# PATH-FOLLOWING CONTROL OF FULLY-ACTUATED SURFACE VESSELS IN THE PRESENCE OF OCEAN CURRENTS [1]

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Abstract: This paper addresses the problem of driving a surface vessel along a desired spatial path. A nonlinear adaptive controller is designed that yields convergence of the trajectories of the closed loop system to the path in the presence of constant unknown ocean currents and parametric model uncertainty. Controller design builds on Lyapunov based techniques and backstepping. The controller derived implicitly compensates for the effect of the ocean current without the need for direct measurements of its velocity. An illustrative simulation example is presented and discussed. *Copyright © 2007 IFAC*

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## 1. INTRODUCTION

Motion control is the key building block in the development of any control architecture for autonomous vehicles involved in the execution of realistic mission scenarios. Motion control systems must yield good performance in the presence of external disturbances and plant uncertainty. This is specially relevant in the case of marine vehicles operating in harsh environments, under the influence of wind, waves, and currents.

This paper addresses a specific problem of motion control that is commonly referred to as path following. Its main contribution is the derivation of a nonlinear, adaptive controller for a fully-actuated marine vessel to steer it along a desired spatial path in the presence of unknown, constant ocean currents and parametric model uncertainty.

In what follows we consider a general class of vessels that can be modelled by equations of the form (see Section 2)

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1) + \boldsymbol{G}_1(\boldsymbol{x}_1)\boldsymbol{x}_2 + \boldsymbol{\phi}_1(\boldsymbol{x}_1)\boldsymbol{b}_1 \,, \tag{1a}$$

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(\boldsymbol{x}_1,\boldsymbol{x}_2) + \boldsymbol{G}_2(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{u} + \boldsymbol{\phi}_2(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{b}_2 \,, \tag{1b}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}_1,\boldsymbol{x}_2) \,, \tag{1c}$$

where $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ are components of the state, $\boldsymbol{u}$ is the control input, the functions $\boldsymbol{f}_1$, $\boldsymbol{f}_2$, and the maps $\boldsymbol{G}_1$, $\boldsymbol{G}_2$ are smooth, and $\boldsymbol{b}_1$, $\boldsymbol{b}_2$ are unknown constant biases or disturbances. It is assumed that $\boldsymbol{G}_1(\boldsymbol{x}_1)$ and $\boldsymbol{G}_2(\boldsymbol{x}_1,\boldsymbol{x}_2)$ are nonsingular for all $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, and that all variables are of the same dimension, that is, $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{u}, \boldsymbol{b}_1, \boldsymbol{b}_2 \in \mathbb{R}^n$. The plant is in strict feedback form (Krstić *et al.* 1995) and represents a class of fully-actuated $n$-degrees-of-freedom ($n$-DOF) mechanical systems. The biases $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are unknown in the sense that their time evolution is neither known in advance, nor

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can it be measured by sensors mounted on the vessel. Equations (1a) and (1b) can be regarded as the *kinematics* and *dynamics* of the vessel, respectively, and (1c) represents the output of the system. Since $u$ has dimension equal to the degrees of freedom of the vessel, the vehicle is said to be fully-actuated.

One particular type of adaptive control problem for model (1) consists in computing a state feedback controller of the form

$$\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \hat{\boldsymbol{b}}), \qquad \text{(control law)} \quad (2)$$

$$\dot{\hat{\boldsymbol{b}}} = \text{function of } (\boldsymbol{x}, \hat{\boldsymbol{b}}), \qquad \text{(adaptation law)} \quad (3)$$

(where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$, $\boldsymbol{b} = (\boldsymbol{b}_1, \boldsymbol{b}_2)$, and $\hat{\boldsymbol{b}}(t) \in \mathbb{R}^{2n}$ denotes the estimate of $\boldsymbol{b}$) whose goal is to drive $\boldsymbol{y}(t)$ to zero or a small neighbourhood of the origin, while keeping all closed-loop signals bounded. For the case of surface vessels, the term $\boldsymbol{b}_1$ captures the influence of the ocean current at a kinematic level, while $\boldsymbol{b}_2$ represents unknown disturbances.

To compensate for the effect of constant unknown biases, it is common practice in linear time-invariant control systems design to include an extra integrator state for each unknown bias since this will guarantee zero steady-state error (under the assumption that the closed-loop system is stable). However, for general nonlinear systems a straightforward extension of this technique is not available, and the *adhoc* inclusion of integral action in a feedback controller may lead to unwanted oscillations in the closed-loop system response (Skjetne and Fossen 2004).

In (Skjetne *et al.* 2005), the effect of ocean currents is modelled as a disturbance at the dynamic level ($\boldsymbol{b}_1$ is considered zero). In this set-up, even a constant current will show up as a time-varying disturbance in the body-axis dynamic equations of motion. For this reason, in this paper an ocean current is modelled as a constant unknown bias in the kinematic equations of motion of the vessel.

