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# IB-DFE receivers with space diversity for CP-assisted DS-CDMA and MC-CDMA systems

Rui Dinis1\*, Paulo Silva2 and António Gusmão3

<sup>1</sup>ISR-IST, Technical University of Lisbon, Portugal <sup>2</sup>ISR-IST/EST, University of Algarve, Portugal <sup>3</sup>CAPS-IST, Technical University of Lisbon, Portugal

#### SUMMARY

Multi-Carrier Code Division Multiple Access (MC-CDMA), currently regarded as a promising multiple access scheme for broadband communications, is known to combine the advantages of an Orthogonal Frequency Division Multiplexing (OFDM)-based, Cyclic Prefix (CP)-assisted block transmission with those of CDMA systems. Recently, it was recognised that DS-CDMA (Direct Sequence) implementations can also take advantage of the benefits of the CP-assisted block transmission approach, therefore enabling an efficient use of Fast Fourier Transform (FFT)-based, chip level Frequency-Domain Equalisation (FDE) techniques. When employing a linear FDE with both MC-CDMA and DS-CDMA, the FDE coefficients can be optimised under the Minimum Mean Squared Error (MMSE) criterion, so as to avoid significant noise enhancement. The residual interference levels can be very high, especially for fully loaded scenarios, since the FDE/MMSE does not perform a perfect channel inversion.

This paper deals with CP-assisted DS-CDMA systems and MC-CDMA systems with frequency-domain spreading. We consider the use of Iterative Block Decision Feedback Equalisation (IB-DFE) FDE techniques as an alternative to conventional, linear FDE techniques, and derive the appropriate IB-DFE parameters in a receiver diversity context. Our performance results show that IB-DFE techniques with moderate complexity allow significant performance gains in both systems, with bit error rate (BER) that can be close to the single-code matched filter bound (MFB) (especially for the CP-assisted DS-CDMA alternative), even with full code usage. Copyright © 2007 John Wiley & Sons, Ltd.

\* Correspondence to: Rui Dinis, IST, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisbon, Portugal. E-mail: rdinis@ist.utl.pt

## 1. INTRODUCTION

It is widely known that a Cyclic Prefix (CP)-assisted block transmission approach, allowing low-complexity Frequency-Domain Equalisation (FDE) receiver techniques, is suitable for high data rate transmission over severely time-dispersive channels. This approach can be employed with either multi-carrier (MC) or singlecarrier (SC) modulations [1, 2]. When adopted in CDMA systems, it leads to Multi-Carrier Code Division Multiple Access (MC-CDMA) implementations [3–5], and also, as recently recognised, quite efficient DS-CDMA implementations [6, 7]. These CP-assisted schemes are especially interesting for multicode and/or downlink transmission, taking advantage of synchronised, orthogonal spreading codes. In fact, since all spreading codes face the same channel, the multicode detection can be efficiently treated as an equalisation problem. Although these lead to suboptimum receiver designs, their complexity is much lower than the optimum receivers (whose complexity grows exponentially with the number of spreading codes [8–10]).

Conventional, linear FDE techniques are known to lead to a significant noise enhancement when a Zero Forcing (ZF) criterion is adopted for restoring orthogonality in channels with deep in-band notches. A simple frequency-

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3 domain matched filtering is also known to lead to a 4 very poor performance. For this reason, an Minimum 5 Mean-Squared Error (MMSE) FDE equaliser is usually 6 preferable [11]. However, an FDE/MMSE does not perform 7 an ideal channel inversion; therefore, when this type of 8 equaliser is employed within CP-assisted CDMA systems, 9 we are not able to keep the different spreading codes 10 fully orthogonalised. This means severe interference levels, 11 especially when different powers are assigned to different 12 codes.

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13 It is well-known that nonlinear equalisers can signif-14 icantly outperform linear equalisers. For this reason, a 15 promising Iterative Block Decision Feedback Equalisation 16 (IB-DFE) approach was proposed for CP-assisted SC 17 schemes [12], with both the feedforward and the feedback 18 parts implemented in the frequency domain (a similar 19 concept was also proposed in Reference [13]). An extension 20 of this approach to SC/FDE receivers with space diversity 21 was also shown to be feasible [14], allowing much better 22 performance than the conventional, linear SC/FDE receiver 23 approach. An appropriate extension to layered space-time 24 SC/FDE receivers for multiple antenna systems was also 25 developed [15, 16].

26 This paper deals with CP-assisted MC-CDMA systems, 27 with frequency-domain spreading, and DS-CDMA sys-28 tems, by considering the use of IB-DFE techniques in 29 space diversity receivers, as an alternative to conventional 30 linear FDE techniques. For MC-CDMA schemes, our 31 receiver design is related to the turbo receiver proposed 32 in Reference [17], although with a much lower signal 33 processing complexity since we do not employ the channel 34 decoder output in the feedback loop. Our receiver for 35 DS-CDMA is related to the turbo receiver proposed 36 in Reference [18], however, with much lower signal 37 processing complexity, especially when severely time-38 dispersive channels are considered due to the fact that 39 we are considering an Fast Fourier Transform (FFT)-40 based frequency-domain implementation (moreover, the 41 channel decoder output is required for the feedback loop in 42 Reference [18]). Moreover, we consider the receiver design 43 with L-order space diversity, while that was not considered 44 in References [17, 18].

