Background Subtraction Based on Rank Constraint for Point Trajectories

Aamir Ahmad, Alessio Del Bue, Pedro Lima
ISR,Instituto Superior Técnico
Av. Rovisco Pais 1, Lisboa, Portugal
{aahmad,adb,pal}@isr.ist.utl.pt

Abstract

This work deals with a background subtraction algorithm for a fish-eye lens camera having 3 degrees of freedom, 2 in translation and 1 in rotation. The core assumption in this algorithm is that the background is considered to be composed of a dominant static plane in the world frame. The novelty lies in developing a rank-constraint based background subtraction for equidistant projection model, a property of the fish-eye lens. A detail simulation result is presented to support the hypotheses explained in this paper.

1. Introduction

One of the most crucial efficiency parameter for image processing algorithms today, is computation time. Often a major chunk of computation time is lost in processing unnecessary pixels which usually are part of the background in an image or frame. Background is usually defined as that part of the world which is static for a long period of time and cannot be labeled as an object of interest. Background Subtraction(BS) is extensively dealt in literature with varying scenarios and conditions but the major constraint common to majority of such algorithms is that the camera is assumed to be fixed with respect to world. Some of the algorithms consider the minor movements caused in the fixed camera due to vibrations. Quite recently, researchers have started to explore the idea of having a BS algorithm for roto-translating cameras. In [1] Yaser *et al.* present a rank-constraint based BS method for cameras having orthographic projection model. The novelty in our work is developing a rank constraint on image streams for equidistant projection and then applying it for BS as Yaser *et al.* do in [1]. Yaser *et al.* use the rank constraint for orthographic projection developed by Tomasi *et al.* in [2]. The striking difference between orthographic and equidistant projection is that the former is affine while the later is a non-linear transformation.

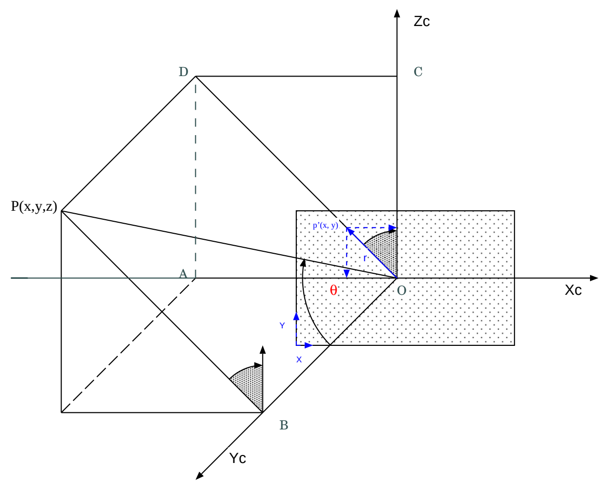


Figure 1. Equidistant Projection

In Section 2, we explain the equidistant projection model and camera & robot setup. In Section 3, the rank constraint method is explained followed by simulation results in Section 4. A comment on future work is made in Section 5.

2. Projection Model & Reference Frames

2.1 Equidistance Projection Model

In order to understand our approach it is essential to overview equidistant projection model. We also explain the frames of reference and the transformations among those frames continuously used throughout the paper.

In equidistant projection model, a point $P(x, y, z)$ in a 3-D world co-ordinate system is projected on the image plane according to (1)

$$r = f\Theta \tag{1}$$

where r is the distance of projected point from the image center, f is the focal length of the fish eye lens and Θ (in radians) is the angle which the ray joining the point $P(x, y, z)$

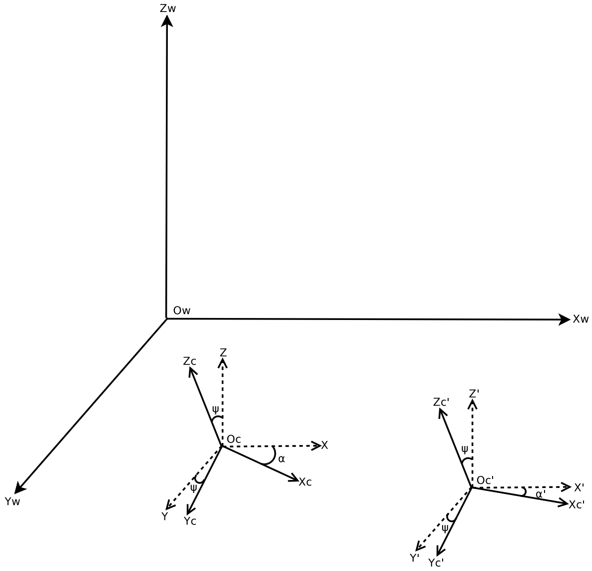


Figure 2. Reference Frame Transformations

in camera frame and the optical center makes with the optical axis of the lens. In Figure 1, the angle θ is the angle between $\vec{PO}$ and $\vec{OY_c}$