The above approach to current modeling is pursued in (Aguiar and Pascoal 2002), where a nonlinear adaptive controller is presented for the problem of dynamic positioning and way-point tracking of an underactuated AUV (in the horizontal plane) in the presence of a constant unknown ocean current disturbance and parametric model uncertainty. An exponential kinematic observer is designed for the current velocity and convergence of the resulting closed-loop system trajectories is proved. The present work departs from that of (Aguiar and Pascoal 2002) in that an adaptive backstepping strategy is used, rather than an observer-based approach.

Other related work can be found in (Refsnes *et al.* 2005), where a strategy for tracking control and station keeping of a 6-DOF fully-actuated vehicle is proposed. A more general type of dynamic disturbance is considered, to take into account slowly varying current forces and moments, and unmodelled dynamics. The bias is modelled as a random process driven by zero-mean Gaussian white noise. The compensation of the bias involves the use of an observer and ensures exponential stability for the closed-loop system.

One particular type of motion control involves steering vehicles to and along desired paths without specifying a temporal law. This is known as the *path-following* control problem. As shown in (Encarnação and Pascoal 2001, Skjetne *et al.* 2005), the solution to this problem is commonly divided into two tasks: a geometric task, where the vehicle is required to converge and remain on the desired path; a dynamic task, that specifies a time, speed, or acceleration assignment along the path. In this paper, the path-following problem is solved for a class of fully-actuated vessels with 3-DOF, subjected to an unknown ocean current (considered at a kinematic level) and parametric model uncertainty, using adaptive nonlinear control based on Lyapunov techniques and backstepping. The same methodology can be employed for the case of 6-DOF autonomous underwater vehicles, as long as the fully-actuated condition is verified.

The paper is organised as follows. Section 2 describes the dynamic model of the autonomous vehicles considered. The problem of path-following is formally stated in Section 3. In Section 4, a strategy for path-following is developed. Section 5 gives an illustrative example where simulation results are presented. Finally, in Section 6 conclusions and directions for future work are presented.

## 2. VEHICLE MODELING

A surface vessel is modelled as a rigid body subject to external forces and torques. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame with its origin at the centre of mass of the vehicle, as represented in Fig. 1. The generalized position of the vehicle is $\boldsymbol{\eta} := (x, y, \psi)$, where $(x, y)$ are the coordinates of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$ and $\psi$ is the orientation of vehicle (yaw angle) that parameterizes the matrix $J(\psi)$, transforming body coordinates into inertial coordinates, given by

$$J := J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Denote by $\boldsymbol{\nu} := (u, v, r)$ the generalized velocity of the vehicle relative to $\{\mathcal{I}\}$, expressed in $\{\mathcal{B}\}$. In general, the fluid is in motion. To take into account this motion, let $\boldsymbol{\nu}_f = (u_f, v_f, r_f)$ be the generalized velocity of the fluid relative to $\{\mathcal{I}\}$

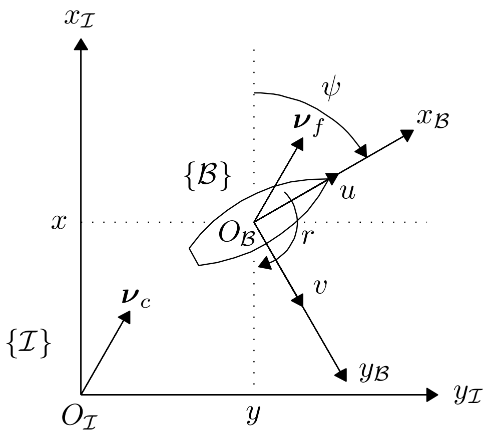


Fig. 1. Inertial and body-fixed coordinate frames.

expressed in $\{\mathcal{B}\}$. Because the fluid is assumed to be irrotational, $r_f = 0$. Let $\boldsymbol{\nu}_c = (u_c, v_c, 0)$ be the velocity of the ocean current, expressed in $\{\mathcal{I}\}$. The velocities $\boldsymbol{\nu}_f$ and $\boldsymbol{\nu}_c$ are related by $\boldsymbol{\nu}_f = J^\top \boldsymbol{\nu}_c$. Assuming the ocean current is constant, $\dot{\boldsymbol{\nu}}_c = 0$. Let $\boldsymbol{\nu}_r := \boldsymbol{\nu} - \boldsymbol{\nu}_f$ denote the relative velocity between the vessel and the fluid. The following kinematic relations apply:

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \,, \tag{4a}$$

$$\dot{J} = rJS \,, \tag{4b}$$

where $S$ is the skew-symmetric matrix

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; ; \; (S^\top = -S) \,.$$

In what follows, we consider a fully-actuated vehicle with equations of motion of the form (Fossen 1994),

