Enhanced PCA-Based Localization Using Depth Maps with Missing Data

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***Abstract*—In this paper a new method for a global self-localization of mobile robots based on a PCA positioning sensor to operate in unstructured environments is proposed and experimentally validated. Unlike many existing systems that use RGB signals to capture information from the environment, in this work a 2D depth sensor is used, allowing the self-localization to be performed under different illumination conditions. However, depth sensors provide measurements corrupted with missing data, due to limitations on the support physic principles exploited (e.g. light that illuminates surfaces with diffuse reflection or wave fading), which severely degrades the performance of the estimation techniques and limits its use. The main goal of this paper is to present a self-localization system for mobile robots based on a PCA positioning sensor that relies on corrupted depth measurements and the corresponding experimental validation. The proposed method allows for the implementation of a global self-localization system for indoor environments with higher accuracy, that provide a Gaussian estimated position error and work in any illumination conditions.**

I. INTRODUCTION

The problem of mobile robots localization with only onboard sensors in indoors environments has been a great challenge to researches in mobile robotics, see [8], [3] and the references therein. To perform this task, usually, mobile robots are equipped with different types of sensors like compasses, accelerometers, gyros, cameras, time of flight cameras and encoders, providing enough information to the measuring system to determine its global pose, i.e., position and orientation in a mapped environment.

Vision is one of the most popular sensors in mobile robotics to provide measurements to solve the localization, due to the large amount of information provided on the environment, extracted from the RGB image [23], [22], [16], [11]. However, in vision systems remains a general limitation related to different environment illumination conditions that decreases the localization systems robustness.

To avoid the above mentioned problem, some localization systems are based on time-of-flight sensors [17]. The use of time-to-flight sensors allows to obtain depth information about the environment and presents a more robust system able to cope with different light conditions. Moreover, the time-of-flight cameras allows the capture of depth images, where the sensor is able to receive a grid with depth information from all field of view [1]. However, it is expensive to implement this type of cameras in many mobile robotic platforms.

Recently, the companies PrimeSense and Microsoft developed a device primarily for video games, called Kinect, that combines a RGB and a depth camera. Due to its low price and a straightforward way to be connected with a computer, the Kinect device became popular in mobile robotics community creating several different applications of mobile robots [25], [12], [13].

A very common problem in depth sensors, including the Kinect depth sensor, is the existence of missing data in signals, caused by IR beams that are not well reflected, not returning to the depth sensor receiver. In [19], [20], a method using the Principal Component Analysis (PCA) methodology is presented to avoid the problem of missing data in signals and its performance is compared with other state-of-the-art algorithms. The PCA [14] is an efficient algorithm that converts the database into an orthogonal space creating a database with a high compression ratio, when compared with the amount of captured data. Moreover, the PCA allows to develop localization systems that do not depend on any predefined structure [15], [2], i.e, does not need to detect any specific features about the environment. In [21], PCA is used for terrain reference navigation of underwater vehicles.

There are different approaches in installing cameras to develop localization systems. The most common solution is to allow placement of cameras to look around to obtain its position [24], [11], while some mobile robots use a single camera looking upward [9], [26]. The use of vision from the ceiling has the advantage that images can be considered without scaling and are static.

While many localization systems uses the information of extraction features to the localization of the mobile robot in a structured environment [18], the use of PCA allows the creation of a localization system with a great compression ratio and without the need of extraction features.

In this work, the main purpose is the experimental validation of [19], [20] avoiding the missing data existing in a ceiling vision localization system performed by a Kinect depth sensor.

This paper is organized as follows: Section II presents the mobile robot platform and the motivation for the use of Kinect in the proposed localization system; in the Section III, the principal component analysis for signals with missing data is detailed. For performance analysis purposes, Section IV presents experimental results of the proposed method, compared with the results achieved using the classical PCA algorithm along a straight line (1D localization); in Section V the results of the proposed method are presented and validated in a 2D localization approach. Finally, Section VI presents some conclusions and unveils future work.