The point $P(x, y, z) \equiv (Px, Py, Pz)$ in camera frame when projected as $p'(x, y) \equiv (p_x, p_y)$ in camera's image frame according to the model equation (1), can be expressed by (2) and (3). We assume that the X and Y axes of image frame coincide with the X and Z axes of camera frame as illustrated in Figure 1.

$$p_x = f \cdot (\arctan \frac{\sqrt{Px^2 + Pz^2}}{Py}) \cdot \frac{Px}{\sqrt{Px^2 + Pz^2}} \quad (2)$$

$$p_y = f \cdot (\arctan \frac{\sqrt{Px^2 + Pz^2}}{Py}) \cdot \frac{Pz}{\sqrt{Px^2 + Pz^2}} \quad (3)$$

2.2 Reference Frames and Transformations

In our approach, the camera is mounted on top of an omni directional robot which roto-translates on a fixed ground plane $Z_w = -k$. The position of the camera is such that Y_c (Y axis of camera frame, which also coincides with the optical axis of the lens) always makes a constant angle Ψ (positional constraint) with the XY plane of the World frame as shown in Figure 2. The center of the camera reference frame always lies on the fixed world plane $Z_w = 0$. The X axis of the camera frame, lying always on the XY plane of the world frame makes a varying angle α with X axis of world frame, corresponding to robot rotation about the Z axis of robot which is always parallel to Z_w.

Points in world frame $P(x, y, z)_w$ when expressed in the camera frame $P'(x, y, z)_c$, are obtained by a translation of origin to camera co-ordinate's origin $O_c(x, y, 0)$ followed by $\alpha\, rad$ rotation about Z axis and eventually $\Psi\, rad$ rotation about X axis.

$$P'(x, y, z)_c = R_z(-\alpha) \cdot R_x(\Psi) \cdot (P(x, y, z)_w - O_c(x, y, 0)) \quad (4)$$

3. Rank Constraint Method

A fixed set of P points are randomly selected on the ground plane $Z_w = -k$, the dominant background plane in the world on which the robot roto-translates. The initial position of the robot and hence the camera is set by randomly choosing the camera focal point $O_c(x, y, 0)$ on $X - Y$ plane on world frame and the angle α. Angle Ψ is fixed to a given value (0.7853982 rad in our experiment). The points are then expressed in the camera frame according to (4) and subsequently projected in camera image frame according to (2) and (3). The next robot position and hence the camera position is obtained according to the trajectory it follows by roto-translating on the ground plane giving rise to a new focal point position and the angle α. Hence the whole process of co-ordinate transformation by (4) and projection by (2) and (3) is repeated for the new camera frame. This is done for a total of F consecutive frames, which eventually gives rise to a set of 2-D trajectories in the moving image frame, one for each point in the initial set P.

A trajectory formed by a point i in the set P can be represented as $\mathbf{w}_i = [x_{1i}^T ... x_{Fi}^T] \in R^{1 \times 2F}$, where $x_{fi} = [p_{x_{fi}}\ p_{y_{fi}}]^T$ in each frame f. The set of these trajectories give rise to a $2F$ x P trajectory matrix henceforth called $W_{2F \times P}$.

$$W_{2F \times P} = [\mathbf{w}_1^T\ \mathbf{w}_2^T\ ....\mathbf{w}_P^T] = \begin{bmatrix} p_{x_{11}} & \cdot & \cdot & p_{x_{1P}} \\ p_{y_{11}} & \cdot & \cdot & p_{y_{1P}} \\ \cdot & \cdot & \cdot & \cdot \\ p_{x_{F1}} & \cdot & \cdot & p_{x_{FP}} \\ p_{y_{F1}} & \cdot & \cdot & p_{y_{FP}} \end{bmatrix} \quad (5)$$

We show that the matrix W is highly rank deficient. Experimentally we find that the rank is 6 for $F > 3$ and $P > 6$.