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{f}(\boldsymbol{\nu}_r) \tag{5}$$

where $M \in \mathbb{R}^{3\times 3}$, $M \succ 0$ denotes a constant symmetric positive definite mass matrix, $C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r$ captures Coriolis and centripetal effects, $\boldsymbol{\tau} := (\tau_u, \tau_v, \tau_r)$ is the generalized control input consisting of forces $\tau_u, \tau_v$ and torque $\tau_r$, and $\boldsymbol{f}(\boldsymbol{\nu}_r)$ represents the hidrodynamic damping forces and torques acting on the body. For the special case of surface vessels, $M$ includes also the so-called hydrodynamic added-mass $M_A$, i.e., $M = M_{RB} + M_A$, where $M_{RB}$ is the rigid-body mass matrix. Equation (5) depends on a set of physical parameters, some of which are often only known with great uncertainty. In fact, while the coefficients of $M$ and $C(\boldsymbol{\nu}_r)$ can be determined with reasonable accuracy, some of the coefficients related to the hydrodynamic damping $\boldsymbol{f}(\boldsymbol{\nu}_r)$ are difficult to measure, so they should be considered unknown or uncertain and estimated by the adaptive controller to be designed. Let $n_p$ be the total number of hydrodynamic coefficients or parameters, and let $\boldsymbol{\varphi} \in \mathbb{R}^{n_p}$ be the vector that represents them. It is assumed that these parameters are constant ($\dot{\boldsymbol{\varphi}} = \mathbf{0}$) and that $\boldsymbol{f}(\boldsymbol{\nu}_r)$ depends linearly on $\boldsymbol{\varphi}$. This implies that $\boldsymbol{f}(\boldsymbol{\nu}_r)$ can be expressed as $\Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi}$ with $\Phi(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_p}$. Therefore, (5) can be rewritten as

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi} \,. \tag{6}$$

Define $\boldsymbol{\varphi}_k \in \mathbb{R}^{n_k}$ as the vector of known parameters and $\boldsymbol{\varphi}_u \in \mathbb{R}^{n_u}$ as the vector of uncertain or unknown parameters, such that $n_p = n_k + n_u$, and

$$\Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi} = \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u \,, \tag{7}$$

where $\Phi_k(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_k}$ and $\Phi_u(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_u}$. Replacing (7) in (6) yields

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u \,.$$

To summarise, the equations of motion of a vessel with parametric model uncertainty, subject to an ocean current, are

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \,, \tag{8a}$$

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u \,. \tag{8b}$$

System (8) belongs to the class of systems that can be written as in (1) by making $\boldsymbol{x}_1 = \boldsymbol{\eta}$ and $\boldsymbol{x}_2 = \boldsymbol{\nu}_r$. In this case, one has $\boldsymbol{\phi}_1(\boldsymbol{x}_1) = I_3$, $\boldsymbol{b}_1 = \boldsymbol{\nu}_c$, $\boldsymbol{\phi}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \Phi_u(\boldsymbol{\nu}_r)$, and $\boldsymbol{b}_2 = \boldsymbol{\varphi}_u$.

## 3. PROBLEM STATEMENT

Before stating the path-following problem, define the estimate of the ocean current, expressed in $\{\mathcal{I}\}$, as $\hat{\boldsymbol{\nu}}_c := (\hat{u}_c, \hat{v}_c, 0)$, where only the first two components need to be estimated since the fluid is assumed irrotacional. The estimates of the uncertain parameters are represented by $\hat{\boldsymbol{\varphi}} \in \mathbb{R}^{n_u}$. The adaptation laws for $\hat{\boldsymbol{\nu}}_c$ and $\hat{\boldsymbol{\varphi}}$ are chosen as functions of state variables, in order to guarantee that the vehicle follows the desired path. It is not required that the estimates converge to the actual values of the associated variables. The problem of path-following is stated formally as follows:

*Path-following problem*: Let $\boldsymbol{\eta}_d(\gamma) \in \mathbb{R}^3$ be a desired path parameterized by a continuous variable $\gamma \in \mathbb{R}$ and $v_d(\gamma) \in \mathbb{R}$ a desired speed assignment. Suppose also that $\boldsymbol{\eta}_d(\gamma)$ is sufficiently smooth and its derivatives (with respect to $\gamma$) are bounded. Design a control law for $\boldsymbol{\tau}$ and adaptation laws for $\hat{\boldsymbol{\nu}}_c$ and $\hat{\boldsymbol{\varphi}}$, such that: all closed-loop signals are bounded; the position of the vehicle converges to the desired path, i.e, $\|\boldsymbol{\eta}(t) - \boldsymbol{\eta}_d(\gamma(t))\| \to 0$ as $t \to +\infty$ (geometric task); and the vehicle satisfies the desired speed assignment along the path, i.e, $|\dot{\gamma}(t) - v_d(\gamma(t))| \to 0$ as $t \to +\infty$ (dynamic task).

## 4. PATH-FOLLOWING

The path-following controller developed here is inspired by the work in (Encarnação and Pascoal 2001) and (Skjetne *et al.* 2005). However, it goes one step further in that it allows for the consideration of unknown ocean currents together with parametric model uncertainty. Backstepping techniques are used to derive the controller, by iteratively introducing control-Lyapunov functions (Krstić *et al.* 1995).

