On the BER Performance of Hierarchical M-QAM Constellations With Diversity and Imperfect Channel Estimation

Nuno M. B. Souto, Francisco A. B. Cercas, Rui Dinis, and João C. M. Silva

***Abstract*—The analytical bit error rate of hierarchical quadrature amplitude modulation formats, which include uniform and nonuniform constellations, over flat Rayleigh fading environments is studied in this paper. The analysis takes into account the effect of imperfect channel estimation and considers diversity reception with both independent identically and nonidentically distributed channels, employing maximal ratio combining.**

***Index Terms*—Channel estimation, diversity, quadrature amplitude modulation (QAM), Rayleigh fading.**

I. Introduction

In THE design of wireless communication networks, the limitation on spectrum resources is an important restriction for achieving high bit rate transmissions. The use of M-ary quadrature amplitude modulation (M-QAM) is considered an attractive technique to overcome this restriction due to its high spectral efficiency, and it has been studied and proposed for wireless systems by several authors [1], [2].

A great deal of attention has been devoted to obtaining analytical expressions for the bit error rate (BER) performance of M-QAM with imperfect channel estimation. A tight upper bound on the symbol error ratio (SER) of 16-QAM with pilot-symbol-assisted modulation (PSAM) in Rayleigh fading channels was presented in [3]. An approximate expression for the BER of 16-QAM and 64-QAM in flat Rayleigh fading with imperfect channel estimates was derived in [4], while in [5], exact expressions were obtained for 16-QAM diversity reception with maximal ratio combining (MRC). The method used in these papers can be extended to general M-QAM constellations, but the manipulations and development required can become quite cumbersome. Recently, exact expressions in [6], were published for the performance of uniform M-QAM constellations with PSAM for identical diversity channels, while in [7], expressions valid for nonidentically distributed channels were derived, though in this case, they were not linked to any specific channel estimation method.

All the studies mentioned before relate to M-QAM uniform constellations that can be regarded as a subset of the more general case of nonuniform M-QAM constellations (also called hierarchical constellations). These constellations can be used as a very simple method to provide unequal bit error protection and to improve the efficiency and flexibility of a network in the case of broadcast transmissions. Nonuniform 16/64-QAM constellations have already been incorporated in the digital video broadcasting-terrestrial (DVB-T) standard [8]. A recursive algorithm for the exact BER computation of hierarchical M-QAM constellations in additive white Gaussian noise (AWGN) and fading channels was presented in [9]. Later on, closed-form expressions were also obtained for these channels [10]. As far as we know, the analytical BER performance of these constellations in Rayleigh channels with imperfect channel estimation has not yet been investigated.

In this paper, we adopt a different method from [4] and [5] to derive general closed-form expressions for the BER performance in Rayleigh fading channels of hierarchical M-QAM constellations with diversity employing an MRC receiver. We consider diversity reception with both independent identically and nonidentically distributed channels. PSAM philosophy with channel estimation that accomplished through a finite impulse response (FIR) filter is assumed.

This paper is organized as follows. Section II describes the model of the communication system, which includes the definition of nonuniform M-QAM constellations, the channel, and the modeling of the channel estimation error. Section III presents the derivation of the BER expressions and Section IV presents some numerical and simulation results. The conclusions are summarized in Section V.

II. System and Channel Model

A. QAM Hierarchical Signal Constellations

In hierarchical constellations, there are two or more classes of bits with different error protection levels and to which different streams of information can be mapped. By using nonuniformly spaced signal points (where the distances along the I- or Q- axis between adjacent symbols can differ depending on their positions), it is possible to modify the different error protection levels. As an example, a nonuniform 16-QAM constellation is shown in Fig. 1. The basic idea is that the constellation can be

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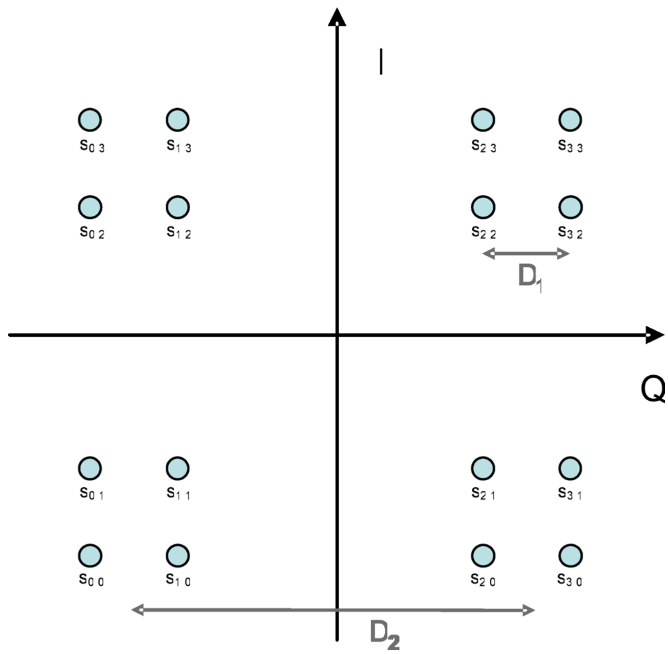

Fig. 1. Nonuniform 16-QAM constellation.

viewed as a 16-QAM constellation if the channel conditions are good enough or as a quadrature phase-shifting keying (QPSK) constellation otherwise. In the latter situation, the received bit rate is reduced by half. This constellation can be characterized by the parameter $k = D_1/D_2 (0 < k \leq 0.5)$. If $k = 0.5$, the resulting constellation corresponds to a uniform 16-QAM. For the general case of an M-QAM constellation, the symbols are defined as

$$s = \sum_{l=1}^{\log_2(\sqrt{M})} \left(\pm\frac{D_l}{2}\right) + \sum_{l=1}^{\log_2(\sqrt{M})} \left(\pm\frac{D_l}{2}\right) j \tag{1}$$

and the number of possible classes of bits with different error protection is $\log_2(M)/2$. In the following derivation, we assume that the parallel information streams are split into two, so that half of each stream goes for the in-phase branch and the other half goes to the quadrature branch of the modulator. The resulting bit sequence for each branch is Gray coded, mapped to the corresponding pluggable authentication modules ($\sqrt{M}$-PAM) constellation symbols, and then, grouped together, forming complex M-QAM symbols. The Gray encoding for each ($\sqrt{M}$-PAM) constellation is performed according to the procedure described in [9]. Firstly, the constellation symbols are represented in a horizontal axis where they are numbered from left to right with integers starting from 0 to $\sqrt{M} - 1$. These integers are then converted into their binary representations, so that each symbol s_j can be expressed as a binary sequence with $\log_2(M)/2$ digits: $b_j^1, b_j^2, \ldots, b_j^{\log_2 M/2}$. The corresponding Gray code $[g_j^1, \ldots, g_j^{\log_2 M/2}]$ is then computed using ($\oplus$ represents modulo-2 addition)

$$g_j^1 = b_j^1, \quad g_j^i = b_j^i \oplus b_j^{i-1}, \quad i = 2, 3, \ldots, \log_2 \frac{M}{2}. \tag{2}$$

