Magnetic Navigation and Tracking of Underwater Vehicles ⋆

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Abstract: This paper proposes novel methods with the potential to improve the performance of navigation and tracking systems in underwater environments. The work relies on well-established methods of potential field inversion and introduces a new analytic formulation designed to stabilize the solution of the inverse problem in real-time applications. The navigation method proposed exploits the terrain information associated with geomagnetic field anomalies, without the need of a priori maps. The procedure can also be applied to track a moving vehicle based on its associated disturbance of the environmental magnetic field. We envision the integration of theses methods in terrain-aided navigation systems, simultaneous localization and mapping algorithms, and tracking applications.

Keywords: Navigation; tracking; magnetic methods; inverse problems; particle filters.

1. INTRODUCTION AND MAIN CONTRIBUTION OF THE PAPER

The execution of long-range and long-term missions by robotic underwater vehicles in a fully autonomous mode is still a challenging problem. Without the aid of external references, the position error of high-grade inertial navigation systems (INS) grows at a minimum rate of 0.1 percent of the distance traveled. Even with the integration of Doppler velocity loggers (DVL) with INS to improve the performance of dead-reckoning navigation systems, the positioning error grows unbounded at a considerable rate. Hence, efficient and affordable navigation methods are under development to afford underwater robotic vehicles the capacity of executing long-range missions with minimum human intervention. Among the novel methods proposed, the terrain aided navigation (TAN) and the Simultaneous localization and mapping (SLAM) approaches have great potential for the implementation of a new generation of reliable and affordable navigation systems. However, a fair assessment of the state of the art shows that TAN and SLAM implementations in the marine environment are still in a experimental phase. SLAM is a method rooted in the mobile robotics community where navigation problems have been solved relying on the extraction of geometric features and prominent landmarks or based on the utilization of artificial beacons. Normally, these conditions cannot be ensured in marine environments. On the other hand, TAN has already proved its efficacy in natural, unstructured environments but requires the existence of prior maps for navigation, a requirement that cannot be fulfilled easily in most applications. The terrain-based approach also assumes that the terrain is sufficiently rich in terms of topography to permit the estimation of position. It is well-known that this assumption is not valid in vast areas of the ocean floor. To solve this problem we proposed in prior works to complement the topographic information with geomagnetic data extracted from the terrain; Teixeira (2007); Teixeira and Pascoal (2008).

It is against this backdrop of ideas that this paper proposes the combination of different analytic methods of geopotential field inversion to implement 3D localization algorithms that can be employed in navigation and tracking. We propose its integration in TAN and SLAM to improve the navigation capabilities of autonomous underwater vehicles. Based on the same methods, we present a tracking procedure that may find applications in civilian and military applications.

2. MAGNETIC METHODS IN NAVIGATION AND TRACKING PROBLEMS

The magnetic field of the Earth is a vector field characterized by very slow variations in its intensity and orientation due to geophysical phenomena in the interior of the planet and by higher frequency fluctuations caused by external influences such as the solar activity. In addition to these large-scale variations, there are local anomalies in terms of magnitude and orientation of the geomagnetic field that are introduced by natural and artificial objects with induced and remanent magnetization. The exploitation of

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these geomagnetic anomalies as a source of information for the navigation of AUVs has been proposed many years ago but the concept still requires practical demonstration.

Implementation issues One of the advantages of magnetic navigation consists in being passive and economical in terms of energy. Magnetic sensors do not emit signals that can be detected and tracked remotely and their typical power consumption is orders of magnitude lower than that required by active sonar systems employed in bathymetric TAN.