*Step 1. Coordinate transformation*: Define the position error in the body-fixed frame as $\boldsymbol{z}_1 := J^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d)$ and the current estimation error as $\tilde{\boldsymbol{\nu}}_c := \boldsymbol{\nu}_c - \hat{\boldsymbol{\nu}}_c$. The dynamic equation of $\boldsymbol{z}_1$ is

$$\begin{aligned}\dot{\boldsymbol{z}}_1 &= \dot{J}^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d) + J^\top(\dot{\boldsymbol{\eta}} - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) \\ &= -rSJ^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d) + J^\top(J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) \\ &= -rS\boldsymbol{z}_1 + \boldsymbol{\nu}_r + J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) + J^\top\tilde{\boldsymbol{\nu}}_c\,, \quad (9)\end{aligned}$$

where $\boldsymbol{\eta}_d^\gamma = \partial\boldsymbol{\eta}_d/\partial\gamma$. Let $\zeta := \dot{\gamma} - \upsilon_d(\gamma)$ represent the "along-path" speed tracking error and rewrite (9) as

$$\dot{\boldsymbol{z}}_1 = -rS\boldsymbol{z}_1 + \boldsymbol{\nu}_r + J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\upsilon_d) + J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta). \quad (10)$$

*Step 2. Convergence of $\boldsymbol{z}_1$*: Define a first control-Lyapunov function as

$$V_1 := \frac{1}{2}\boldsymbol{z}_1^\top\boldsymbol{z}_1\,,$$

whose time derivative is

$$\dot{V}_1 = \boldsymbol{z}_1^\top\left(\boldsymbol{\nu}_r + J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\upsilon_d)\right) + \boldsymbol{z}_1^\top J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta), \quad (11)$$

where the fact that $\boldsymbol{z}_1^\top S\boldsymbol{z}_1 = 0$ for all $\boldsymbol{z}_1$ is used. Define the velocity error $\boldsymbol{z}_2 := \boldsymbol{\nu}_r - \boldsymbol{\alpha}$, where $\boldsymbol{\alpha} := -J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\upsilon_d) - K_1\boldsymbol{z}_1$ and rewrite (11) as

$$\dot{V}_1 = -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 + \boldsymbol{z}_1^\top\boldsymbol{z}_2 + \boldsymbol{z}_1^\top J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta)\,.$$

*Step 3. Backstepping for $\boldsymbol{z}_2$*: First, the time derivative of $\boldsymbol{\alpha}$ is decomposed in three terms

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\sigma} + \boldsymbol{\alpha}^\gamma\dot{\gamma} - K_1J^\top\tilde{\boldsymbol{\nu}}_c\,,$$

where the functions $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}^\gamma$ are defined as

$$\begin{aligned}\boldsymbol{\sigma} &:= -K_1(\boldsymbol{\nu}_r - rS\boldsymbol{z}_1 + J^\top\hat{\boldsymbol{\nu}}_c) \\ &\quad + rSJ^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\upsilon_d) - J^\top\dot{\hat{\boldsymbol{\nu}}}_c\,, \quad (12) \\ \boldsymbol{\alpha}^\gamma &:= K_1J^\top\boldsymbol{\eta}_d^\gamma + J^\top(\boldsymbol{\eta}_d^{\gamma^2}\upsilon_d + \boldsymbol{\eta}_d^\gamma\upsilon_d^\gamma)\,, \quad (13)\end{aligned}$$

with $\boldsymbol{\eta}_d^{\gamma^2} = \partial^2\boldsymbol{\eta}_d/\partial\gamma^2$ and $\upsilon_d^\gamma = \partial\upsilon_d/\partial\gamma$. Notice that $\boldsymbol{\sigma}$ depends on $\dot{\hat{\boldsymbol{\nu}}}_c$ that is still undefined. The dynamic equation of $\boldsymbol{z}_2$ is then

$$\begin{aligned}M\dot{\boldsymbol{z}}_2 &= \boldsymbol{\tau} - C\boldsymbol{\nu}_r + \Phi_k\boldsymbol{\varphi}_k + \Phi_u\boldsymbol{\varphi}_u - M\left(\boldsymbol{\sigma} + \boldsymbol{\alpha}^\gamma\dot{\gamma}\right) \\ &\quad + MK_1J^\top\tilde{\boldsymbol{\nu}}_c\,.\end{aligned}$$

The uncertain parameters estimation error is defined as $\tilde{\boldsymbol{\varphi}} := \boldsymbol{\varphi}_u - \hat{\boldsymbol{\varphi}}$. Define a second control-Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{z}_2^\top M\boldsymbol{z}_2 = \frac{1}{2}\boldsymbol{z}_1^\top\boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M\boldsymbol{z}_2\,,$$

whose time derivative is

$$\begin{aligned}\dot{V}_2 &= -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 + \boldsymbol{z}_1^\top J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta) + \tilde{\boldsymbol{\varphi}}^\top\Phi_u^\top\boldsymbol{z}_2 \\ &\quad + \boldsymbol{z}_2^\top\big(\boldsymbol{z}_1 + \boldsymbol{\tau} - C\boldsymbol{\nu}_r + \Phi_k\boldsymbol{\varphi}_k + \Phi_u\hat{\boldsymbol{\varphi}} \quad (14) \\ &\quad - M(\boldsymbol{\sigma} + \boldsymbol{\alpha}^\gamma\upsilon_d)\big) - \boldsymbol{z}_2^\top M\boldsymbol{\alpha}^\gamma\zeta + z_2^\top MK_1J^\top\tilde{\boldsymbol{\nu}}_c.\end{aligned}$$