45 The paper is organised as follows: Section 2 describes the 46 CP-assisted CDMA schemes to be considered and the basic 47 linear FDE principles. The IB-DFE receiver techniques 48 are addressed in Section 3, where their parameters are 49 derived. A set of performance results, in the CDMA context, 50 is presented in Section 4, and Section 5 is concerned 51 with the conclusions and complementary remarks of this 52 paper.

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2. CP-ASSISTED CDMA SYSTEMS WITH LINEAR FDE

In this section we describe the CP-assisted DS-CDMA and MC-CDMA systems to be considered, involving a multicode transmission with constant spreading factor (the extension to Variable Spreading Factor (VSF) schemes [19] is straightforward). In both cases, the receiver can be based on a linear FDE (see Figure 1A, where an L-branch space diversity receiver is considered). As with other CP-assisted techniques, after removing the cyclic extension, the received time-domain block associated to each diversity branch,  $\{y_n^{(l)}; n = 0, 1, \dots, N-1\}, l = 1, 2, \dots, L$ , is passed to the frequency domain, leading to the block  $\{Y_k^{(l)}; k =$ 0, 1, ..., N - 1}, with N denoting the length of the useful part of the block<sup>†</sup>. When the cyclic extension is longer than the overall channel impulse response, the samples  $Y_k^{(l)}$  can be written as

$$Y_k^{(l)} = H_k^{(l)} S_k + N_k^{(l)}$$
(1)

where  $H_k^{(l)}$  and  $N_k^{(l)}$  denote the channel frequency response and the noise term for the *k*th frequency and the *l*th diversity branch, respectively, and  $\{S_k; k = 0, 1, \dots, N-1\} = DFT$  $\{s_n; n = 0, 1, \dots, N-1\}, \text{ with } \{s_n; n = 0, 1, \dots, N-1\}$ 1} denoting the transmitted time-domain block. For a linear FDE, the frequency-domain samples at its output are given by

$$\tilde{S}_k = \sum_{l=1}^{L} F_k^{(l)} Y_k^{(l)}$$
(2)

where the set  $\{F_k^{(l)}; k = 0, 1, \dots, N-1\}$  denotes the FDE coefficients associated to the lth diversity branch. By setting

$$F_{k}^{(l)} = \frac{H_{k}^{*(l)}}{\sum_{l=1}^{L} \left| H_{k}^{(l)} \right|^{2}}$$
(3)

we could invert completely the channel effects (ZF criterion) while actually implementing an approximate Maximal Ratio Combining (MRC) rule. As an alternative, we could optimise these coefficients under the MMSE

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<sup>&</sup>lt;sup>†</sup> For MC-CDMA, N is the number of subcarriers; for DS-CDMA, N is the number of chips per block.



Figure 1. Linear FDE receiver structure with *L*-branch space diversity (A) and transmission models for MC-CDMA (B) and DS-CDMA (C).

(4)

criterion, leading to Reference [11]

 $F_{k}^{(l)} = \frac{H_{k}^{*(l)}}{\alpha + \sum_{l=1}^{L} \left| H_{k}^{(l)} \right|^{2}}$ 

where

$$\alpha = \frac{\sigma_{\rm N}^2}{\sigma_{\rm S}^2} \tag{5}$$

with  $\sigma_N^2$  denoting the variance of the noise terms (supposed to be identical in all diversity branches) and  $\sigma_S^2$  denoting the variance of the data symbols.

## 2.1. MC-CDMA

Let us consider an MC-CDMA scheme. The frequencydomain block to be transmitted is  $\{S_k; k = 0, 1, ..., N - 1\}$ , where N = KM, with K denoting the spreading factor and M the number of data symbols per spreading code. The frequency-domain symbols are given by

$$S_k = \sum_{p=1}^P \xi_p S_{k,p} \tag{6}$$

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where *P* is the number of spreading codes and  $\xi_p$  is an appropriate weighting coefficient for power control purposes (the power associated to the *p*th spreading code is proportional to  $\xi_p^2$ ). { $S_{k,p}$ ; k = 0, 1, ..., N - 1} is an interleaved version of { $S'_{k,p}$ ; k = 0, 1, ..., N - 1} (rectangular  $K \times M$  interleaver, so that different chips associated with a given data symbol are spaced by *M* subcarriers).

$$S'_{k,p} = C_{k,p} A_{\lfloor k/K \rfloor, p} \tag{7}$$

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is the *k*th chip for the *p*th spreading code ( $\lfloor x \rfloor$  denotes 'larger integer not higher that *x*'). { $A_{m,p}$ ; m = 0, 1, ..., M - 1} denotes the block of data symbols associated to the *p*th spreading code and { $C_{k,p}$ ; k = 0, 1, ..., N - 1} is the corresponding spreading sequence. An orthogonal spreading is assumed throughout this paper, with  $C_{k,p}$  belonging to a Quaternary Phase Shift Keying (QPSK) constellation. Without loss of generality, it is assumed that  $|C_{k,p}| = 1$ . At the receiver side, the  $A_{k,p}$  coefficients are estimated from

$$\tilde{A}_{m,p} = \sum_{k' \in \Psi_m} \tilde{S}_{k'} C^*_{k',p} \tag{8}$$

with  $\Psi_m = \{m, m + M, \dots, m + (K - 1)M\}$  denoting the set of frequencies employed to transmit the *m*th data symbol of each spreading code and  $\tilde{S}_k$  given by

Equation (2) (see Figure 1B). The data estimates  $\hat{A}_{m,p}$  are the 'hard decisions' associated to  $\tilde{A}_{m,p}$ .