II. MODEL PLATFORM

The experimental validation of the positioning system proposed in this paper is performed resorting to a low cost mobile

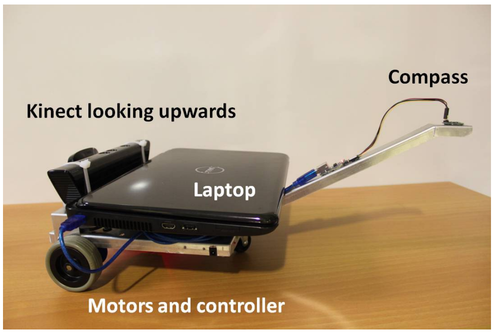

Fig. 1. Mobile platform equipped with kinect sensor and compass

robotic platform [4], with the configuration of a Dubins car. A Microsoft Kinect is installed on the platform, pointing upwards to the ceiling, and a digital compass, located on the extension arm (robot rear part) to avoid the magnetic interference from the motors (see Fig. 1)

The Kinect includes a RGB camera with a VGA resolution (640×480 pixels) using 8 bits and a 2D depth sensor (640×480 pixels) with 11 bits of resolution. The use of this sensor for mobile robots localization could combine the capture of a RGB image and a depth map about the environment, obtaining RGB-D images, as shown in Fig. 2. This image depicts the ceiling view captured by the Kinect installed onboard the mobile robot. Note that it is possible to observe both the 3D shape of the existing technical installations in the ceiling and its color.

The robot moves in an environment indoors in buildings with some information (e.g. building-related systems such as HVAC, electrical and security systems, etc.). It is possible to use the signals captured by a Kinect looking upward (RGB image, depth map or both) by an algorithm that can provide mobile robot global position in the environment.

Due to limitations found in image-based mobile robot localization approaches, regarding illumination changes, and aiming the development of an efficient self-localization solution that can work in places with variation on the level of illumination, only the Kinect depth signal is used, resorting to an adaptation to the method proposed in [21], [6], [7], [5] to the problem at hand.

However, as it is possible to observe in Fig. 2, due to geometry and properties of some objects, several waves are

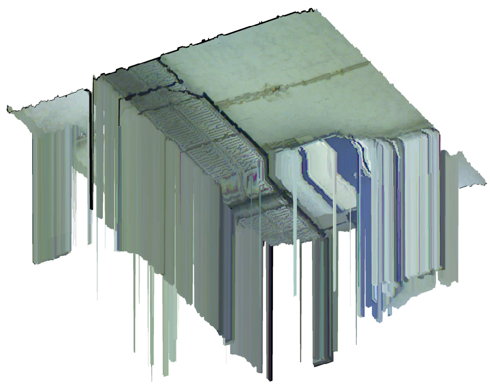

Fig. 2. RGB-D image of the ceiling view obtained by the kinect installed onboard the mobile robot

not well reflected and, thus, can not be understood by the depth sensor receiver. In the case of Kinect, such a problem results in the appearance of points with null distance ($0mm$) inside the data array with the depth values (distances to various points in the plane), that may lead to erroneous results in the localization system. In this paper an extension of a PCA-based position approach will be presented aiming to cope with the illumination variability problem common to usual vision systems, in both cases to be validated experimentally.

III. PCA FOR SIGNALS WITH MISSING DATA

PCA [14] is a methodology based on the Karhunen-Loève (KL) transformation that is often used in applications that need data compression, like image and voice processing, data mining, exploratory data analysis and pattern recognition. The data reduction is obtained through the use of a database eigenspace approximation by the best fit eigenvectors. This technique makes the PCA an algorithm that has a high compression ratio and requires reduced computational resources. The PCA algorithm is used as the mobile robot position sensor in [6], [7] .

The PCA eigenspace is created based on a set of M stochastic signals $\mathbf{x}_i \in \mathbb{R}^N$, $i = 1, \ldots, M$ acquired by a Kinect depth sensor installed onboard the mobile robot, considering an area with N mosaics in two dimensional space, $N = N_x N_y$, where N_x and N_y are the number of mosaics in x and y axis, respectively.