The main problem to solve in magnetic-based navigation is the suppression of electromagnetic noise or the mitigation of its effects on the measured data used for localization. A global source of magnetic noise is the solar activity that affects the magnetosphere of our planet and manifests in the form of periodic diurnal variations of the Earth's magnetic field, superimposed by random components of higher frequency. This which noise affects equally any measurements obtained in a given region, can be filtered out by taking differential measurements of the magnetic field using magnetic gradiometers instead of single magnetometers. In robotic platforms, the electrical thrusters and other components of the vehicle constitute important sources of electromagnetic noise. This issue assumes special relevance in applications like the present one, that require magnetic measurements with very high precision. A typical solution to this problem consists in the placement of magnetic sensors as far away as possible from the sources of noise but this may not be practical to implement in small robotic vehicles. We are currently studying solutions that involve well-known calibration techniques to compensate the disturbances introduced by the vehicle and its payload, combined with methods based on temporal correlation, spectral analysis, and band-pass filtering to mitigate the electrical noise induced by the thrusters/actuators.

Besides underwater navigation, we envision tracking of underwater objects or vehicles as another interesting application of the methods proposed here. The tracking system proposed consists basically in a vector gradiometer which senses the anomalous magnetic field vector caused by a passing vehicle. The corresponding gradient measurements are processed by an inversion algorithm to estimate its position and velocity vector. In these conditions, the tracking problem can be solved with a relatively simplified sensor set-up since the magnetic observer may be deployed at a fixed location thus eliminating the need for the continuous acquisition of its relative localization and orientation parameters. In this type of application, local sources of electromagnetic noise can be easily avoided.

2.1 Alternative methods of magnetic navigation

A survey of the literature reveals that two main methods have been proposed in the last decades for magnetic navigation of underwater vehicles. The first approach is basically an extension of the terrain-aided navigation concept with exploitation of geopotential fields. It consists in matching a set of scalar or vectorial field measurements performed by a moving platform with a magnetic signature of the terrain. A different approach that has emerged more recently consists in the application of inverse methods to localize the sources of the local magnetic anomalies and estimate the position of the vehicle relative to these sources; see e.g. Kumar et al. (2005); Pei and Yeo (2006); Nara et al. (2006); Birsan (2011). This method can be used in the context of TAN but shows considerable promise for integration in SLAM algorithms due to its ability to localize accurately point-like features that do not have to be mapped a priori. The increasing interest in this class of methods may be attributed in part to the emergence in the last few years of sensors such as superconducting quantum interference devices (SQUID) and spin exchange relaxation-free (SERF) magnetometers. These ultra-high sensitivity devices open the possibility of exploring in practice some concepts that could not be tested with the technologies previously available.

A few reports on the utilization of magnetic inversion methods for underwater vehicle tracking have been published recently; however, to the best of our knowledge no experimental results are available; see e.g. Birsan (2006).

3. THEORETICAL FOUNDATIONS AND IMPLEMENTATION OF THE PROPOSED APPROACH

The approach to magnetic navigation proposed here borrows from the theories of classical electrodynamics and geopotential field inversion; see Jackson (1975) and Blakely (1995). It is also inspired by related studies in the geophysics and navigation domains; see, e.g. Wynn et al. (1975); Pedersen and Rasmussen (1990); Reid et al. (1990); Zhang et al. (2000); Schmidt et al. (2004); Kumar et al. (2005); Allen et al. (2005); Heath (2007). This class of methods assumes the existence of anomalous magnetic dipoles in the environment whose sources can be localized using a set of very precise measurements of the magnetic field vector and its gradients. Its potential in terms of navigation is justified by the fact that many geological features in the ocean floor generate magnetic dipoles of large magnitude that can be treated as landmarks and processed by navigation methods such as TAN and SLAM.

3.1 Analytic inversion of magnetic field anomalies

To introduce the method, we consider the problem of localizing a magnetic object characterized by a dipole moment $\mathbf{m} = m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + m_z\hat{\mathbf{z}}$. See Telford et al. (1998) and the references therein for an introduction to basic concepts in geomagnetism. The magnetic field observed at a point P localized relatively to the dipole center by the vector $\mathbf{r} = r_x\hat{\mathbf{x}} + r_y\hat{\mathbf{y}} + r_z\hat{\mathbf{z}}$ with modulus $r = |\mathbf{r}|$, is

$$B_i = \frac{\mu_0}{4\pi}\left[\frac{3(\mathbf{m} \bullet \mathbf{r})}{r^5} r_i - \frac{m_i}{r^3}\right] , \quad (i = x, y, z) \ . \qquad (1)$$