B. Received Signal Model

Let us consider a transmission over an L diversity branch flat Rayleigh fading channel where the branches can have different average powers. Assuming perfect carrier and symbol synchronization, each received signal sample can be modeled as

$$r_k = \alpha_k \cdot s + n_k, \qquad k = 1, \ldots, L \tag{3}$$

where α_k is the channel coefficient for diversity branch k, s is the transmitted symbol, and n_k represents additive white thermal noise. Both α_k and n_k are modeled as zero mean complex Gaussian random variables with $E[|\alpha_k|^2] = 2\sigma_{\alpha_k}^2$ ($2\sigma_{\alpha_k}^2$ is the average fading power of the kth diversity branch) and $E[|n_k|^2] = 2\sigma_n^2 = N_0$ ($N_0/2$ is the noise power spectral density). Due to the Gaussian nature of α_k and n_k, the probability density function (pdf) of the received signal sample r_k, conditioned on the transmitted symbol s, is also Gaussian with zero mean. The receiver performs MRC of the L received signals. As a result of the mapping employed, the I and Q branches are symmetric (the BER is the same), and so, our derivation can be developed using only the decision variable for the I branch, i.e.,

$$z_{\mathrm{re}} = \mathrm{Real}\left\{\sum_{k=1}^{L} r_k \hat{\alpha}_k^*\right\}. \tag{4}$$

C. Channel Estimation

In this analysis, we consider a PSAM philosophy [3] where the transmitted symbols are grouped in N-length frames with one pilot symbol periodically inserted into the data sequence. The channel estimates for the data symbols can be computed by means of an interpolation with an FIR filter of length W, which uses the received pilot symbols of the previous $\lfloor (W-1)/2 \rfloor$ and subsequent $\lfloor W/2 \rfloor$ frames. Several FIR filters were proposed in the literature, such as the optimal Wiener filter interpolator [3], the low pass sinc interpolator [11], or the low-order Gaussian interpolator [12]. According to this channel estimation procedure, the estimate $\hat{\alpha}_k$ is a zero-mean complex Gaussian variable.

Assuming that the channel's corresponding autocorrelation and cross-correlation functions can be expressed as in [13], the variance of the channel estimate for symbol t ($t = 2, \ldots, N$; $t = 1$ corresponds to the pilot symbol position) in frame u can be written as

$$\begin{aligned} &E\left[|\hat{\alpha}_k((u-1)N+t)|^2\right] \\ &\quad = 2\sigma_{\alpha_k}^2 \sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} \sum_{i=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} \\ &\qquad \times h_t^{j-u+1} h_t^{i-u+1} J_0(2\pi f_D |i-j| N T_s) \\ &\qquad + \frac{1}{|S_p|^2} N_0 \sum_{j=-\lfloor (W-1)/2 \rfloor + u-1}^{\lfloor W/2 \rfloor + u-1} (h_t^{j-u+1})^2 \end{aligned} \tag{5}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the Doppler frequency, h_t^{j-u+1} are the interpolation coefficients of the FIR, filter and S_p is a pilot symbol. The second-order moment of r_k and $\hat{\alpha}_k$, which will be required further

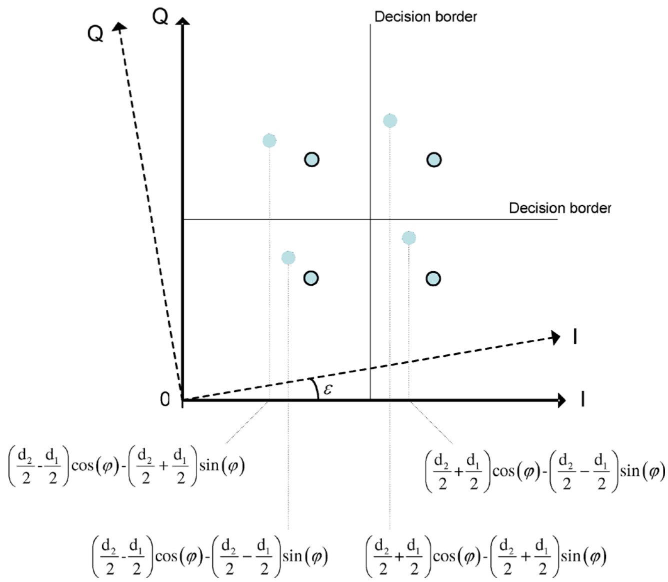

Fig. 2. Impact in a 16-QAM constellation (upper right quadrant shown) of cross-quadrature interference originated by imperfect channel estimation. Only the case of phase error is shown.

ahead, is given by

$$\begin{aligned} &E\left[r_k((u-1)N+t)\hat{\alpha}_k^*((u-1)N+t)\right| s] \\ &= 2\sigma_{\alpha_k}^2 s \sum_{j=-\lfloor (W-1)/2\rfloor+u-1}^{\lfloor W/2\rfloor+u-1} \\ &\quad \times h_t^{j-u+1} J_0\left(2\pi f_D \left|(u-1-j)N+t-1\right| T_s\right). \end{aligned} \quad (6)$$

III. BER Performance Analysis

To accomplish this analysis, we start by deriving the bit error probability for each type of bit i_m $(m = 1, \ldots, \log_2(M)/2)$ in a constellation. This error probability depends on the position t in the transmitted frame, which means that it is necessary to average the bit error probability over all the positions in the frame to obtain the overall BER. Although the BER performance of an M-QAM constellation in a Rayleigh channel with perfect channel estimation can be analyzed by simply reducing it to a $\sqrt{M}$-PAM constellation, this simplification is not possible in the presence of an imperfect channel estimation. In fact, since the channel estimates are not perfect, a residual phase error will be present in the received symbols even after channel compensation at the receiver. This phase error adds interference from the quadrature components to the in-phase components and vice versa, as shown in Fig. 2.