# 2.2. DS-CDMA

Let us consider now a DS-CDMA scheme. The transmitted block of chips is  $\{s_n; n = 0, 1, ..., N - 1\}$ , once again N = KM, K is the spreading factor and M is the number of data symbols for each spreading code. The overall 'chip' symbols  $s_n$  are given by

$$s_n = \sum_{p=1}^{P} \xi_p s_{n,p} \tag{9}$$

where  $\xi_p$  is an weighting coefficient, proportional to the transmitted power for the *p*th user, and

S

$$a_{n,p} = c_{n,p} a_{\lfloor n/K \rfloor, p} \tag{10}$$

is the *n*th chip for the *p*th user.  $\{a_m; m = 0, 1, ..., M - 1\}$ denotes the block of data symbols associated to the *p*th user and  $\{c_{n,p}; n = 0, 1, ..., N - 1\}$  denotes the corresponding spreading sequence. As with MC-CDMA, an orthogonal spreading and  $|c_{n,p}| = 1$  are assumed.

In this case, the FDE receiver could estimate the data symbols from

$$\tilde{a}_{m,p} = \sum_{n'=mK}^{mK+K-1} \tilde{s}_{n'} c^*_{n',p}$$
(11)

with  $\{\tilde{s}_n; n = 0, 1, ..., N - 1\} = \text{IDFT}\{\tilde{S}_k; k = 0, 1, ..., N - 1\}$  (see Figure 1C). The data estimates  $\hat{a}_{m,p}$  are the 'hard decisions' associated to  $\tilde{a}_{m,p}$ .

## 3. ITERATIVE BLOCK DECISION FEEDBACK EQUALISATION FOR CP-ASSISTED CDMA

#### 3.1. Receiver structure

Figure 2 presents the receiver structures that we are considering in this paper, where the linear FDE is replaced



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by an IB-DFE. In both cases, for a given iteration i, the output samples are given by

$$\tilde{S}_{k}^{(i)} = \sum_{l=1}^{L} F_{k}^{(l,i)} Y_{k}^{(l)} - B_{k}^{(i)} \hat{S}_{k}^{(i-1)}$$
(12)

where  $\{F_k^{(l,i)}; k = 0, 1, ..., N-1\}$  (l = 1, 2, ..., L) and  $\{B_k^{(i)}; k = 0, 1, ..., N-1\}$  denote the feedforward and the feedback equaliser coefficients, respectively, optimised so as to maximise the 'overall signal-to-noise plus interference', as described in the following. The block  $\{\hat{S}_k^{(i-1)}; k = 0, 1, ..., N-1\}$  is an estimate of the transmitted block  $\{S_k^{(i-1)}; k = 0, 1, ..., N-1\}$ , obtained from the data estimates of the (i-1)th iteration,  $\{\hat{A}_{k,p}^{(i-1)}; k = 0, 1, ..., M-1\}$  in the MC-CDMA case and IDFT  $\{\hat{a}_{n,p}^{(i-1)}; n = 0, 1, ..., M-1\}$  in the DS-CDMA case, as in Equations (6) and (7) or Equations (9) and (10), respectively.

The data estimates are the hard decisions associated to the despreaded samples,  $\{\tilde{A}_{k,p}^{(i-1)}; k = 0, 1, ..., M - 1\}$ in the MC-CDMA case and  $\{\tilde{a}_{n,p}^{(i-1)}; n = 0, 1, ..., M - 1\}$ in the DS-CDMA case, and are given by Equations (8) and (11), respectively. It should be pointed out that 'soft estimates' could be employed in the feedback loop instead of 'hard estimates'; since the performances are similar, unless we use the channel decoder output in the feedback loop [20], we just considered 'hard estimates'.

## 3.2. Computation of the receiver parameters

If there were no intersymbol interference (ISI) at the output of the feedforward filter, the 'overall' channel frequency response  $\sum_{l=1}^{L} F_k^{(l,i)} H_k^{(l)}$  would be constant. Therefore, the ISI component in the frequency domain is associated to the difference between the average channel frequency response after the feedforward filter, defined as

$$\gamma^{(i)} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=1}^{L} F_k^{(l,i)} H_k^{(l)}$$
(13)

and its actual value. If we have reliable estimates of the transmitted block, the feedback filter can then be used to remove this residual ISI.