The gradient of the vector field is a tensor defined by

$$\mathbf{T} = \begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial z} \end{bmatrix} [\, B_x \;\; B_y \;\; B_z \,] = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \qquad (2)$$

and using (1), each element B_{ij} of $\mathbf{T}$ has the form (see Heath (2007))

$$B_{ij} = \frac{\partial B_i}{\partial j} = \frac{\mu_0}{4\pi}\left[3\delta_{ij}\frac{(\mathbf{m} \bullet \mathbf{r})}{r^5} + 3\frac{m_i}{r^5}r_j + 3\frac{m_j}{r^5}r_i - 15\frac{(\mathbf{m} \bullet \mathbf{r})}{r^7}r_i r_j\right] \quad (3)$$

which, using the scaled moment $\mathbf{M} = 3\mu_0\mathbf{m}/\left(4\pi r^4\right)$ and $\mathbf{n} = \mathbf{r}/r$, can be expressed in the more compact form

$$B_{ij} = -5(\mathbf{M} \bullet \mathbf{n})n_i n_j + M_i n_j + M_j n_i + (\mathbf{M} \bullet \mathbf{n})\delta_{ij} \ . \quad (4)$$

3.2 Alternative methods of geomagnetic field inversion

We consider two distinct approaches to geomagnetic field inversion: *analytic inversion*, which encompasses deterministic methods based on eigenanalysis of the magnetic gradient tensor (EGT) and tensor Euler deconvolution (TED), and *probabilistic inversion* including Monte Carlo inversion (MCI) methods.

Analytic inversion The analytic inversion of the gradient tensor expression (4) provides four solutions for the pair of normalized parameters $\mathbf{M}$ and $\mathbf{n}$; only two of these solutions have physical meaning and only one of them is real; see e.g. Wynn (1997) and Heath (2007). Although a proof of uniqueness of the solution for $\mathbf{M}$ and $\mathbf{n}$ exists, it does not provide a form of determining the unique solution. In practice, if the halfspace of source location is known the set of solutions may be reduced to two but the real solution can only be determined by tests to verify the solutions that produce the corresponding gradient tensor and vector field. Hence, the method requires measurements of the magnetic field vector B at the same position. The method does not provide solutions for $\mathbf{m}$ and $\mathbf{r}$ but only for the scaled moment $\mathbf{M}$ and direction vector $\mathbf{n}$. We consider that estimation of the magnetization parameters of the sources is not relevant for navigation although we admit that it may be relevant for object identification in tracking applications, as suggested in some of the aforementioned publications.

Tensor Euler deconvolution is an inversion method that provides direct estimates of the location of magnetic sources of geopotential fields that verify Euler's homogeneity equation, cf. Zhang et al. (2000). It can be applied not only to dipoles but also to other types of magnetic sources. The basic requirement for its application is knowledge of the structural index associated to the anomaly, a scalar parameter that represents the rate of decay of the field intensity with the distance from the magnetic source. The main disadvantage of the method consists in its high sensitivity to noise in the measured data and the consequent instability of the solutions.

We propose in the sequel an analytic inversion method that combines tensor Euler deconvolution with eigenanalysis of the magnetic gradient tensor. It takes advantage of the uniqueness of solutions obtained with TED and uses EGT to introduce a restriction on the solution space that constrains the solutions obtained with Euler deconvolution. This approach dispenses with the need of a priori information on the magnetic parameters of landmarks used for navigation or the vehicle magnetic parameters in the case of tracking.