An explicit closed-form expression for the bit error probability of generalized nonuniform QAM constellations in AWGN and Rayleigh channels was derived in [10]. It is possible to adapt this expression for the case of imperfect channel estimation, which is a more general case where the constellation cannot be simply analyzed as a PAM constellation. In this situation, assuming equiprobable transmitted symbols, i.e., $P(s_{j,f}) = 1/M$, the average BER can be computed as

$$\begin{aligned} P_b(i_m) &= \frac{1}{(N-P)}\frac{2}{\sqrt{M}}\sum_{t=1}^{N-P}\sum_{j=0}^{\sqrt{M}/2-1}\Bigg[1-g_j^k+(-1)^{g_j^k} \\ &\quad \times \sum_{l=1}^{2^{(k-1)}}\Bigg[(-1)^{l+1}\frac{2}{\sqrt{M}}\times\sum_{f=0}^{\sqrt{M}/2-1} \\ &\quad \times \mathrm{Prob}\left\{z_{\mathrm{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2 \,|s_{j,f}, t\right\}\Bigg]\Bigg]. \end{aligned} \quad (7)$$

where

$$\mathbf{b}_m(l) = \frac{\mathbf{d_s}((2l-1)\,2^{1/2\cdot\log_2 M-m})+\mathbf{d_s}((2l-1)\,2^{1/2\cdot\log_2 M-m}+1)}{2}$$

and

$$\mathbf{d_s}(j) = \sum_{i=1}^{1/2\cdot\log_2 M}\left(2b_j^i-1\right)D_{1/2\cdot\log_2 M-i+1}.$$

Therefore, to calculate the analytical BER, it is necessary to obtain an expression for $\mathrm{Prob}\{z_{\mathrm{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s_{j,f}, t\}$, i.e., the probability that the decision variable is smaller than the decision border $\mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2$, conditioned on the fact that the transmitted symbol was $s_{j,f}$ and its position in the frame is t. Note that, due to the existing symmetries in the constellations, we only need to compute $\mathrm{Prob}\{z_{\mathrm{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s_{j,f}, t\}$ over the constellation symbols in one of the quadrants. In the following derivations, we will drop indexes t, i (in-phase branch label of the constellation symbol), and f (quadrature branch label of the constellation symbol), and we replace $\mathbf{b}_m(l)$ by w, for simplicity of notation. To avoid the explicit dependency of the decision borders on the channel estimate, the probability expression can be rewritten as

$$\begin{aligned} &\mathrm{Prob}\left(z_{\mathrm{re}} < w\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s\right) \\ &\quad = \mathrm{Prob}\left(\mathrm{Real}\left\{\sum_{k=1}^{L}(r_k - w\hat{\alpha}_k)\hat{\alpha}_k^*\right\} < 0|s\right) \\ &\quad = \mathrm{Prob}(z'_{\mathrm{re}} < 0|s) \end{aligned} \quad (8)$$

where the modified variables $r'_k = r_k - w\cdot\hat{\alpha}_k$ and $z'_{\mathrm{re}} = \sum_{k=1}^{L} z'_{\mathrm{re}_k} = \sum_{k=1}^{L}\mathrm{Real}\{r'_k\hat{\alpha}_k^*\}$ are defined. Since r_k and $\hat{\alpha}_k$ are complex random Gaussian variables and w is a constant, r'_k also has a Gaussian distribution. The second moment of r'_k, the cross moment of r'_k and $\hat{\alpha}_k$, and the respective cross-correlation coefficient are given by

$$\begin{aligned} E[|r'_k|^2\,|s] &= 2\,|s|^2\,\sigma_{\alpha_k}^2 + N_0 - 2w\,\mathrm{Re}\left\{E\left[r_k\hat{\alpha}_k^*|s\right]\right\} \\ &\quad + w^2E[|\hat{\alpha}_k|^2] \end{aligned} \quad (9)$$

$$E\left[r'_k\hat{\alpha}_k^*|s\right] = E\left[r_k\hat{\alpha}_k^*|s\right] - wE[|\hat{\alpha}_k|^2] \quad (10)$$

$$\mu'_k = \frac{E\left[r'_k\hat{\alpha}_k^*|s\right]}{\sqrt{E[|r'_k|^2\,|s]E[|\hat{\alpha}_k|^2\,|s]}} = |\mu'_k|\,e^{-\varepsilon'_k\cdot j}. \quad (11)$$

We will first compute the pdf of z'_{re} conditioned on s for each individual reception branch. We start by writing the pdf of z'_k ($z'_k = r'_k \hat{\alpha}^*_k$) conditioned on s [14]

$$p\left(z'_k|s\right) = \frac{1}{2\pi\sigma^2_{\hat{\alpha}_k}\,\sigma^2_{r'_k}\left(1-|\mu'_k|^2\right)} \times \exp\left[\frac{|\mu'_k|\left(z'_{\mathrm{re}_k}\ \cos\ \varepsilon'_k + z'_{\mathrm{im}_k}\ \sin\ \varepsilon'_k\right)}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}\right] \times K_0\left[\frac{\sqrt{z'^2_{\mathrm{re}_k} + z'^2_{\mathrm{im}_k}}}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}\right] \tag{12}$$

where $K_0\left(\cdot\right)$ denotes the modified Hankel function of order zero and

$$\sigma^2_{r'_k} = \frac{E[|r'_k|^2|s]}{2}.$$

Integrating (12) over z'_{im_k} ($z'_{\mathrm{im}_k} = \mathrm{Imag}\left\{z'_k\right\}$) yields the marginal pdf of the decision variable $z_{\mathrm{re}'_k}$

$$p(z_{\mathrm{re}'_k}|s) = \int_{-\infty}^{+\infty} p\left(z'_k|s\right)\ dz_{\mathrm{im}'_k} = \frac{1}{F_k}\exp[G_k z_{\mathrm{re}'_k}]\exp[-H_k||z'_{\mathrm{re}_k}||] \tag{13}$$

where

$$F_k = 2\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\sqrt{1-|\mu'_k|^2\left(\sin\varepsilon'_k\right)^2},$$

$$G_k = \frac{|\mu'_k|\ \cos\ \varepsilon'_k}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}, \quad H_k = \frac{\sqrt{1-|\mu'_k|^2\left(\sin\ \varepsilon'_k\right)^2}}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}.$$

The decision variable z'_{re} is the sum of L independent random variables with pdfs similar to (13). According to [15], the resulting pdf can be computed through the inverse Fourier transform of the product of the individual characteristic functions. The characteristic function of the pdf defined in (13) is obtained by applying the Fourier transform, which results in

$$\Psi_k\left(\upsilon j\right) = -\frac{2H_k}{F_k\left(\upsilon j - G_k - H_k\right)\left(\upsilon j - G_k + H_k\right)}. \tag{14}$$

If we divide the L diversity branches into L' different sets, where each set k contains ρ_k received branches with equal powers satisfying $L = \sum_{k=1}^{L'}\rho_k$, then the resulting characteristic function is given by the product of the individual characteristic functions, and can be written as

$$\Psi\left(\upsilon j\right) = \prod_{k=1}^{L'}\left(\frac{-2H_k}{F_k}\right)^{\rho_k} \times \frac{1}{\left(\upsilon j - G_k - H_k\right)^{\rho_k}\left(\upsilon j - G_k + H_k\right)^{\rho_k}}. \tag{15}$$