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Therefore, the equalised frequency-domain samples associated to each iteration,  $\tilde{S}_k^{(i)}$ , can be written as

$$\tilde{S}_{k}^{(i)} = \gamma^{(i)} S_{k} + \varepsilon_{k}^{\text{Eq}(i)} \tag{14}$$

where  $\varepsilon_k^{\text{Eq}(i)} = \tilde{S}_k^{(i)} - \gamma^{(i)}S_k$  denotes an overall error that includes both the channel noise and the residual ISI. In the same way, the corresponding time-domain samples can be written as

$$\tilde{s}_n^{(i)} = \gamma^{(i)} s_n + \varepsilon_n^{\text{eq}(i)} \tag{15}$$

where the block { $\varepsilon_n^{\text{eq}(i)}$ ; n = 0, 1, ..., N - 1} is the IDFT of the block { $\varepsilon_k^{\text{Eq}(i)}$ ; k = 0, 1, ..., N - 1}. The forward and backward IB-DFE coefficients,

The forward and backward IB-DFE coefficients,  $\{F_k^{(l,i)}; k = 0, 1, ..., N-1\}$  (l = 1, 2, ..., L) and  $\{B_k^{(i)}; k = 0, 1, ..., N-1\}$ , respectively, are chosen so as to maximise the 'signal-to-noise plus interference ratio', SNIR, defined as

$$\mathrm{SNIR}^{(i)} = \frac{|\gamma^{(i)}|^2 E\left[|S_k|^2\right]}{E\left[\left|\varepsilon_k^{\mathrm{Eq}(i)}\right|^2\right]}$$
(16)

The frequency-domain estimates,  $\hat{S}_k^{(i)}$ , can be written as

$$\hat{S}_{k}^{(i)} = \rho^{(i)} S_{k} + \Delta_{k}^{(i)}$$
(17)

where the correlation coefficient  $\rho^{(i)}$  is given by

$$\rho^{(i)} = \frac{E\left[\hat{s}_n^{(i)} s_n^*\right]}{E\left[|s_n|^2\right]} = \frac{E\left[\hat{S}_k^{(i)} S_k^*\right]}{E\left[|S_k|^2\right]}$$
(18)

and  $\Delta_k^{(i)}$  denotes a zero-mean error term. Since it is assumed that  $E[\Delta_k^{(i)}S_{k'}^{(i)*}] \approx 0$  for  $k' \neq k$ ,

$$E\left[\left|\Delta_{k}^{(i)}\right|^{2}\right] \approx \left(1 - \left(\rho^{(i)}\right)^{2}\right) E\left[|S_{k}|^{2}\right]$$
(19)

The coefficient  $\rho^{(i-1)}$ , which can be regarded as the blockwise reliability of the decisions used in the feedback loop (from the previous iteration), is crucial for the good performance of the proposed receivers, can be estimated from the samples  $\tilde{a}_{n,p}$  as described in the next subsection.

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By combining Equations (1), (12) and (17), we obtain

$$\tilde{S}_{k}^{(i)} = \sum_{l=1}^{L} F_{k}^{(l,i)} \left( H_{k}^{(l)} S_{k} + N_{k}^{(l)} \right) 
- B_{k}^{(i)} \left( \rho^{(i-1)} S_{k} + \Delta_{k}^{(i-1)} \right) 
= \gamma^{(i)} S_{k} + \left( \sum_{l=1}^{L} F_{k}^{(l,i)} H_{k}^{(l)} - \gamma^{(i)} - \rho^{(i-1)} B_{k}^{(i)} \right) S_{k} 
- B_{k}^{(i)} \Delta_{k}^{(i-1)} + \sum_{l=1}^{L} F_{k}^{(l,i)} N_{k}^{(l)}$$
(20)

This means that  $\tilde{S}_k^{(i)}$  has four terms: a 'signal' component,  $\gamma^{(i)}S_k$ , and three 'noise' components. The first component in the last equality of Equation (20) is the residual ISI, the second component accounts for the errors in  $\hat{s}_n^{(i-1)}$  and the final component is concerned to the channel noise.

The maximisation of the SNIR (16) is equivalent to the minimisation of

$$E\left[\left|\varepsilon_{k}^{\mathrm{Eq}(i)}\right|^{2}\right]$$

$$= E\left[\left|\sum_{l=1}^{L}F_{k}^{(l,i)}H_{k}^{(l)} - \gamma^{(i)} - \rho^{(i-1)}B_{k}^{(i)}\right|^{2}\right]E\left[|S_{k}|^{2}\right]$$

$$+ E\left[\left|B_{k}^{(i)}\Delta_{k}^{(i-1)}\right|^{2}\right] + E\left[\left|\sum_{l=1}^{L}F_{k}^{(l,i)}N_{k}^{(l)}\right|^{2}\right]$$

$$= E\left[\left|\sum_{l=1}^{L}F_{k}^{(l,i)}H_{k}^{(l)} - \gamma^{(i)} - \rho^{(i-1)}B_{k}^{(i)}\right|^{2}\right]2\sigma_{\mathrm{S}}^{2}$$

$$+ E\left[\left|B_{k}^{(i)}\right|^{2}\right]\left(1 - \left(\rho^{(i-1)}\right)^{2}\right)2\sigma_{\mathrm{S}}^{2}$$

$$+ \sum_{l=1}^{L}E\left[\left|F_{k}^{(l,i)}\right|^{2}\right]2\sigma_{\mathrm{N}}^{2}$$
(21)