Monte Carlo inversion This probabilistic approach has demonstrated high potential of application to tracking problems; see, e.g. the results reported in Birsan (2011). It integrates forward modeling of the magnetic dipole in a Monte Carlo simulation procedure. Despite its versatility, MCI requires prior information on the magnetic parameters that can be difficult or impossible to obtain in many practical applications. This limitation, which is not so relevant in inversion problems that can be solved offline, becomes manifest in real-time applications involving unknown scenarios and moving objects. In the present work we introduce a new version of the method that uses a simplified sensor set-up that can contribute to facilitate its implementation and improve its robustness in practical applications.

3.3 Inversion methods proposed

Analytic inversion According to the fundamental laws of electromagnetism, a magnetic field in a non-magnetic media and in the absence of electro-magnetic time-dependent effects is irrotational and has zero divergence. It is also known that in these conditions the magnetic field verifies Laplace's equation. Representing the magnetic field vector by $B = [\, B_x \ B_y \ B_z \,]^{\mathrm{T}}$, these properties are expressed through the following equations, respectively:

$$\nabla \times B = 0 \quad (5)$$

$$\nabla \bullet B = 0 \quad (6)$$

$$\nabla^2 B = \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} = 0 \ . \quad (7)$$

Using the above properties, it is easy to verify that tensor matrix (2) is symmetric and has trace equal to zero. Hence, a set of five measurements of the first-order gradients of each vector component is sufficient to build a gradient tensor of the field.

Tensor Euler deconvolution can be used for magnetic source localization since in general magnetic field anomalies verify Euler's homogeneity equation, i.e.

$$x\frac{\partial B}{\partial x} + y\frac{\partial B}{\partial y} + z\frac{\partial B}{\partial z} = -n(B - B_0) \quad (8)$$

where n denotes the degree of homogeneity (or *structural index* in geophysical nomenclature), B represents the total magnetic field vector and B_0 represents the regional field which can be known a priori and must be subtracted from the total field to compute the anomalous field vector [1]. For dipoles, (8) applies with $n = 3$ and an inversion formula using the gradient tensor is

$$\mathbf{r} = -3\mathbf{T}^{-1}(B - B_0) \ . \quad (9)$$

It is well known that the inversion problem is typically ill-posed and highly sensitive to errors in measurement data;

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[1] We emphasize that the Euler equation which applies to a local magnetic anomaly cannot be applied to the total ambient field which does not behave as a homogeneous function of the same degree.

Tarantola (2005). Commonly, solutions to this problem include the application of regularization techniques such as Tikhonov regularization; Golub et al. (1999). The method we introduce, designated *TED+EGT*, constrains the solutions through the inclusion in (9) of an additional equation derived from eigenanalysis of the gradient tensor. The results obtained with this approach in computer simulations demonstrate the superior performance of the new method in terms of stability of solutions compared with those obtained by simple tensor Euler deconvolution.

Since the gradient tensor is a real symmetric matrix, it is possible to find an orthogonal set $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ of eigenvectors of $\mathbf{T}$. The eigenvector ($\mathbf{b}_3$ by our convention) corresponding to the smallest eigenvalue, is perpendicular to the $\mathbf{m}-\mathbf{r}$ plane, its direction being given by the external product $\mathbf{m} \times \mathbf{r}$; Heath (2007). Hence, it becomes trivial to show that the solution space for $\mathbf{r}$ is restricted to the plane defined by the two orthogonal vectors $\mathbf{b}_1$ and $\mathbf{b}_2$, as illustrated in Fig. 1. We immediately conclude that the solutions for the localization vector $\mathbf{r}$ must conform to the plane defined by $\mathbf{b}_3 \bullet \mathbf{r} = 0$, or

$$b_{3x} r_x + b_{3y} r_y + b_{3z} r_z = 0 \quad . \tag{10}$$

Including (10) in the set of equations represented by (9) we obtain the novel inversion formula expressed by the overdetermined system of equations

$$\begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \\ b_{3x} & b_{3y} & b_{3z} \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = -3 \begin{bmatrix} B_x - B_{0x} \\ B_y - B_{0y} \\ B_z - B_{0z} \\ 0 \end{bmatrix} \tag{11}$$

that can be solved using the Moore-Penrose pseudoinverse. The solution thus obtained is expressed in the referential of the sensor used to acquire the elements of the gradient tensor.