Using the feedback law

$$\boldsymbol{\tau} = -\boldsymbol{z}_1 - K_2\boldsymbol{z}_2 + C\boldsymbol{\nu}_r - \Phi_k\boldsymbol{\varphi}_k - \Phi_u\hat{\boldsymbol{\varphi}} + M(\boldsymbol{\sigma} + \boldsymbol{\alpha}^\gamma\upsilon_d) \quad (15)$$

with $K_2 \succ 0$, and substituting in (14), yields

$$\dot{V}_2 = -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2\boldsymbol{z}_2 + \boldsymbol{\rho}^\top\tilde{\boldsymbol{\nu}}_c + \boldsymbol{\pi}^\top\tilde{\boldsymbol{\varphi}} + \mu\zeta, \quad (16)$$

where the following auxiliary functions are used: $\mu = -(\boldsymbol{\eta}_d^\gamma)^\top J\boldsymbol{z}_1 - (\boldsymbol{\alpha}^\gamma)^\top M\boldsymbol{z}_2$, $\boldsymbol{\pi} = \Phi_u^\top\boldsymbol{z}_2$ and $\boldsymbol{\rho} = J(\boldsymbol{z}_1 + K_1M\boldsymbol{z}_2)$.

*Step 4. Feedback law for $\ddot{\gamma}$*: Define a third control-Lyapunov function

$$V_3 := V_2 + \frac{1}{2\lambda\beta}\zeta^2 = \frac{1}{2}\boldsymbol{z}_1^\top\boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M\boldsymbol{z}_2 + \frac{1}{2\lambda\beta}\zeta^2, \quad (17)$$

where $\lambda, \beta > 0$. Its time derivative is

$$\begin{aligned}\dot{V}_3 &= -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2\boldsymbol{z}_2 + \boldsymbol{\rho}^\top\tilde{\boldsymbol{\nu}}_c + \boldsymbol{\pi}^\top\tilde{\boldsymbol{\varphi}} \\ &\quad + \left(\frac{1}{\lambda\beta}\dot{\zeta} + \mu\right)\zeta\,. \quad (18)\end{aligned}$$

Making

$$\dot{\zeta} = -\lambda\zeta - \lambda\beta\mu \quad (19)$$

and replacing in (18), yields

$$\dot{V}_3 = -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2\boldsymbol{z}_2 + \boldsymbol{\rho}^\top\tilde{\boldsymbol{\nu}}_c + \boldsymbol{\pi}^\top\tilde{\boldsymbol{\varphi}} - \frac{1}{\beta}\zeta^2. \quad (20)$$

Taking into account that $\dot{\zeta} = \ddot{\gamma} - \upsilon_d^\gamma\dot{\gamma}$, the feedback control law for $\ddot{\gamma}$ becomes

$$\ddot{\gamma} = -\lambda\zeta - \lambda\beta\mu + \upsilon_d^\gamma\dot{\gamma}. \quad (21)$$

*Step 5. Adaptation laws for $\hat{\boldsymbol{\nu}}_c$ and $\hat{\boldsymbol{\varphi}}$*: Define a fourth control-Lyapunov function as

$$\begin{aligned}V_4 &= V_3 + \frac{1}{2}\tilde{\boldsymbol{\nu}}_c^\top\Sigma^+\tilde{\boldsymbol{\nu}}_c + \frac{1}{2}\tilde{\boldsymbol{\varphi}}^\top\Gamma^{-1}\tilde{\boldsymbol{\varphi}} \\ &= \frac{1}{2}\boldsymbol{z}_1^\top\boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M\boldsymbol{z}_2 + \frac{1}{2\lambda\beta}\zeta^2 + \frac{1}{2}\tilde{\boldsymbol{\nu}}_c^\top\Sigma^+\tilde{\boldsymbol{\nu}}_c \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{\varphi}}^\top\Gamma^{-1}\tilde{\boldsymbol{\varphi}}\,, \quad (22)\end{aligned}$$

where $\Gamma \succ 0$ and $\Sigma^+$ is the pseudoinverse of

$$\Sigma = \begin{bmatrix}\Sigma_1 & 0 \\ 0 & 0\end{bmatrix}$$

with $\Sigma_1 \in \mathbb{R}^{2\times 2}$ and $\Sigma_1 \succ 0$. The last row and column of $\Sigma$ are both zero because the last component of $\boldsymbol{\nu}_c$ is always zero (irrotacional fluid). From $\dot{\tilde{\boldsymbol{\nu}}}_c = -\dot{\hat{\boldsymbol{\nu}}}_c$ and $\dot{\tilde{\boldsymbol{\varphi}}} = -\dot{\hat{\boldsymbol{\varphi}}}$, the time derivative of (22) becomes

$$\begin{aligned}\dot{V}_4 &= -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2\boldsymbol{z}_2 - \frac{1}{\beta}\zeta^2 \\ &\quad + \tilde{\boldsymbol{\nu}}_c^\top\left(\boldsymbol{\rho} - \Sigma^+\dot{\hat{\boldsymbol{\nu}}}_c\right) + \tilde{\boldsymbol{\varphi}}^\top\left(\boldsymbol{\pi} - \Gamma^{-1}\dot{\hat{\boldsymbol{\varphi}}}\right). \quad (23)\end{aligned}$$