50 conditioned to a given  $\gamma^{(i)}$ , where  $2\sigma_{\rm S}^2 = E[|S_k|^2]$ . 51 The optimum receiver coefficients can be obtained by 52 employing the Lagrangian multipliers method. For this 

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purpose, we can define the Lagrangian function

$$J = E\left[\left|\varepsilon_{k}^{\text{Eq}(i)}\right|^{2}\right] + \lambda^{(i)}\left(\frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=1}^{L}F_{k}^{(l,i)}H_{k}^{(l)} - 1\right)(22)$$

and assume that the optimisation is carried out under  $\gamma^{(i)} = 1$ . The optimum receiver coefficients are obtained by solving the following set of L + 2 equations

$$rac{\partial J}{\partial F_k^{(l,i)}}$$

$$= 4\sigma_{\rm S}^2 H_k^{(l)*} \left( \sum_{l'=1}^L F_k^{(l',i)} H_k^{(l')} - 1 - \rho^{(i-1)} B_k^{(i)} + \frac{\lambda^{(i)}}{2\sigma_{\rm S}^2 N} \right)$$

$$+4\sigma_{\rm N}^2 F_k^{(l,i)} = 0, \quad l = 1, 2, \dots, L$$
(23)

 $\frac{\partial J}{\partial P^{(i)}}$ 

$$= -4\sigma_{\rm S}^2 \rho^{(i-1)} \left( \sum_{l'=1}^{L} F_k^{(l',i)} H_k^{(l')} - 1 - \rho^{(i-1)} B_k^{(i)} \right)$$

$$+4\sigma_{\rm S}^2(1-(\rho^{(i-1)})^2)B_k^{(i)}=0 \tag{24}$$

and

$$\frac{\partial J}{\partial \lambda^{(i)}} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{l=1}^{L} F_k^{(l,i)} H_k^{(l)} - 1 = 0$$
(25)

As expected, Equation (25) is equivalent to  $\gamma^{(i)} = 1$ . The remaining equations can be rewritten in the form

$$H_k^{(l)*}\left(\sum_{l'=1}^L F_k^{(l',i)} H_k^{(l')} - 1 - \rho^{(i-1)} B_k^{(i)} + \frac{\lambda^{(i)}}{2\sigma_{\rm S}^2 N}\right)$$

$$+ \alpha F_k^{(l',i)} = 0, \quad l = 1, 2, \dots, L$$
 (26)

and

$$\rho^{(i-1)} \left( \sum_{l'=1}^{L} F_k^{(l',i)} H_k^{(l')} - 1 - \rho^{(i-1)} B_k^{(i)} \right)$$

$$= (1 - (\rho^{(i-1)})^2) B_k^{(i)}$$
(27)

with  $\alpha$  given by Equation (5).

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 From Equation (27), the optimum values of  $B_k^{(i)}$  are

$$B_k^{(i)} = \rho^{(i-1)} \left( \sum_{l'=1}^{L} F_k^{(l',i)} H_k^{(l')} - 1 \right)$$
(28)

By substituting Equation (28) in Equation (26), we get the set of L equations

$$\left(1 - (\rho^{(i-1)})^2\right) H_k^{(l)*} \sum_{l'=1}^L F_k^{(l',i)} H_k^{(l')} + \alpha F_k^{(l,i)}$$

$$= \left(1 - (\rho^{(i-1)})^2 - \frac{\lambda^{(i)}}{2\sigma_{\rm S}^2 N}\right) H_k^{(l)*}, \quad l = 1, 2, \dots, L$$

$$(29)$$

It can be easily verified by substitution that the solutions of Equation (26) are

$$F_{k}^{(l,i)} = \frac{K_{F}^{(l)} H_{k}^{(l)*}}{\alpha + (1 - (\rho^{(i-1)})^{2}) \sum_{l'=1}^{L} \left| H_{k}^{(l')} \right|^{2}}, \quad l = 1, 2, \dots, L$$
(30)

where the normalisation constant

$$K_F^{(i)} = 1 - \left(\rho^{(i-1)}\right)^2 - \frac{\lambda^{(i)}}{2\sigma_{\rm S}^2 N} \tag{31}$$

ensures that  $\gamma^{(i)} = 1$ .

These feedforward coefficients can be used in Equation (28) for obtaining the feedback coefficients  $B_k^{(i)}$ .

Clearly, for the first iteration (i = 0), no information exists about  $S_k$  and the correlation coefficient in Equation (30) is zero. This means that  $B_k^{(0)} = 0$  and

$$F_{k}^{(l,0)} = \frac{K_{F}^{(0)} H_{k}^{(l)*}}{\alpha + \sum_{l'=1}^{L} \left| H_{k}^{(l')} \right|^{2}}, \quad l = 1, 2, \dots, L \quad (32)$$

corresponding to the optimum frequency-domain equaliser coefficients under the MMSE criterion [11, 21]. After that first iteration, if the residual bit error rate (BER) is not too high (at least for the spreading codes with higher transmit power), we can use the feedback coefficients to eliminate a significant part of the residual interference.