One important requisite of the envisioned applications is the ability of the methods to reject localization solutions which have no physical meaning. To achieve this, the localization algorithms implement heuristics based on combinations of the following criteria: 1-the magnitude of the differential signals measured in the presence of a magnetic anomaly must exceed a predefined threshold; 2-the output of a difference operator is used to reject localization solutions that correspond to unfeasible velocities of the vehicle/object; 3-the condition number of the gradient tensor used by the inversion method must not exceed a predefined limit.

Monte Carlo inversion The Monte Carlo inversion procedure that we implement is illustrated by the simulated tracking of a vehicle which is assumed to move in the horizontal plane with unknown velocity vector and unknown but approximately constant altitude. The procedure uses a Partice Filter to estimate the state of the system which includes the 3-dimensional position $\mathbf{r} = [r_x, r_y, r_z]^T$ and the 2D velocity $\mathbf{v} = [v_x, v_y]^T$ of the vehicle based on measurements of gradients of the vector field.

The state and measurement vectors are respectively

$$\mathbf{x} = [r_x, r_y, r_z, v_x, v_y]^T$$

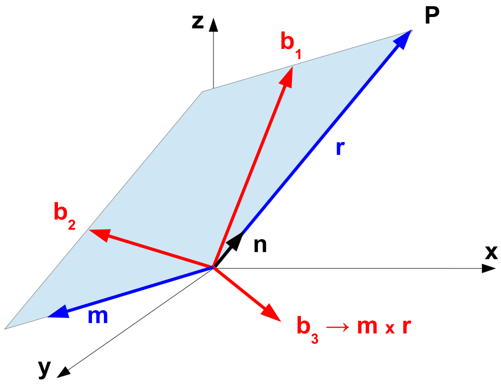

Fig. 1. Magnetic dipole vector parameters and eigenvectors of the gradient tensor.

and

$$\mathbf{z} = [B_{xx}, B_{xy}, B_{xz}, B_{yy}, B_{yz}]^T \quad .$$

The state-space and measurement models of the system are respectively:

$$\mathbf{x}_k = \mathbf{\Phi}\, \mathbf{x}_{k-1} + \mathbf{w}_k \tag{12}$$

$$\mathbf{z} = \mathbf{\Gamma}(\mathbf{x}_k) + \mathbf{v}_k \tag{13}$$

where $\mathbf{w}_k$ and $\mathbf{v}_k$ denote the process and measurement model sequences, respectively. The state-transition matrix $\mathbf{\Phi}$ assumes that the velocity vector is constant. The nonlinear measurement function $\mathbf{\Gamma}(.)$ is derived from the model of the gradient tensor (3) to obtain the set of five gradient components used by the particle filter. Notice that the present Monte Carlo formulation only applies if the dipole moment of the moving object is known. Otherwise, the state vector must be augmented to include the three vector components of that parameter and the estimation procedure must provide estimates of the corresponding values.

4. NAVIGATION AND TRACKING SIMULATIONS

This section describes the results of simulations aimed at showing the efficacy of the methods developed for navigation and tracking.

4.1 Configuration of simulations

The scenario, including the geomagnetic anomalies and the trajectories used in the simulations of navigation and tracking, is represented in Fig.2. The magnetic environment is based on real magnetic data which was collected for trials of magnetic navigation at the experimental site in *Parque das Nações*, Lisbon. The main parameters used in the simulations are presented in Table 1 and the distinct combinations of inversion methods and types of sensors applied are listed in Table 2, together with the maximum estimation errors corresponding to the validated solutions.