Using the laws

$$\begin{aligned}\dot{\hat{\boldsymbol{\nu}}} &= \Sigma\boldsymbol{\rho}\,, \quad (24a) \\ \dot{\hat{\boldsymbol{\varphi}}} &= \Gamma\boldsymbol{\pi}\,, \quad (24b)\end{aligned}$$

in (23) yields

$$\dot{V}_4 = -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2\boldsymbol{z}_2 - \frac{1}{\beta}\zeta^2\,. \quad (25)$$

As noted before, the function $\boldsymbol{\sigma}$ depends on $\dot{\hat{\boldsymbol{\nu}}}_c$. Replacing (24a) in (12) gives

$$\boldsymbol{\sigma} = -K_1(\boldsymbol{\nu}_r - rS\boldsymbol{z}_1 + J^\top\hat{\boldsymbol{\nu}}_c) + rSJ^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma \upsilon_d) - J^\top\Sigma\boldsymbol{\rho}\,.$$

Using (22) as candidate Lyapunov function, which has a negative semidefinite time derivative (25), and resorting to LaSalle's invariance principle (see, e.g., (Khalil 2002)), it is possible to prove global asymptotic stability of the closed-loop system. This is formally stated next.

*Theorem 1.* The controller formed by the control law (15) and the adaptation laws (24), solves the path-following problem. Moreover, $\hat{\boldsymbol{\nu}}_c(t)$ tends to $\boldsymbol{\nu}_c$ asymptotically.

**PROOF.** Let $\boldsymbol{q} := (\boldsymbol{z}_1, \boldsymbol{z}_2, \tilde{\boldsymbol{\nu}}_c, \zeta)$ and $\boldsymbol{z} := (\boldsymbol{q}, \tilde{\boldsymbol{\varphi}})$. Showing that $\boldsymbol{q}$ tends asymptotically to zero and that $\tilde{\boldsymbol{\varphi}}$ is bounded (in fact it tends to a constant value) for the closed-loop system, proves the theorem. The closed-loop system dynamics of $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are

$$\dot{\boldsymbol{z}}_1 = -rS\boldsymbol{z}_1 + \boldsymbol{z}_2 - K_1\boldsymbol{z}_1 - J^\top\boldsymbol{\eta}_d^\gamma\zeta + J^\top\tilde{\boldsymbol{\nu}}_c\,, \tag{26a}$$

$$M\dot{\boldsymbol{z}}_2 = -\boldsymbol{z}_1 - K_2\boldsymbol{z}_2 - M\boldsymbol{\alpha}^\gamma\zeta + MK_1J^\top\tilde{\boldsymbol{\nu}}_c + \Phi_u\tilde{\boldsymbol{\varphi}}\,. \tag{26b}$$