In the common PCA-based approaches, the eigenspace of the set of acquired data is characterized by the corresponding mean $\mathbf{m}_x = \frac{1}{M}\sum_{i=1}^{M}\mathbf{x}_i$. However, usually these signals obtained by sensors are corrupted with missing data. In the case of the depth map provided by Kinect, the points where failures occurred in the depth data reception are marked with a null distance ($0mm$). Therefore, the existence of missing data in signal $\mathbf{x}_i$ corrupts the PCA mean value computation creating an orthogonal space with erroneous data.

To solve the position estimation problem when data with missing data is used, a mean substitution method is applied to the PCA position sensor, as described below. Thus, a vector $\mathbf{l}$ with length N consisting of boolean values is used to mark the real and missed data of a signal $\mathbf{x}_i$. Then, considering the j^{th} component of acquired signal $\mathbf{x}_i$, the index $\mathbf{l}_i(j)$ is set to 1 if the signal $\mathbf{x}_i(j)$ is available and it is set to 0 if there is a missing data.

Hence, to avoid the negative impact of the sensor signals missing data in PCA-based approaches performance, an extension to this methodology is proposed in this paper, where instead of considering all values of the M stochastic signals to compute the previously mentioned mean value $\mathbf{m}_x$, only the correct data is used and the value corresponding to missing data is neglected. Thus, the mean data is computed as follows:

$$\mathbf{m}_x(j) = \frac{1}{c(j)}\sum_{i=1}^{M}\mathbf{l}_i(j)\mathbf{x}_i(j), \;\; j = 1, ..., N \tag{1}$$

where $c(j)$ is the number of j^{th} components for a set of M signals $\mathbf{x}_i \in \mathbb{R}^N$, $i = 1, \ldots, M$ without missing data. The counter $\mathbf{c}$ is a vector with length N defined by:

$$\mathbf{c} = \sum_{i=1}^{M}\mathbf{l}_i \tag{2}$$

In order to apply the mean substitution method to the PCA algorithm, all missing data presented in the acquired database is replaced by the mean value of the corresponding component, i.e., if there is a missing data in the j^{th} component of the i^{th} signal, the missing value $\mathbf{x}_i(j)$ is replaced with the value of $\mathbf{m}_x(j)$. After this substitution, the decomposition of the $\mathbf{x}_i(j)$ into the orthogonal space follows the PCA algorithm classical approach, i.e. $\mathbf{v} = \mathbf{U}^T(\mathbf{x} - \mathbf{m}_x)$. Matrix $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \dots\ \mathbf{u}_N\ ]$ should be composed by the N orthogonal column vectors of the basis, verifying the eigenvalue problem

$$\mathbf{R}_{xx}\mathbf{u}_j = \lambda_j \mathbf{u}_j,\ \ j = 1, ..., N, \quad (3)$$

Assuming that the eigenvalues are ordered, i.e. $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$, the choice of the first $n \ll N$ principal components leads to stochastic signals approximation given by the ratio on the covariances associated with the components, i.e. $\sum_n \lambda_n / \sum_N \lambda_N$.

During the mission, before the projection of the depth image into the orthogonal space, the mean substitution should be followed in order to eliminate the problem caused by missing data in the signal $\mathbf{x}$, i.e, all j^{th} component of the signal $\mathbf{x}_i$ should be replaced by the corresponding mean value $\mathbf{m}_x(j)$.

The robot position $\hat{x}$ and $\hat{y}$ is still obtained by finding on a given neighborhood δ, the mosaic whose eigenvector is nearest of the acquired signal decomposed into the orthogonal space:

$$\forall_i \|[\hat{x}\ \hat{y}]^T - [x_i\ y_i]^T\|_2 < \delta,\ \ r_{\text{PCA}} = \min_i\ \ \|\mathbf{v} - \mathbf{v}_i\|_2; \quad (4)$$

Given the mosaic i that verifies this condition, its center coordinates $[x_i\ y_i]^T$ is selected as the robot position obtained by the PCA-based sensor.