4.2 Navigation using analytic inversion

The method of 3D localization used in the simulations of magnetic navigation relies on the analytic inversion procedures introduced in subsection 3.3. This approach

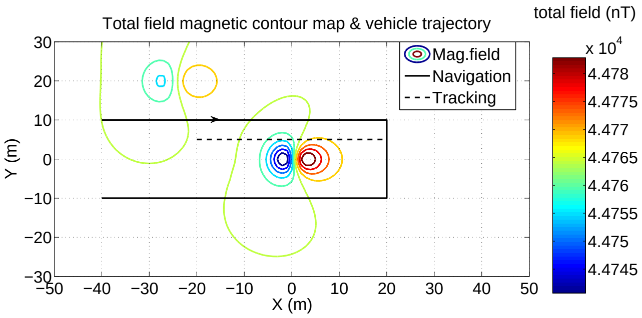

Fig. 2. Scenario and trajectories used in navigation and tracking simulations.

Table 1. Simulations parameters

Parameter	symbol	value and units
Gradiometer baseline	b	1m
Fluxgate sensor noise intensity	σ_f^2	$(1\text{nT})^2$
SQUIDs sensor noise intensity	σ_s^2	$(0.1\text{pT})^2$
Altitude of the vehicles	z	5m
Num. of Monte Carlo simulations	M	100
Num. of particles in PF	N	600
Velocity of underwater vehicles	V	1ms^{-1}

Table 2. Methods, sensors, and max. errors

Problem	Inversion	SQUID	FluxGate
Navigation	Analytic	1.6m	n.a.
Tracking	Analytic	0.13m	1.7m
Tracking	Monte Carlo	0.4m RMS	0.7m RMS

provides the localization of the vehicle relatively to the magnetic landmarks observed in real time. It is obvious that when the magnetic anomalies are not mapped a priori, the output of this method must be integrated with additional navigation data to implement an absolute navigation solution. The localization error obtained in the simulations is shown in Fig. 3; this figure compares the error obtained by tensor Euler deconvolution with the error obtained by the novel method that combines TED with EGT. The results show that the methods are equivalent when the sensor is deployed at relatively short distances from a well-defined dipole. However, the TED method becomes unstable at larger distances from the dipole sources and in the presence of multi-poles. In these circumstances, the TED+EGT inversion is clearly superior. The present study focuses on the problem of 3D navigation using magnetic landmarks that are not previously mapped. In this context, analytic methods such as those discussed above are advantageous since they do not require prior information on the magnetic dipoles used for navigation.

4.3 Tracking using analytic and Monte Carlo inversion

The results of tracking simulations are plotted in Figs. 4 and 5. In these simulations we applied the TED+EGT analytic inversion and the Monte Carlo inversion procedure. The results obtained indicate that analytic inversion methods can provide very good localization results if implemented with very high-sensitivity sensors. However, they become much less reliable, when implemented with standard sensors such as fluxgate magnetometers. On the contrary, the Monte Carlo inversion method analyzed showed to be highly robust to measurement noise allowing for the implementation of less expensive tracking systems and achieving results that approximate those obtained with more expensive sensors. The results obtained illustrate the performance of the system using magnetic field anomalies of low intensity.

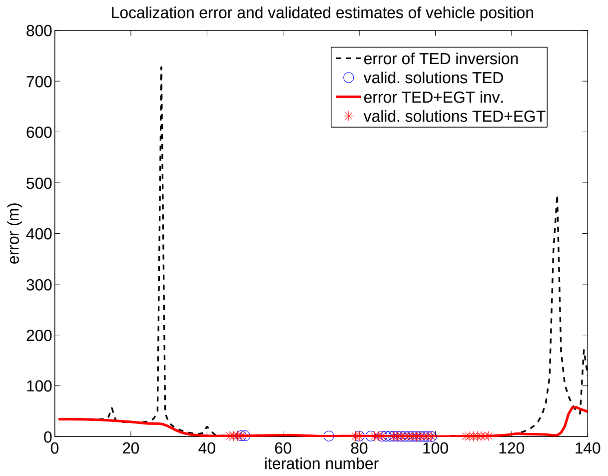

Fig. 3. Comparison of 3D position errors obtained in navigation with conventional Euler deconvolution and with the new method (TED+EGT) with corresponding validated solutions.