The pdf of z'_{re} is obtained by decomposing (15) as a sum of simple fractions (according to the method proposed in [16]) and applying the inverse Fourier transform. The desired probability can now be computed by integrating this pdf from $-\infty$ to 0 as follows

$$\mathrm{Prob}\left(z'_{\mathrm{re}} < 0\right) = \int_{-\infty}^{0} p\left(z'_{\mathrm{re}}\right)dz'_{\mathrm{re}} = \prod_{k=1}^{L'}\left(\frac{-2H_k}{F_k}\right)^{\rho_k} \times \sum_{k=1}^{L'}\sum_{i=1}^{\rho_k}\left(-1\right)^i A^1_{k,i}\left(G_k + H_k\right)^{-i} \tag{16}$$

where

$$A^1_{k,i} = \frac{1}{\left(\rho_k - i\right)!}\left[\frac{\partial^{\rho_k - i}}{\partial s^{\rho_k - i}}\left(\prod_{\substack{j=1\\ j\neq k}}^{L'}\frac{1}{\left(s - G_j - H_j\right)^{\rho_j}} \times \prod_{j=1}^{L'}\frac{1}{\left(s - G_j + H_j\right)^{\rho_j}}\right)\right]_{s=G_k+H_k}. \tag{17}$$

If all the diversity branches have equal powers, i.e., $\rho_k = L$, then $L' = 1$ and F_k, G_k, and H_k are all equal (index k can be dropped) leading to

$$\mathrm{Prob}\left(z'_{\mathrm{re}} < 0\right) = \frac{1}{\left(2FH\right)^L}\sum_{k=1}^{L}\binom{2L-k-1}{L-k}\left(\frac{2H}{G+H}\right)^k \tag{18}$$

which is the same result achieved in [6], though written in a different format. However, if all the diversity branches are different, i.e., $\rho_k = 1$ for any k and $L' = L$, then

$$A^1_{l,1} = \prod_{\substack{j=1\\ j\neq l}}^{L}\frac{1}{\left(G_l + H_l - G_j - H_j\right)^{\theta_j}} \times \prod_{j=1}^{L}\frac{1}{\left(G_l + H_l - G_j + H_j\right)^{\theta_j}} \tag{19}$$

and

$$\mathrm{Prob}\left(z_{\mathrm{re}'} < 0\right) = \prod_{k=1}^{L}\left(\frac{-2H_k}{F_k}\right) \times \sum_{k=1}^{L}\frac{-A^1_{k,1}}{\left(G_k + H_k\right)}. \tag{20}$$

IV. Numerical Results

To verify the validity of the derived expressions, some simulations were run using the Monte Carlo method. The results obtained are plotted as a function of E_S/N_0 (E_S—symbol energy) in Figs. 3 and 4. Fig. 3 presents a nonuniform 16-QAM ($k = 0.3$) transmission with three equal branch diversity reception, while Fig. 4 presents a nonuniform 64-QAM ($k_1 = 0.4$ and $k_2 = 0.4$) transmission with four diversity branches and different relative powers ([0 dB -3 dB -6 dB -9 dB]). In both cases, the frame size is $N = 16$, the pilot symbols are $S_p = 1 + j$ (transmitted with the same power level of the data symbols), a sinc interpolator [11] is employed with $W = 15$, and

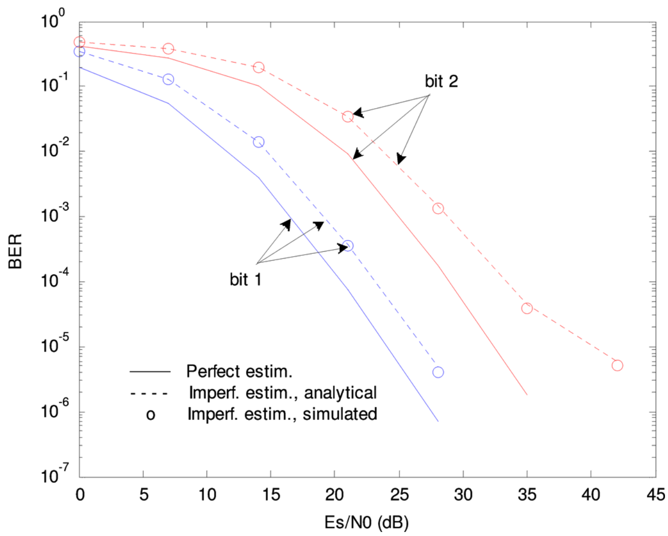

Fig. 3. Analytical and simulated BER performances of nonuniform 16-QAM ($k = 0.3$) with three equal diversity branches; $f_d T_s = 1.5 \times 10^{-2}$; sinc interpolation with $W = 15$ and $N = 16$.

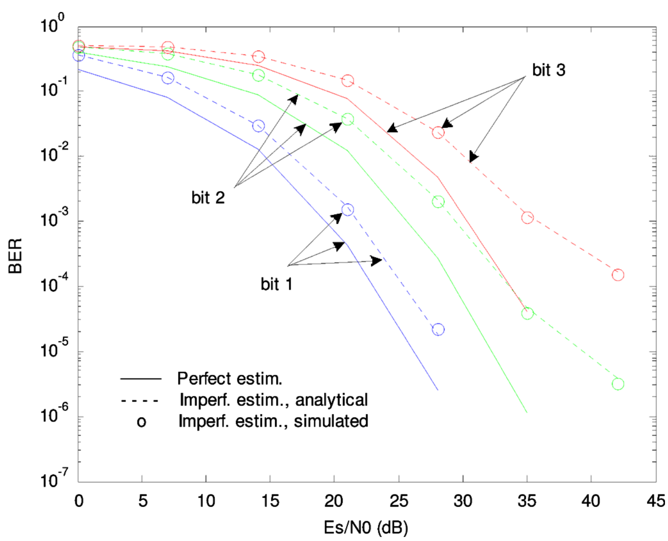

Fig. 4. Analytical and simulated BER performances of nonuniform 64-QAM ($k_1 = 0.4$ and $k_2 = 0.4$) with four diversity branches and different relative powers ([0 dB −3 dB −6 dB −9 dB]); $f_d T_s = 1.5 \times 10^{-2}$; sinc interpolation with $W = 15$ and $N = 16$.

Rayleigh fading is considered with $f_d T_s = 1.5 \times 10^{-2}$. The analytical results computed using expressions (7), (18), and (20), as well as curves corresponding to perfect channel estimation are drawn in both figures. In both cases, it is clear that the analytical results accurately match the simulated ones. We can also notice in both figures a considerable difference between the performance with perfect channel estimation and with imperfect channel estimation, the existence of irreducible BER floors (more noticeable in the least protected bit since they appear at higher BER values) being visible. This is a consequence of the effect of the time-varying fading channel that results in a reduced quality of the channel estimates. Moreover, these figures also show that the nonuniformity of the constellations clearly results in differentiated performances for the different bit classes.

V. Conclusion

In this paper, we have derived general analytical expressions for the evaluation of the exact BER performance for the individual bit classes of any nonuniform square M-QAM constellation, in the presence of imperfect channel estimation. These expressions can be applied to Rayleigh fading environments, with either equal or unequal receiving diversity branches, and MRC receivers.