Then, the mean substitution approach is used when there is missing data in the depth signals coming from the Kinect sensor. Just like during the creation of the PCA eigenspace, it must be done before the application of the PCA algorithm, i.e, all j^{th} component of the signal $\mathbf{x}_i$ should be replaced by the corresponding mean value $\mathbf{m}_x(j)$.

IV. EXPERIMENTAL RESULTS ALONG A STRAIGHT LINE (1D LOCALIZATION)

As concept validation, the proposed self-localization method is initially developed considering a straight line (1D), based on the model proposed in [6] and detailed in the appendix A. Thus, to create the PCA eigenspace, a set of 31 depth images are captured along a straight line with 3 m of length (sampling ratio of 0.1 m). Considering that the Kinect depth sensor has a resolution of 640 by 480 points, and with the purpose of reducing the amount of data stored in PCA eigenspace, the depth images are compressed with a ratio of $100 : 1$, and transformed into vectors $\mathbf{x}_i \in \mathbb{R}^{3072}$, $i = 1, \dots, 31$.

The mobile robot follows along a straight line with constant velocity and the position estimation is obtained using the model proposed in [6], see Appendix A for details. The position estimates are based on data obtained from the onboard sensors and the commands to the actuators, assuming constant values between sampling times (zero order hold assumption).

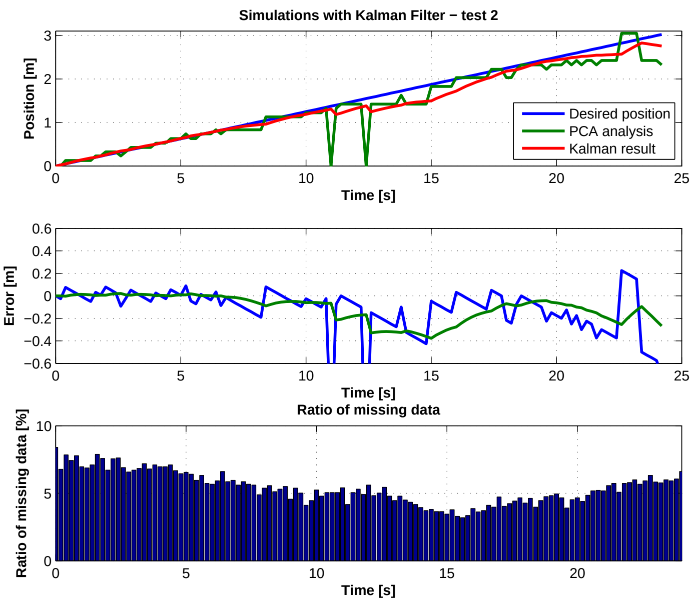

Fig. 3. Results of PCA-based positioning sensor and localization estimates from Kalman filter

A. Monte Carlo Performance Tests

To assess the mobile robot self-localization methodology proposed in this paper, a Monte Carlo test composed of 10 experiments as described above is repeated. Images are captured with a frequency of 5 Hz to be processed by the PCA-based positioning sensor; Fig. 3 gives the localization results obtained in one of those experiments. The results show that the PCA algorithm provides a reasonable approximation to the real robot localization. However, due to the existence of missing data, the position obtained by the PCA algorithm often gives incorrect results.

Analyzing Fig. 3, it is possible to see that the obtained position often reaches errors greater than 0.1 m (distance at which the images are acquired to the eigenspace). Figure 4 shows the results of three tests, where it is possible to see the existence of large perturbations in the results of the PCA-based position sensor. Thus, the fusion of this position sensor with by a Kalman filter (KF), detailed in Appendix A, do not