Magnitude of magnetic field anomalies The magnetization vector employed in the tracking simulations corresponds to the magnetic field of a small AUV. The largest magnetic field anomaly used in simulations of navigation is equivalent to the magnetic anomaly produced by a metallic drum of 100 Kg. In these conditions, the ability to track a moving object is limited to a few tens of meters; magnetic navigation based on dipole identification is also limited to the same range. However, the ranges of dipole detection increase significantly in the presence of larger magnetic anomalies.

Sensor setup for Monte Carlo inversion It is important to remark that the Monte Carlo inversion procedure proposed here does not depend on measurements of the regional magnetic field vector. In contrast to the localization methods reported in the aforementioned literature, only the gradients required to build the gradient tensor must be measured, as represented in the measurement vector defined in Section 3.3. This is an important result since the range of the regional field intensity is several orders of magnitude larger than the differential intensity that can be measured directly by some types of gradiometers. Vector sensors with large dynamic ranges are characterized by reduced resolution and measurements of large magnitude vectors are much more sensitive to orientation errors. Additionally, the absolute value of the regional field is affected by diurnal variations of the Earth's magnetic field that may be difficult to compensate. Hence, the method proposed introduces an important practical advantage relatively to other reported implementations.

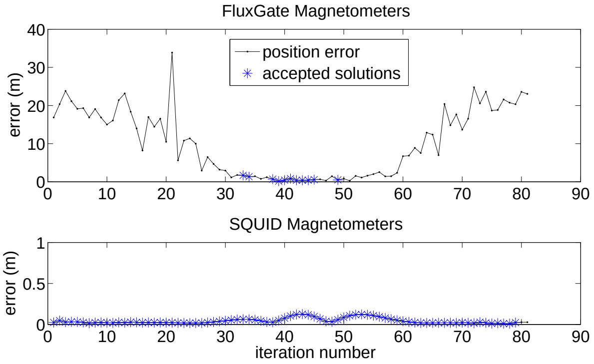

Fig. 4. Tracking errors and validated solutions obtained by TED+EGT analytic inversion with SQUID and Fluxgate sensors.

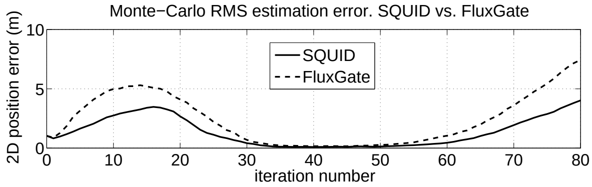

Fig. 5. Comparison of the RMS tracking errors obtained in 100 simulations by Monte Carlo inversion with SQUID and Fluxgate sensors.

5. CONCLUSIONS AND FUTURE WORK

The main objective of the work reported in the paper is the exploration of methods that can contribute to increase the robustness of navigation and tracking algorithms and the simplification of the required instrumentation. In this sense, computer simulations constitute valuable tools that afford the designers of navigation systems fundamental guidelines for practical implementations.

The paper introduces a new analytic method of potential field inversion which is demonstrated to achieve superior results in terms of localization of magnetic dipoles. This procedure is combined with simple heuristics based on the vehicles kinematics and with basic algebraic principles to filter out localization solutions which have no physical meaning. The method produces accurate estimates of position that can be used in real-time for autonomous navigation or vehicle tracking. Its main disadvantage is the requirement of very-high sensitivity sensors which are complex to install in moving platforms and may be overly expensive. As an alternative to the former method, the paper proposes the utilization of a Monte Carlo inversion procedure that proves to be very robust to measurement noise, allowing for the implementation of tracking systems with conventional magnetic sensors. The implementation proposed specifies a simplified sensor setup which contributes to the robustness of the tracking algorithm. This method is currently being adapted to implement a magnetic localization algorithm integrated in a terrain aided navigation system for AUVs. The experimental validation of these methods is part of the work in progress in the context of the research project ATLAS-Geo (http://atlas-geo.web.ua.pt).

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