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When  $\rho \approx 1$  (after several iterations and/or moderateto-high SNRs), we have an almost full cancellation of the 'inter-code' interference through these coefficients, while the feedforward coefficients perform an approximate matched filtering.

It should be noted that, when L = 1 (no diversity) the IB-DFE parameters derived above become identical to those given in Reference [12]. It should also be noted that the feedforward coefficients can take the form

$$F_k^{(l,i)} = H_k^{(l)*} G_k^{(i)}, \quad l = 1, 2, \dots, L$$
(33)

with

$$G_{k}^{(i)} = \frac{K_{F}^{(i)}}{\alpha + (1 - (\rho^{(i-1)})^{2}) \sum_{l'=1}^{L} \left| H_{k}^{(l')} \right|^{2}}$$
(34)

This means that the bank of feedforward filters can be replaced by a bank of matched filters which implement an ideal MRC, followed by a single feedforward filter characterised by the set of coefficients  $\{G_k^{(i)}; k = 0, 1, \dots, N-1\}$ .

# 3.3. Calculation of $\rho_p$

In this subsection we show how one can obtain an estimate of the correlation coefficient. Assuming uncorrelated data blocks, it can be easily shown that

$$\rho^{(i-1)} = \sum_{p=1}^{P} \xi_p^2 \rho_p^{(i-1)} \tag{35}$$

with

$$\rho_{p}^{(i-1)} = \frac{E\left[\hat{A}_{k,p}A_{k,p}^{*}\right]}{E\left[|A_{k,p}|^{2}\right]} = \frac{E\left[\hat{a}_{n,p}a_{n,p}^{*}\right]}{E\left[|a_{n,p}|^{2}\right]}$$
(36)

denoting the correlation coefficient associated to the *p*th user. For a DS-CDMA scheme  $\rho_p^{(i)}$  can be obtained as follows (a similar approach could be employed for MC-CDMA schemes).

Let us assume that the transmitted symbols  $a_{n,p}$  belong to a QPSK constellation (the generalisation to other constellations is straightforward). In this case,

$$a_{n,p} = a_{n,p}^{\mathrm{I}} + ja_{n,p}^{\mathrm{Q}} = \pm d \pm jd$$
 (37)

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(38)

(42)

where  $a_{n,p}^{I} = \operatorname{Re}\{a_{n,p}\}$  and  $a_{n,p}^{Q} = \operatorname{Im}\{a_{n,p}\}$  are the inphase and quadrature components of  $a_{n,p}$ , respectively, and d = D/2, with D corresponding to the minimum Euclidean distance (for the sake of simplicity, in the following we will ignore the dependency with the iteration number *i*). In this case,

 $E\left[|a_{n,p}|^2\right] = \frac{D^2}{4}$ For an unbiased FDE ( $\gamma = 1$ ), the time-domain samples

at the output of the FDE are

$$\tilde{a}_{n,p} = \tilde{a}_{n,p}^{I} + j\tilde{a}_{n,p}^{Q} = a_{n,p} + v_{n,p}$$
(39)

where  $\tilde{a}_{n,p}^{I} = \operatorname{Re}\{\tilde{a}_{n,p}\}, \ \tilde{a}_{n,p}^{Q} = \operatorname{Im}\{\tilde{a}_{n,p}\}$  and  $\nu_{n,p}$  is the overall noise component. We will assume that  $v_{n,p}$  is approximately Gaussian-distributed<sup>‡</sup>, with  $E[v_{n,p}] = 0$ . Moreover, the SNR for detection purposes is

$$\text{SNIR}_{p}^{\text{eq}} = \frac{E\left[|a_{n,p}|^{2}\right]}{E\left[|v_{n,p}|^{2}\right]} = \frac{P|\xi_{p}|^{2}}{\sum\limits_{p'=1}^{P}|\xi_{p'}|^{2}}\text{SNIR}$$
(40)

with SNIR given by Equation (16), that is  $SNIR_p$  is higher for the users with higher assigned power.

The symbol estimates can be written as

$$\hat{a}_n = a_{n,p} + \varepsilon_{n,p}^{\mathrm{I}} + j\varepsilon_{n,p}^{\mathrm{Q}} \tag{41}$$

where the error coefficients  $\varepsilon_{n,p}^{I}$  (or  $\varepsilon_{n,p}^{Q}$ ) are zero if there is no error in  $a_{n,p}^{I}$  (or  $a_{n,p}^{Q}$ ) and  $\pm D$  otherwise. This means that  $\varepsilon_{n,p}^{I}$  and  $\varepsilon_{n,p}^{Q}$  are random variables, both taking the values 0 and  $\pm D$  with probabilities  $1 - P_{e,p}$  and  $P_{e,p}$ , respectively. Therefore,

where  $P_{e,p}$  denotes the BER associated to the *p*th user, which can be approximated by

 $\rho_p = 1 - 2P_{e,p}$ 

$$P_{e,p} \approx Q\left(\sqrt{\mathrm{SNIR}_p^{\mathrm{eq}}}\right) \tag{43}$$

for QPSK constellations.