Let $V(\boldsymbol{z}) = V_4(\boldsymbol{z}_1, \boldsymbol{z}_2, \tilde{\boldsymbol{\nu}}_c, \tilde{\boldsymbol{\varphi}}, \zeta)$, where $V_4(\cdot)$ is defined in (22), be a candidate Lyapunov function. The function $V : \mathbb{R}^p \to \mathbb{R}$, where $p = 9 + n_u$ ($\tilde{\boldsymbol{\nu}}_c$ has dimension two), is a differentiable, radially unbounded and positive definite function. The time derivative of $V(\boldsymbol{z})$ is given by (25) which is a negative semidefinite function. Therefore, it can be concluded that the closed-loop system is stable, that is, all variables are bounded. To guarantee convergence of $\boldsymbol{q}$ to zero, it is necessary to analyse the set of points where $\dot{V}(\boldsymbol{z})$ is zero. Let $\Omega_c = \{\boldsymbol{z} \in \mathbb{R}^p : V(\boldsymbol{z}) \le c\}$ be a sublevel set of $V(\boldsymbol{z})$. Because $\dot{V}(\boldsymbol{z}) \le 0$ and $V(\boldsymbol{z})$ is radially unbounded, the set $\Omega_c$ is compact and positively invariant, i.e., a solution that starts in $\Omega_c$ stays in $\Omega_c$ for all $t \ge 0$. Let $\mathcal{E} = \{\boldsymbol{z} \in \Omega_c : \dot{V}(\boldsymbol{z}) = 0\} = \{\boldsymbol{z} \in \Omega_c : \boldsymbol{z}_1 = \boldsymbol{z}_2 = \boldsymbol{0} \wedge \zeta = 0\}$ be the set of points where $\dot{V}(\boldsymbol{z})$ is zero, and let $\boldsymbol{z}(t)$ be a solution that belongs to $\mathcal{E}$. By definition, $\boldsymbol{z}_2(t) \equiv \boldsymbol{0}$ implies $\boldsymbol{\nu}_r \equiv \boldsymbol{\alpha}$. Also, $\boldsymbol{z}_1(t) \equiv \boldsymbol{0}$ implies $\dot{\boldsymbol{z}}_1(t) \equiv \boldsymbol{0}$. Replacing in (26a), yields $J^\top\tilde{\boldsymbol{\nu}}_c(t) \equiv \boldsymbol{0} \Rightarrow \tilde{\boldsymbol{\nu}}_c(t) \equiv \boldsymbol{0}$ because rank $J = 3$. On the other hand, $\boldsymbol{z}_2(t) \equiv \boldsymbol{0}$ implies $\dot{\boldsymbol{z}}_2(t) \equiv \boldsymbol{0}$. Replacing in (26b), gives $\Phi_u(\boldsymbol{\alpha}(t))\tilde{\boldsymbol{\varphi}}(t) \equiv \boldsymbol{0}$. Using (24b), yields $\dot{\hat{\boldsymbol{\varphi}}}(t) \equiv \boldsymbol{0}$ which implies $\hat{\boldsymbol{\varphi}}(t) \equiv$ constant. Therefore, $\tilde{\boldsymbol{\varphi}}(t) \equiv$ constant. Hence, only solutions that belong to the set $\mathcal{M} = \{\boldsymbol{z} \in \Omega_c : \boldsymbol{q} = \boldsymbol{0} \wedge \tilde{\boldsymbol{\varphi}} = \boldsymbol{a}\}$, where $\boldsymbol{a} \in \mathbb{R}^{n_u}$ is a constant vector that verifies $\Phi_u(\boldsymbol{\alpha}(t))\boldsymbol{a} = \boldsymbol{0}$, can remain identically in $\mathcal{E}$. Notice that $\boldsymbol{a}$ is not necessarily the zero vector because $\Phi_u(\boldsymbol{\alpha}(t))$ may not be full rank.

Table 1. Physical parameters.

| Type of parameter | Symbol | Value | Units |
|---|---|---|---|
| Mass* | $m_u$ | 421.8 | Kg |
| | $m_v$ | 1008.1 | Kg |
| Moment of Inertia* | $I_r$ | 690.5 | $\mathrm{Kgm}^2$ |
| Hydrodynamic | $X_u$ | −0.5 | Kgm/s |
| Damping | $X_{\lvert u\rvert u}$ | −0.5 | Kg/m |
| | $Y_{\lvert r\rvert v}$ | −339.0 | Kg |
| | $Y_{vv}$ | −121.2 | Kg/m |
| | $N_r$ | −0.26 | $\mathrm{Kgm}^2/\mathrm{s}$ |
| | $N_{\lvert r\rvert r}$ | −1764.2 | $\mathrm{Kgm}^2$ |

*added mass terms included.

Using (Khalil 2002, Theorem 4.4), we conclude that any solution starting in $\Omega_c$ approximates $\mathcal{M}$ when $t \to \infty$, hence $\|\boldsymbol{q}(t)\| \to 0$ when $t \to \infty$. Because $V(\boldsymbol{z})$ is radially unbounded, the result is global, since given an initial state $\boldsymbol{z}(t_0)$, the constant $c$ can be chosen arbitrarily large so that $\boldsymbol{z}(t_0) \in \Omega_c$. Since $\|\boldsymbol{\eta}(t) - \boldsymbol{\eta}_d(\gamma(t))\| = \|\boldsymbol{z}_1(t)\| \to 0$ and $|\dot{\gamma}(t) - \upsilon_d(\gamma(t))| = |\zeta(t)| \to 0$, the path-following problem is solved. Moreover, because $\tilde{\boldsymbol{\nu}}_c(t)$ tends to zero asymptotically, the estimate $\hat{\boldsymbol{\nu}}_c(t)$ tends asymptotically to $\boldsymbol{\nu}_c$. □

## 5. AN ILLUSTRATIVE EXAMPLE

Consider a vehicle whose equations of motion can be written as in (8), with

$$M = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_v & 0 \\ 0 & 0 & I_r \end{bmatrix},\; C(\boldsymbol{\nu}_r) = \begin{bmatrix} 0 & -m_v r & 0 \\ m_u r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{\varphi} = \left(X_u, X_{|u|u}, Y_{|r|v}, Y_{vv}, N_r, N_{|r|r}\right),$$

$$\Phi(\boldsymbol{\nu}_r) = \begin{bmatrix} u_r & |u_r|u_r & 0 & 0 & 0 & 0 \\ 0 & 0 & |r|v_r & v_r v_r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & |r|r \end{bmatrix}^\top.$$