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On the BER Performance of Hierarchical M-QAM Constellations With Diversity and Imperfect Channel Estimation

Nuno M. B. Souto, Francisco A. B. Cercas, Rui Dinis, and João C. M. Silva

***Abstract*—The analytical bit error rate of hierarchical quadrature amplitude modulation formats, which include uniform and nonuniform constellations, over flat Rayleigh fading environments is studied in this paper. The analysis takes into account the effect of imperfect channel estimation and considers diversity reception with both independent identically and nonidentically distributed channels, employing maximal ratio combining.**

***Index Terms*—Channel estimation, diversity, quadrature amplitude modulation (QAM), Rayleigh fading.**

I. Introduction

In THE design of wireless communication networks, the limitation on spectrum resources is an important restriction for achieving high bit rate transmissions. The use of M-ary quadrature amplitude modulation (M-QAM) is considered an attractive technique to overcome this restriction due to its high spectral efficiency, and it has been studied and proposed for wireless systems by several authors [1], [2].

A great deal of attention has been devoted to obtaining analytical expressions for the bit error rate (BER) performance of M-QAM with imperfect channel estimation. A tight upper bound on the symbol error ratio (SER) of 16-QAM with pilot-symbol-assisted modulation (PSAM) in Rayleigh fading channels was presented in [3]. An approximate expression for the BER of 16-QAM and 64-QAM in flat Rayleigh fading with imperfect channel estimates was derived in [4], while in [5], exact expressions were obtained for 16-QAM diversity reception with maximal ratio combining (MRC). The method used in these papers can be extended to general M-QAM constellations, but the manipulations and development required can become quite cumbersome. Recently, exact expressions in [6], were published for the performance of uniform M-QAM constellations with PSAM for identical diversity channels, while in [7], expressions valid for nonidentically distributed channels were derived, though in this case, they were not linked to any specific channel estimation method.

All the studies mentioned before relate to M-QAM uniform constellations that can be regarded as a subset of the more general case of nonuniform M-QAM constellations (also called hierarchical constellations). These constellations can be used as a very simple method to provide unequal bit error protection and to improve the efficiency and flexibility of a network in the case of broadcast transmissions. Nonuniform 16/64-QAM constellations have already been incorporated in the digital video broadcasting-terrestrial (DVB-T) standard [8]. A recursive algorithm for the exact BER computation of hierarchical M-QAM constellations in additive white Gaussian noise (AWGN) and fading channels was presented in [9]. Later on, closed-form expressions were also obtained for these channels [10]. As far as we know, the analytical BER performance of these constellations in Rayleigh channels with imperfect channel estimation has not yet been investigated.

In this paper, we adopt a different method from [4] and [5] to derive general closed-form expressions for the BER performance in Rayleigh fading channels of hierarchical M-QAM constellations with diversity employing an MRC receiver. We consider diversity reception with both independent identically and nonidentically distributed channels. PSAM philosophy with channel estimation that accomplished through a finite impulse response (FIR) filter is assumed.

This paper is organized as follows. Section II describes the model of the communication system, which includes the definition of nonuniform M-QAM constellations, the channel, and the modeling of the channel estimation error. Section III presents the derivation of the BER expressions and Section IV presents some numerical and simulation results. The conclusions are summarized in Section V.

II. System and Channel Model

A. QAM Hierarchical Signal Constellations

In hierarchical constellations, there are two or more classes of bits with different error protection levels and to which different streams of information can be mapped. By using nonuniformly spaced signal points (where the distances along the I- or Q- axis between adjacent symbols can differ depending on their positions), it is possible to modify the different error protection levels. As an example, a nonuniform 16-QAM constellation is shown in Fig. 1. The basic idea is that the constellation can be

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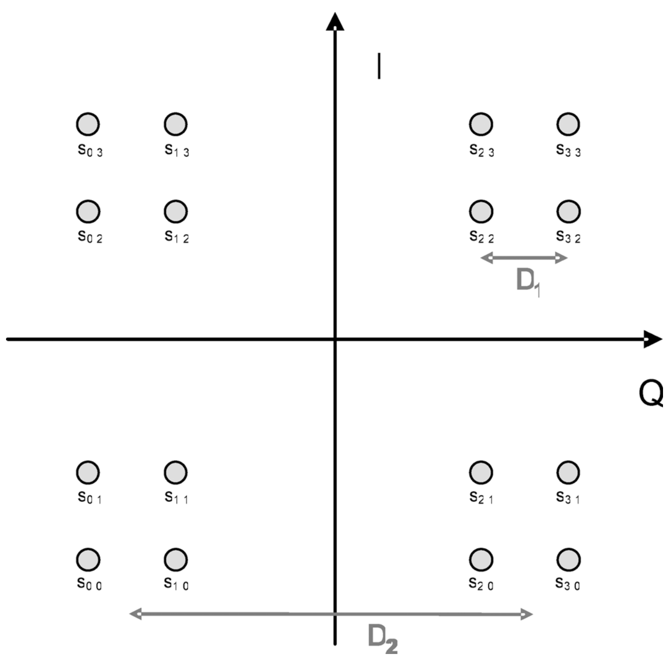

Fig. 1. Nonuniform 16-QAM constellation.

viewed as a 16-QAM constellation if the channel conditions are good enough or as a quadrature phase-shifting keying (QPSK) constellation otherwise. In the latter situation, the received bit rate is reduced by half. This constellation can be characterized by the parameter $k = D_1/D_2 (0 < k \leq 0.5)$. If $k = 0.5$, the resulting constellation corresponds to a uniform 16-QAM. For the general case of an M-QAM constellation, the symbols are defined as

$$s = \sum_{l=1}^{\log_2(\sqrt{M})} \left(\pm\frac{D_l}{2}\right) + \sum_{l=1}^{\log_2(\sqrt{M})} \left(\pm\frac{D_l}{2}\right) j \qquad (1)$$

and the number of possible classes of bits with different error protection is $\log_2(M)/2$. In the following derivation, we assume that the parallel information streams are split into two, so that half of each stream goes for the in-phase branch and the other half goes to the quadrature branch of the modulator. The resulting bit sequence for each branch is Gray coded, mapped to the corresponding pluggable authentication modules ($\sqrt{M}$-PAM) constellation symbols, and then, grouped together, forming complex M-QAM symbols. The Gray encoding for each ($\sqrt{M}$-PAM) constellation is performed according to the procedure described in [9]. Firstly, the constellation symbols are represented in a horizontal axis where they are numbered from left to right with integers starting from 0 to $\sqrt{M} - 1$. These integers are then converted into their binary representations, so that each symbol s_j can be expressed as a binary sequence with $\log_2(M)/2$ digits: $b_j^1, b_j^2, \ldots, b_j^{\log_2 M/2}$. The corresponding Gray code $[g_j^1, \ldots, g_j^{\log_2 M/2}]$ is then computed using ($\oplus$ represents modulo-2 addition)

$$g_j^1 = b_j^1, \quad g_j^i = b_j^i \oplus b_j^{i-1}, \quad i = 2, 3, \ldots, \log_2 \frac{M}{2}. \qquad (2)$$