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## 4. PERFORMANCE RESULTS

In this section we present a set of performance results concerning the proposed receiver structure. We consider the downlink transmission, with each spreading code intended to a given user. It is assumed that N = 256 (similar results could be obtained for other high values of N) and the data symbols are selected from a QPSK constellation under a Gray mapping rule. For both DS-CDMA and MC-CDMA, we consider an orthogonal spreading (Hadamard-Walsh sequences plus pseudo-random scrambling sequences with the same chip rate) and the power amplifier at the transmitter is assumed to be linear. The radio channel is characterised by the power delay profile type C for HIPERLAN/2 (HIgh PERformance Local Area Network) [22], with uncorrelated Rayleigh fading on the different paths. The subcarrier separation is 0.2 MHz. Perfect synchronisation and channel estimation are assumed in all cases. The number of users is P = K, that is we are assuming a fully loaded system. For the sake of comparisons, we included the Matched Filter Bound (MFB) performance, defined as

$$P_{b,\text{MFB}} = E\left[Q\left(\sqrt{\frac{2E_{b}}{N_{0}}}\frac{1}{N}\sum_{k}\sum_{l}\left|H_{k}^{(l)}\right|^{2}}\right)\right] \quad (44)$$

where the expectation is taken over a large number of channels and  $E[|H_k^{(l)}|^2] = 1$ . For MC-CDMA systems, the optimum single user (SU) performance, achievable with a simple MRC receiver, is given by

$$P_{b,\mathrm{SU}} = E\left[Q\left(\sqrt{\frac{2E_{\mathrm{b}}}{N_{0}}\frac{1}{N}\sum_{k\in\Psi_{m}}\sum_{l}\left|H_{k}^{(l)}\right|^{2}}\right)\right] \quad (45)$$

where the expectation is over all channel realisations and all data symbols. Clearly,  $P_{b,MFB} = P_{b,SU}$  when K = N; for K < N,  $P_{b,SU}$  is typically worse than  $P_{b,MFB}$ .

Let us first assume that there is no power control at the BS, that is all users have the same power (this means that  $\xi_p$  is constant).

In Figure 3 we compare semi-analytical BER values, given by Equation (43), with simulated ones for a DS-CDMA system with N = K = 256 and L = 1 (similar behaviours were observed for MC-CDMA systems). Clearly, the semi-analytical BER values are very close to the simulated ones for the first iteration; for the remaining iterations, the theoretical values are slightly optimistic. (For L > 1 the semi-analytical BER values are even closer to the simulated ones.) Figure 4 shows the evolution of

<sup>‡</sup>This assumption is reasonable under severely time-dispersive channel conditions.

10<sup>0</sup>





Figure 3. Semi-analytical (dashed line) and simulated (solid line) BER results when L = 1, for a given number of iterations.

the correlation factor  $\rho$ , together with the corresponding estimates (given by Equation (42)), using an estimated BER obtained from the SNIR, as in Equation (43). Clearly, the  $\rho$ estimates are very close to the true  $\rho$  values for the first iteration. When the number of iterations is increased,  $\rho$ becomes slightly overestimated when the noise levels are high. For moderate-to-low noise levels, the  $\rho$  estimates are still very accurate. The high accuracy of the  $\rho$  estimates is a consequence of the approximated BER values given by Equation (43) being close to the true ones (see also Figure 3).

Figures 5 and 6 concern MC-CDMA and DS-CDMA schemes, respectively, once again with N = K = 256.



Figure 4. Evolution of  $\rho$ , when L = 1 (semi-analytical (dashed line) and simulated (solid line) results).

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Figure 5. MC-CDMA BER performance when K = 256 (M = 1) and P = 256 users, with the same assigned power.

Clearly, the iterative procedure allows a significant improvement relatively to the conventional linear FDE (first iteration). Moreover, the achievable performance is close to the MFB after three iterations. It was also observed that the performance is similar for MC-CDMA and DS-CDMA schemes. This is not surprising, since for K = N all the available bandwidth is used to transmit each data symbols in both cases.

Let us consider now that K = P = 256 and the power assigned to K/2 = 128 users is 6 dB below the power assigned to the other K/2 = 128 users. Clearly, the lowpower users face strong interference levels. Figure 7



Figure 6. DS-CDMA BER performance when K = 256 (M = 1) and P = 256 users, with the same assigned power.

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Figure 7. DS-CDMA BER performance with K/2 = 128 lowpower users and K/2 = 128 high-power users. 

presents the BER for a DS-CDMA system (similar results were observed for MC-CDMA systems), expressed as a function of the  $E_{\rm b}/N_0$  of high-power users (6 dB below the  $E_{\rm b}/N_0$  of low-power users). Once again, the iterative receiver allows significant performance improvements. From this figure, it is clear that performance gains associated to the iterative procedure are higher for low-power users and the corresponding BERs are closer to the MFB than for high-power users (the performance of high-power users are still a few dB from the MFB after three iterations). This is explained as follows: the BER is much lower for high-power users, allowing an almost perfect interference cancellation



Figure 8. Average BER performance when K = 16 (M = 64) and P = 16 users, with the same assigned power.