Matrices $\Phi_k$ and $\Phi_u$ are partitions of $\Phi$ compatible with the choice of uncertain parameters $\boldsymbol{\varphi}_u = (X_{|u|u}, Y_{|r|v}, Y_{vv}, N_{|r|r})$. The physical parameters of the vessel are given in Table 1. In the simulation presented, the desired path is a lemniscate centered at the origin and has "width" $a = 40\,\mathrm{m}$. The path-following controller gains are $K_1 = 0.04I_3$, $K_2 = 200I_3$, $\lambda = 0.05$ and $\beta = 0.01$. The initial conditions of the vehicle are $(x, y, \psi) = (30\,\mathrm{m}, 0, \pi/2\,\mathrm{rad})$ and $u = v = r = 0$ (vehicle at rest). The desired speed assignment is given by $\upsilon_d(\gamma) = 0.5 - 0.2\sin(2\gamma/a)[\mathrm{m/s}]$. The ocean current is set to $\boldsymbol{\nu}_c = (1, -1, 0)[\mathrm{m/s}]$. The adaptation gains are $\Sigma = \mathrm{diag}(10^{-3}, 10^{-3}, 0)$ and $\Gamma = \mathrm{diag}(10, 10^4, 2\times10^2, 10^7)$. The estimates of both the current and the parameters are initialised with a 30% error around their true values. Fig. 2 illustrates the trajectory of each vehicle. As can be seen, the vehicle converges to the desired path, thus compensating the effect of the ocean current. This is corroborated in Fig. 3 where the position and orientation errors given by $\|p(t) - p_d(\gamma(t))\|$ and $\psi(t) - \psi_d(\gamma(t))$, respectively, converge asymptotically to zero. As mentioned before, the esti-

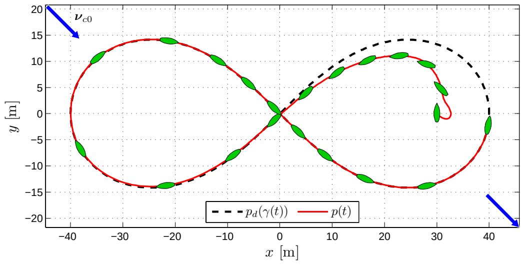


Fig. 2. Desired and actual vehicle trajectory.

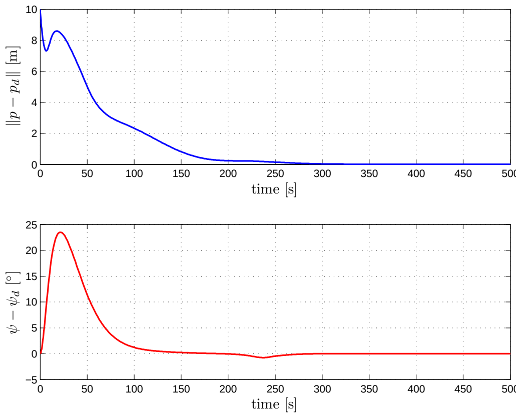


Fig. 3. Temporal evolution of the position and orientation errors.

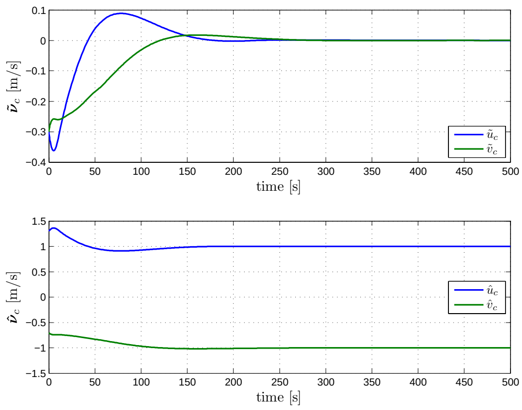


Fig. 4. Temporal evolution of the ocean current's estimation error and estimate.

mate of the ocean current converges asymptotically to its actual value as shown in Fig. 4. Fig. 5 shows the temporal evolution of the (signed) relative parameter estimation error $\tilde{\varphi}^{\text{rel}} = [\tilde{\varphi}_i^{\text{rel}}]_{n_u \times 1}$, where each component is defined as

$$\tilde{\varphi}_i^{\text{rel}} := \frac{\tilde{\varphi}_i}{\varphi_{ui}} \times 100\,[\%] \;\; \text{for} \;\; i = 1, 2, \ldots, n_u \,.$$

As noted earlier, the estimation errors of some uncertain parameters do not tend to zero.

## 6. CONCLUSIONS AND FUTURE WORK

A nonlinear adaptive control law was derived for a class of fully-actuated vehicles in the presence of constant unknown ocean currents and parametric model uncertainty. Controller design was rooted in Lyapunov based techniques and backstepping. Convergence of the closed-loop system was formally proved and an illustrative example with simulation results was presented and discussed. Further work is required to extend the results to the case of time-varying ocean currents and to underactuated vehicles.

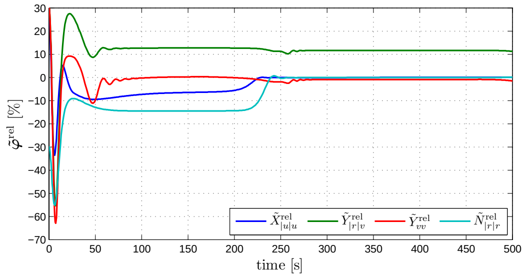


Fig. 5. Temporal evolution of the relative parameter estimation error.

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