B. Received Signal Model

Let us consider a transmission over an L diversity branch flat Rayleigh fading channel where the branches can have different average powers. Assuming perfect carrier and symbol synchronization, each received signal sample can be modeled as

$$r_k = \alpha_k \cdot s + n_k, \qquad k = 1, \ldots, L \qquad (3)$$

where α_k is the channel coefficient for diversity branch k, s is the transmitted symbol, and n_k represents additive white thermal noise. Both α_k and n_k are modeled as zero mean complex Gaussian random variables with $E[|\alpha_k|^2] = 2\sigma_{\alpha_k}^2$ ($2\sigma_{\alpha_k}^2$ is the average fading power of the kth diversity branch) and $E[|n_k|^2] = 2\sigma_n^2 = N_0$ ($N_0/2$ is the noise power spectral density). Due to the Gaussian nature of α_k and n_k, the probability density function (pdf) of the received signal sample r_k, conditioned on the transmitted symbol s, is also Gaussian with zero mean. The receiver performs MRC of the L received signals. As a result of the mapping employed, the I and Q branches are symmetric (the BER is the same), and so, our derivation can be developed using only the decision variable for the I branch, i.e.,

$$z_{\text{re}} = \text{Real}\left\{\sum_{k=1}^{L} r_k \hat{\alpha}_k^*\right\}. \qquad (4)$$

C. Channel Estimation

In this analysis, we consider a PSAM philosophy [3] where the transmitted symbols are grouped in N-length frames with one pilot symbol periodically inserted into the data sequence. The channel estimates for the data symbols can be computed by means of an interpolation with an FIR filter of length W, which uses the received pilot symbols of the previous $\lfloor (W-1)/2 \rfloor$ and subsequent $\lfloor W/2 \rfloor$ frames. Several FIR filters were proposed in the literature, such as the optimal Wiener filter interpolator [3], the low pass sinc interpolator [11], or the low-order Gaussian interpolator [12]. According to this channel estimation procedure, the estimate $\hat{\alpha}_k$ is a zero-mean complex Gaussian variable.

Assuming that the channel's corresponding autocorrelation and cross-correlation functions can be expressed as in [13], the variance of the channel estimate for symbol t ($t = 2, \ldots, N$; $t = 1$ corresponds to the pilot symbol position) in frame u can be written as

$$\begin{aligned} & E\left[|\hat{\alpha}_k((u-1)N+t)|^2\right] \\ &= 2\sigma_{\alpha_k}^2 \sum_{j=-\lfloor (W-1)/2\rfloor+u-1}^{\lfloor W/2\rfloor+u-1} \sum_{i=-\lfloor (W-1)/2\rfloor+u-1}^{\lfloor W/2\rfloor+u-1} \\ &\quad \times h_t^{j-u+1} h_t^{i-u+1} J_0(2\pi f_D |i-j| N T_s) \\ &\quad + \frac{1}{|S_p|^2} N_0 \sum_{j=-\lfloor (W-1)/2\rfloor+u-1}^{\lfloor W/2\rfloor+u-1} (h_t^{j-u+1})^2 \end{aligned} \qquad (5)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, f_D is the Doppler frequency, h_t^{j-u+1} are the interpolation coefficients of the FIR, filter and S_p is a pilot symbol. The second-order moment of r_k and $\hat{\alpha}_k$, which will be required further

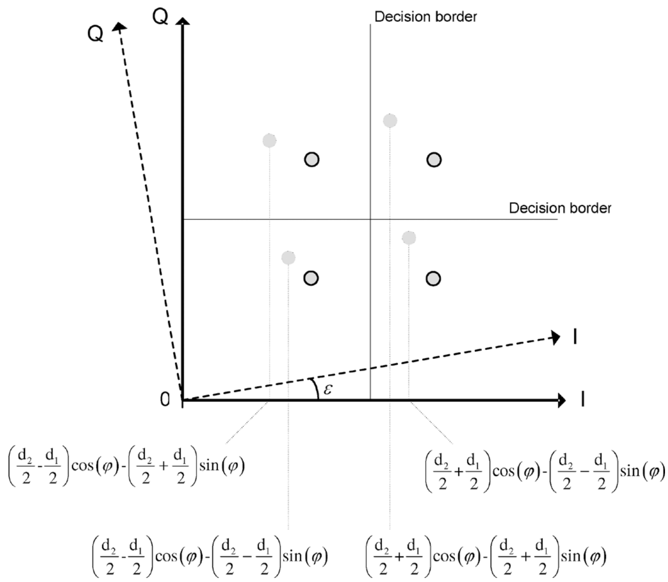

Fig. 2. Impact in a 16-QAM constellation (upper right quadrant shown) of cross-quadrature interference originated by imperfect channel estimation. Only the case of phase error is shown.

ahead, is given by

$$E\left[r_k((u-1)N+t)\hat{\alpha}_k^*((u-1)N+t)|\,s\right] = 2\sigma_{\alpha_k}^2 s \sum_{j=-\lfloor (W-1)/2\rfloor+u-1}^{\lfloor W/2\rfloor+u-1} \times h_t^{j-u+1} J_0\left(2\pi f_D\,|(u-1-j)N+t-1|\,T_s\right). \quad (6)$$

III. BER Performance Analysis

To accomplish this analysis, we start by deriving the bit error probability for each type of bit i_m $(m = 1, \ldots, \log_2(M)/2)$ in a constellation. This error probability depends on the position t in the transmitted frame, which means that it is necessary to average the bit error probability over all the positions in the frame to obtain the overall BER. Although the BER performance of an M-QAM constellation in a Rayleigh channel with perfect channel estimation can be analyzed by simply reducing it to a $\sqrt{M}$-PAM constellation, this simplification is not possible in the presence of an imperfect channel estimation. In fact, since the channel estimates are not perfect, a residual phase error will be present in the received symbols even after channel compensation at the receiver. This phase error adds interference from the quadrature components to the in-phase components and vice versa, as shown in Fig. 2.