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 $10^{\circ}$ - : DS-CDMA : MC-CDMA · : MFB 딾 10<sup>-</sup>

10-

Figure 9. BER performances for a Fourier spreading (with no scrambling), when N = K = 256 and P = 256 spreading codes, with the same assigned power.

 $E_{b}/N_{0}(dB)$ 

Iter.

of their effects on low-power users; the higher BERs for the low-power users preclude an appropriate interference cancellation when we detect high-power users.

It should also be noted that, for K < N, the performance of MC-CDMA schemes is worse, since just a fraction 1/Mof the frequencies is used for the transmission of a given data symbol. This is not the case of DS-CDMA, where all frequencies can be used for transmitting each data symbol, regardless of the spreading factor. For instance, Figure 8 concerns the case where K = 16 (i.e. M = 64), the same power is assigned to all spreading sequences and we have P = 16 users (i.e. a fully loaded system). Although the iterative procedure allows gains of about 2 dB, the achievable performance is similar with two or three iteration, and still far from the MFB and the SU performance (the SU performance is slightly worse that the MFB when K < N).

However, it should be noted that this does not mean necessarily a weakness of the MC-CDMA schemes with small spreading factors (small K). The comparison between DS-CDMA and MC-CDMA schemes should take into



Figure 10. Transmission models for N = K = P = 256 and Fourier spreading/despreading.

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Q2 95

Q4

Q3 91

Q4

Q2 65

Q3 63

1

account other aspects, such as the envelope fluctuations of the transmitted signals and the impact of the channel coding (one might expect larger coding gains for MC-CDMA schemes, especially when a small *K* is combined with interblock interleaving).

## 5. CONCLUSIONS AND COMPLEMENTARY REMARKS

In this paper we considered the use of IB-DFE techniques for CP-assisted DS-CDMA and MC-CDMA systems. With these IB-DFE techniques, the results of the first iteration correspond to those of the conventional, linear, FDE/MMSE technique; the subsequent iterations provide a performance enhancement, thanks to the iterative cancellation of residual interference. Since the feedback loop takes into account not just the hard decisions for each block, but also an 'overall block reliability', the error propagation problem is significantly reduced. Therefore, the proposed receivers have excellent performance, that can be close to the MFB performance, especially for DS-CDMA schemes. Moreover, their implementation is much less complex than that of receivers employing frequency-domain turbo-equalisation [23].

It should be noted that the type of spreading adopted can have a significant impact on the performance of CP-assisted CDMA schemes. As an extreme example (see Figure 9), for M = 1, full code usage under equal power conditions and a Fourier spreading/despreading with no complementary scrambling, the MC-CDMA scheme considered in this paper is equivalent to a CP-assisted SC scheme [24] (see Figure 10), and our receiver reduces to the IB-DFE receiver described in Reference [14]. On the other hand, for M = 1, full code usage under equal power conditions and a Fourier spreading/despreading with no complementary scrambling, the DS-CDMA scheme considered in this paper is equivalent to an Orthogonal Frequency Division Multiplexing (OFDM) scheme (then there is no advantage in using the IB-DFE receiver).

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## **AUTHORS' BIOGRAPHIES**

**Rui Dinis** received his Ph.D. from Instituto Superior Técnico (IST), Technical University of Lisbon, Portugal, in 2001. Since 2001, he has been a Professor at IST. He was a member of the research centre CAPS/IST (Centro de Análise e Processamento de Sinais) from 1992 to 2005. Since 2005, he is a member of the research centre ISR/IST (Instituto de Sistemas e Robótica). He has been involved in several research projects in the broadband wireless communications area. His main research interests include modulation, equalisation and channel coding.

Paulo Silva received his M.Sc. degree from Instituto Superior Tcnico (IST), Technical University of Lisbon, Portugal, in 1999. He is now preparing his Ph.D. thesis at IST. Since 1999, he has been a Professor with Escola Superior de Tecnologia (EST), University of Algarve, Portugal. He was a member of the research centre CAPS/IST (Centro de Análise e Processamento de Sinais) from 1995 to 2005. Since 2005 he is a member of the research centre ISR/IST (Instituto de Sistemas e Robótica). His main research interests concern spread spectrum techniques and multiuser detection.

António Gusmão received his Ph.D. from Instituto Superior Técnico (IST), Technical University of Lisbon, Portugal, in 1987. Since 1987, he has been a Professor at IST, with both teaching and research activities in the digital communications area, on which he has published several tens of journal and conference papers. He has also been a member of the Digital Communications Group of CAPS/IST and the leader of that group since 1998. Since 1992, he has been involved in European projects concerning the development of broadband wireless communication systems. His main research interests concern modelling and simulation of communication systems, modulation and coded modulation issues and signal processing for broadband wireless communications.

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