4. Simulation Results

According to the setup explained in Section 2 we artificially generate trajectories for the robot and hence the camera to construct the matrix W. In simulation 1, the camera focal point $O_c(x, y, 0)$ translates 2 units in each direction (X_w and Y_w) per frame while the angle α increases $1°$ per frame. In simulation 2, the camera focal point $O_c(x, y, 0)$

#	TM in Simulation 1	TM in Simulation 2
1	8561.343377	9139.599419
2	265.144898	273.098248
3	10.636111	6.259409
4	0.512902	0.315992
5	0.015853	0.007201
6	0.000932	0.001058

Table 1. Singular Values for Trajectory Matrix

translates 2 units in X_w direction per frame while the angle α increases 1° per frame. Subsequently we applied singular value decomposition(SVD) method to obtain the rank of W in simulation. Since the number of non-zero singular values of Σ (diagonal matrix with non-negative real numbers on the diagonal) obtained after SVD is the rank, we approximate the singular values which fall below a set threshold ($\frac{SingularValue}{MaxSingularValue} < 10^{-7}$) to 0. The extremely small non-zero singular values result because of floating point approximation in simulation. After running a large number of simulations for different sets of points and camera motion path, we conclude experimentally that the rank can be safely approximated to 6. Singular values for the trajectory matrices for two different camera roto-translation paths are presented in table 1 and the world frame & camera frame trajectories can be visualized in Figure 3.

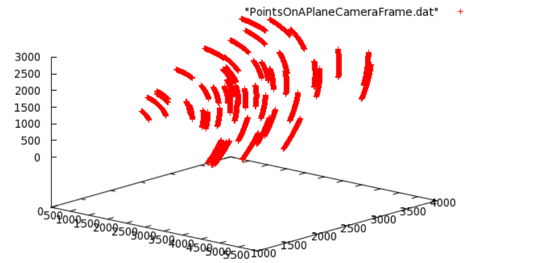
5. Future Work

We are in process of applying this rank constraint method to the Background Subtraction technique developed in [1] to perform BS. We are also currently working on developing a mathematical proof for this rank constraint. We further aim to apply this background subtraction for tracking balls in real time on soccer field by our RoboCup Middle Sized League robots, which will eventually be the test bed of our work. In this test bed the field plane is the dominant ground plane.

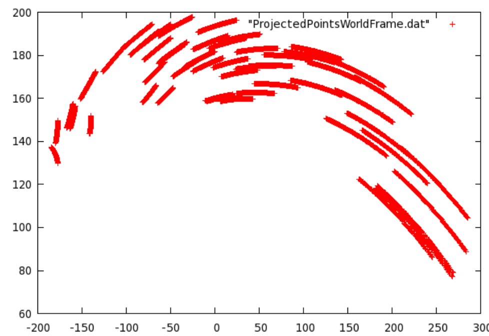
References

[1] Y. Sheikh, O. Javed, and T. Kanade. Background subtraction for freely moving cameras. *IEEE International Conference on Computer Vision (ICCV)*, 2009.

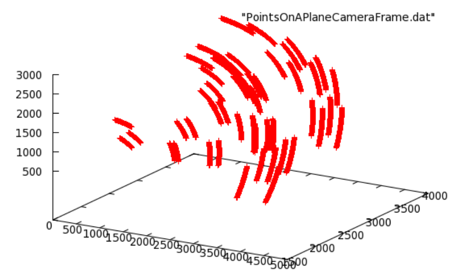
[2] C. Tomasi and T. Kanade. Shape and motion from image streams under orthography:a factorization method. *International Journal of Computer Vision*, pages 137–154, 1992.



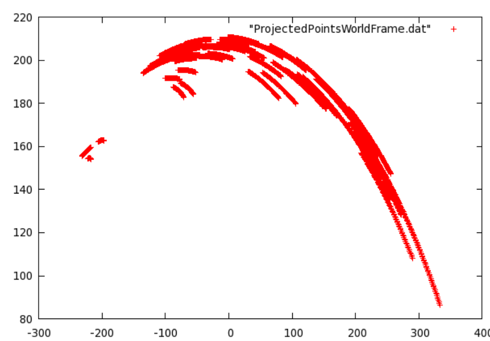
Simulation 1 : Camera frame apparent trajectories of fixed world frame points



Simulation 1 : Image frame trajectories of projected points



Simulation 2: Camera frame apparent trajectories of fixed world frame points



Simulation 2: Image frame trajectories of projected points

In Simulation 1 the camera translates 2 units per degree of freedom in translation and 1 unit in rotation per frame
In Simulation 2 the camera translates 2 units for 1 degree of freedom in translation and 1 unit in rotation per frame

Figure 3. World frame and Image frame Trajectories