An explicit closed-form expression for the bit error probability of generalized nonuniform QAM constellations in AWGN and Rayleigh channels was derived in [10]. It is possible to adapt this expression for the case of imperfect channel estimation, which is a more general case where the constellation cannot be simply analyzed as a PAM constellation. In this situation, assuming equiprobable transmitted symbols, i.e., $P(s_{j,f}) = 1/M$, the average BER can be computed as

$$P_b(i_m) = \frac{1}{(N-P)}\frac{2}{\sqrt{M}} \sum_{t=1}^{N-P} \sum_{j=0}^{\sqrt{M}/2-1} \Bigg[1 - g_j^k + (-1)^{g_j^k} \times \sum_{l=1}^{2^{(k-1)}} \Bigg[(-1)^{l+1}\frac{2}{\sqrt{M}} \times \sum_{f=0}^{\sqrt{M}/2-1} \times \text{Prob}\left\{z_{\text{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2\,|s_{j,f}, t\right\}\Bigg]\Bigg]. \quad (7)$$

where

$$\mathbf{b}_m(l) = \frac{\mathbf{d_s}((2l-1)\,2^{1/2\cdot\log_2 M-m}) + \mathbf{d_s}((2l-1)\,2^{1/2\cdot\log_2 M-m}+1)}{2}$$

and

$$\mathbf{d_s}(j) = \sum_{i=1}^{1/2\cdot\log_2 M} \left(2b_j^i - 1\right) D_{1/2\cdot\log_2 M-i+1}.$$

Therefore, to calculate the analytical BER, it is necessary to obtain an expression for $\text{Prob}\{z_{\text{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s_{j,f}, t\}$, i.e., the probability that the decision variable is smaller than the decision border $\mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2$, conditioned on the fact that the transmitted symbol was $s_{j,f}$ and its position in the frame is t. Note that, due to the existing symmetries in the constellations, we only need to compute $\text{Prob}\{z_{\text{re}} < \mathbf{b}_m(l)\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s_{j,f}, t\}$ over the constellation symbols in one of the quadrants. In the following derivations, we will drop indexes t, i (in-phase branch label of the constellation symbol), and f (quadrature branch label of the constellation symbol), and we replace $\mathbf{b}_m(l)$ by w, for simplicity of notation. To avoid the explicit dependency of the decision borders on the channel estimate, the probability expression can be rewritten as

$$\begin{aligned}\text{Prob}\left(z_{\text{re}} < w\sum_{k=1}^{L}|\hat{\alpha}_k|^2|s\right) &= \text{Prob}\left(\text{Real}\left\{\sum_{k=1}^{L}(r_k - w\hat{\alpha}_k)\hat{\alpha}_k^*\right\} < 0|s\right)\\ &= \text{Prob}(z'_{\text{re}} < 0|s)\end{aligned} \quad (8)$$

where the modified variables $r'_k = r_k - w\cdot\hat{\alpha}_k$ and $z'_{\text{re}} = \sum_{k=1}^{L} z'_{\text{re}_k} = \sum_{k=1}^{L}\text{Real}\{r'_k\hat{\alpha}_k^*\}$ are defined. Since r_k and $\hat{\alpha}_k$ are complex random Gaussian variables and w is a constant, r'_k also has a Gaussian distribution. The second moment of r'_k, the cross moment of r'_k and $\hat{\alpha}_k$, and the respective cross-correlation coefficient are given by

$$\begin{aligned}E[|r'_k|^2\,|s] &= 2\,|s|^2\,\sigma_{\alpha_k}^2 + N_0 - 2w\,\text{Re}\left\{E\left[r_k\hat{\alpha}_k^*|s\right]\right\}\\ &\quad + w^2E[|\hat{\alpha}_k|^2]\end{aligned} \quad (9)$$

$$E\left[r'_k\hat{\alpha}_k^*|s\right] = E\left[r_k\hat{\alpha}_k^*|s\right] - wE[|\hat{\alpha}_k|^2] \quad (10)$$

$$\mu'_k = \frac{E\left[r'_k\hat{\alpha}_k^*|s\right]}{\sqrt{E[|r'_k|^2\,|s]E[|\hat{\alpha}_k|^2\,|s]}} = |\mu'_k|\,e^{-\varepsilon'_k\cdot j}. \quad (11)$$

We will first compute the pdf of $z'_{\rm re}$ conditioned on s for each individual reception branch. We start by writing the pdf of z'_k ($z'_k = r'_k \hat{\alpha}^*_k$) conditioned on s [14]

$$p\left(z'_k|s\right) = \frac{1}{2\pi\sigma^2_{\hat{\alpha}_k}\,\sigma^2_{r'_k}\left(1-|\mu'_k|^2\right)} \times \exp\left[\frac{|\mu'_k|\left(z'_{{\rm re}_k}\ \cos\,\varepsilon'_k + z'_{{\rm im}_k}\ \sin\,\varepsilon'_k\right)}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}\right] \times K_0\left[\frac{\sqrt{z'^2_{{\rm re}_k}+z'^2_{{\rm im}_k}}}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}\right] \quad (12)$$

where $K_0\left(\cdot\right)$ denotes the modified Hankel function of order zero and

$$\sigma^2_{r'_k} = \frac{E[|r'_k|^2|s]}{2}.$$

Integrating (12) over $z'_{{\rm im}_k}$ ($z'_{{\rm im}_k} = {\rm Imag}\left\{z'_k\right\}$) yields the marginal pdf of the decision variable $z_{{\rm re}'_k}$

$$p(z_{{\rm re}'_k}|s) = \int_{-\infty}^{+\infty} p\left(z'_k|s\right)\ dz_{{\rm im}'_k} = \frac{1}{F_k}\exp[G_k z_{{\rm re}'_k}]\exp[-H_k||z'_{{\rm re}_k}||] \quad (13)$$

where

$$F_k = 2\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\sqrt{1-|\mu'_k|^2\left(\sin\varepsilon'_k\right)^2},$$

$$G_k = \frac{|\mu'_k|\ \cos\,\varepsilon'_k}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}, \quad H_k = \frac{\sqrt{1-|\mu'_k|^2\left(\sin\ \varepsilon'_k\right)^2}}{\sigma_{r'_k}\,\sigma_{\hat{\alpha}_k}\left(1-|\mu'_k|^2\right)}.$$

The decision variable $z'_{\rm re}$ is the sum of L independent random variables with pdfs similar to (13). According to [15], the resulting pdf can be computed through the inverse Fourier transform of the product of the individual characteristic functions. The characteristic function of the pdf defined in (13) is obtained by applying the Fourier transform, which results in

$$\Psi_k\left(\upsilon j\right) = -\frac{2H_k}{F_k\left(\upsilon j - G_k - H_k\right)\left(\upsilon j - G_k + H_k\right)}. \quad (14)$$

If we divide the L diversity branches into L' different sets, where each set k contains ρ_k received branches with equal powers satisfying $L = \sum_{k=1}^{L'}\rho_k$, then the resulting characteristic function is given by the product of the individual characteristic functions, and can be written as

$$\Psi\left(\upsilon j\right) = \prod_{k=1}^{L'}\left(\frac{-2H_k}{F_k}\right)^{\rho_k} \times \frac{1}{\left(\upsilon j - G_k - H_k\right)^{\rho_k}\left(\upsilon j - G_k + H_k\right)^{\rho_k}}. \quad (15)$$

