powers ([0dB -3dB -6dB -9dB]). $f_dT_s$=1x10$^{-3}$, basic channel estimator with $P$=1 and $N$=15.

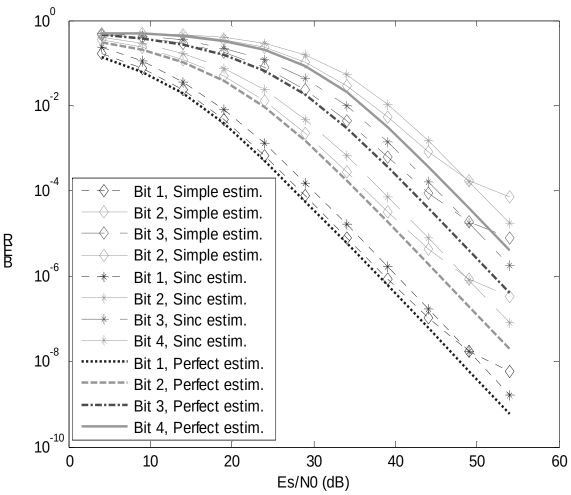


Figure 4 – BER performance of hierarchical 256-QAM ($k_1$= $k_2$= $k_3$=0.4), with 2 diversity branches using basic channel estimation (P=4, N=18) and also a low pass sinc channel estimation filter (K=20, N=18) for $f_dT_s$=1x10$^{-4}$.

shown that using hierarchical constellations it is possible to improve the performance of some of the bit streams at the cost of degrading the performance of the others.

It was shown that poor channel estimation can compromise the reception especially for the least protected bit streams, which were shown to be more sensitive. Using the derived expressions it was verified that for very slow fading channels, a basic channel estimator that employs a small number of pilot symbols for estimating the channel and then uses this estimate for all the data symbols in the frame, can be sufficient for obtaining good performances. If the fading rate is faster, then it is necessary to use the channel estimates of the pilot symbols in several adjacent frames and some interpolation algorithm, like the low pass sinc interpolator, to obtain an acceptable performance.

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