The pdf of $z'_{\rm re}$ is obtained by decomposing (15) as a sum of simple fractions (according to the method proposed in [16]) and applying the inverse Fourier transform. The desired probability can now be computed by integrating this pdf from $-\infty$ to 0 as follows

$${\rm Prob}\left(z'_{\rm re} < 0\right) = \int_{-\infty}^{0} p\left(z'_{\rm re}\right)dz'_{\rm re} = \prod_{k=1}^{L'}\left(\frac{-2H_k}{F_k}\right)^{\rho_k} \times \sum_{k=1}^{L'}\sum_{i=1}^{\rho_k}\left(-1\right)^i A^1_{k,i}\left(G_k + H_k\right)^{-i} \quad (16)$$

where

$$A^1_{k,i} = \frac{1}{\left(\rho_k - i\right)!}\left[\frac{\partial^{\rho_k - i}}{\partial s^{\rho_k - i}}\left(\prod_{\substack{j=1\\ j\neq k}}^{L'}\frac{1}{\left(s - G_j - H_j\right)^{\rho_j}} \times \prod_{j=1}^{L'}\frac{1}{\left(s - G_j + H_j\right)^{\rho_j}}\right)\right]_{s=G_k+H_k}. \quad (17)$$

If all the diversity branches have equal powers, i.e., $\rho_k = L$, then $L' = 1$ and F_k, G_k, and H_k are all equal (index k can be dropped) leading to

$${\rm Prob}\left(z'_{\rm re} < 0\right) = \frac{1}{\left(2FH\right)^L}\sum_{k=1}^{L}\binom{2L-k-1}{L-k}\left(\frac{2H}{G+H}\right)^k \quad (18)$$

which is the same result achieved in [6], though written in a different format. However, if all the diversity branches are different, i.e., $\rho_k = 1$ for any k and $L' = L$, then

$$A^1_{l,1} = \prod_{\substack{j=1\\ j\neq l}}^{L}\frac{1}{\left(G_l + H_l - G_j - H_j\right)^{\theta_j}} \times \prod_{j=1}^{L}\frac{1}{\left(G_l + H_l - G_j + H_j\right)^{\theta_j}} \quad (19)$$

and

$${\rm Prob}\left(z_{{\rm re}'} < 0\right) = \prod_{k=1}^{L}\left(\frac{-2H_k}{F_k}\right) \times \sum_{k=1}^{L}\frac{-A^1_{k,1}}{\left(G_k + H_k\right)}. \quad (20)$$

IV. Numerical Results

To verify the validity of the derived expressions, some simulations were run using the Monte Carlo method. The results obtained are plotted as a function of E_S/N_0 (E_S—symbol energy) in Figs. 3 and 4. Fig. 3 presents a nonuniform 16-QAM ($k = 0.3$) transmission with three equal branch diversity reception, while Fig. 4 presents a nonuniform 64-QAM ($k_1 = 0.4$ and $k_2 = 0.4$) transmission with four diversity branches and different relative powers ([0 dB -3 dB -6 dB -9 dB]). In both cases, the frame size is $N = 16$, the pilot symbols are $S_p = 1 + j$ (transmitted with the same power level of the data symbols), a sinc interpolator [11] is employed with $W = 15$, and

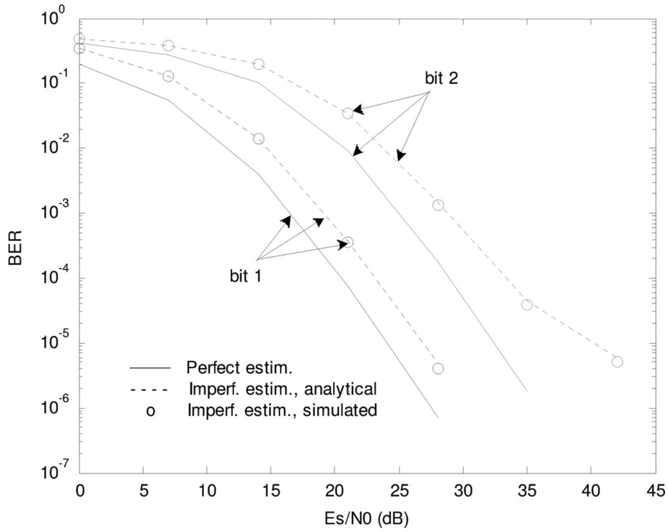

Fig. 3. Analytical and simulated BER performances of nonuniform 16-QAM ($k = 0.3$) with three equal diversity branches; $f_d T_s = 1.5 \times 10^{-2}$; sinc interpolation with $W = 15$ and $N = 16$.

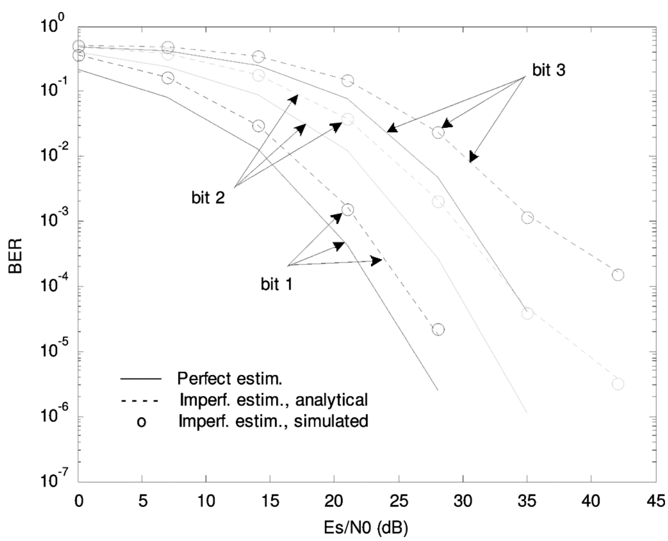

Fig. 4. Analytical and simulated BER performances of nonuniform 64-QAM ($k_1 = 0.4$ and $k_2 = 0.4$) with four diversity branches and different relative powers ([0 dB −3 dB −6 dB −9 dB]); $f_d T_s = 1.5 \times 10^{-2}$; sinc interpolation with $W = 15$ and $N = 16$.

Rayleigh fading is considered with $f_d T_s = 1.5 \times 10^{-2}$. The analytical results computed using expressions (7), (18), and (20), as well as curves corresponding to perfect channel estimation are drawn in both figures. In both cases, it is clear that the analytical results accurately match the simulated ones. We can also notice in both figures a considerable difference between the performance with perfect channel estimation and with imperfect channel estimation, the existence of irreducible BER floors (more noticeable in the least protected bit since they appear at higher BER values) being visible. This is a consequence of the effect of the time-varying fading channel that results in a reduced quality of the channel estimates. Moreover, these figures also show that the nonuniformity of the constellations clearly results in differentiated performances for the different bit classes.

V. Conclusion

In this paper, we have derived general analytical expressions for the evaluation of the exact BER performance for the individual bit classes of any nonuniform square M-QAM constellation, in the presence of imperfect channel estimation. These expressions can be applied to Rayleigh fading environments, with either equal or unequal receiving diversity branches, and MRC receivers.

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