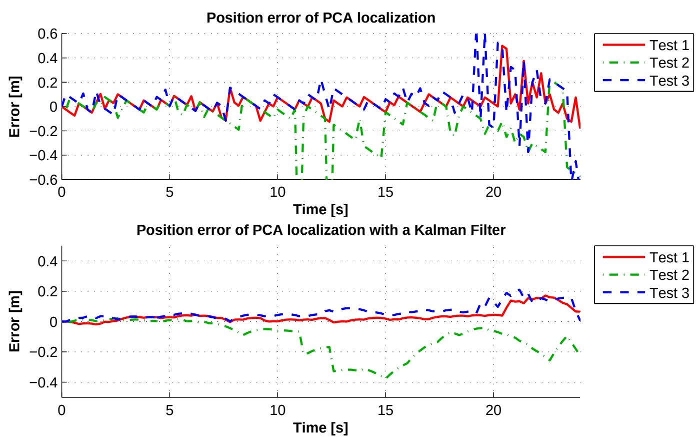

Fig. 4. Localization errors of tests along a straight line

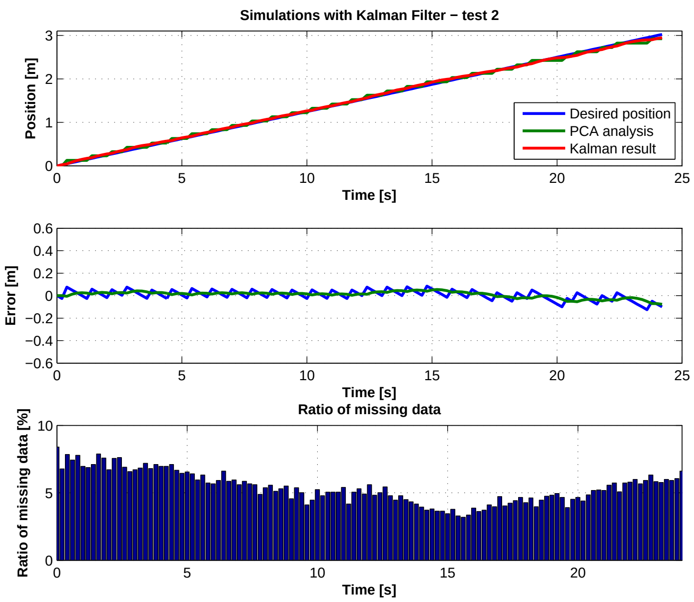

Fig. 5. Results of PCA-based positioning sensor and localization estimates from Kalman filter - new method

always achieve results with an acceptable position errors. As it is possible to see, the position error obtained by the PCA-based position sensor often exceeds 0.1 m.

B. Monte Carlo Performance Tests with Missing Data Correction

Following the methodology proposed in Section III, successful tests are made to check the enhanced performance of the localization system in presence of missing data. Thus, to validate this extension to the PCA-based approach, the same acquired depth data has been considered.

Comparing Fig. 5 with Fig. 3, it is possible to observe that the proposed method is able to eliminate the existing missing data and provide a position value with better accuracy. Analyzing the results presented in Fig. 6, it is possible to see that the results now obtained present position errors smaller than 0.1 m. Once the proposed method is able to compute the robot position with a better accuracy, its fusion with the

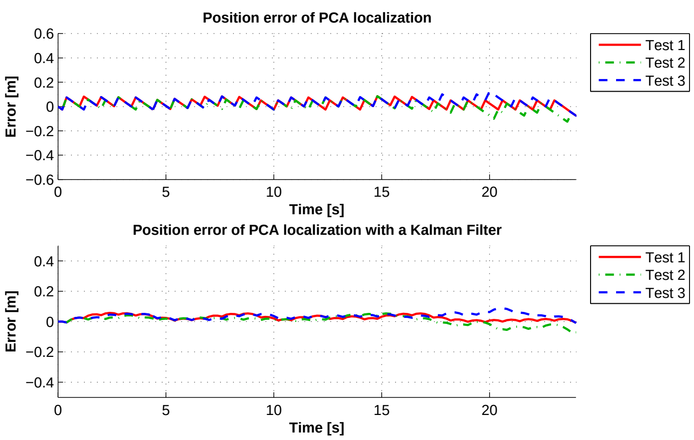

Fig. 6. Localization errors of tests along a straight line - new method

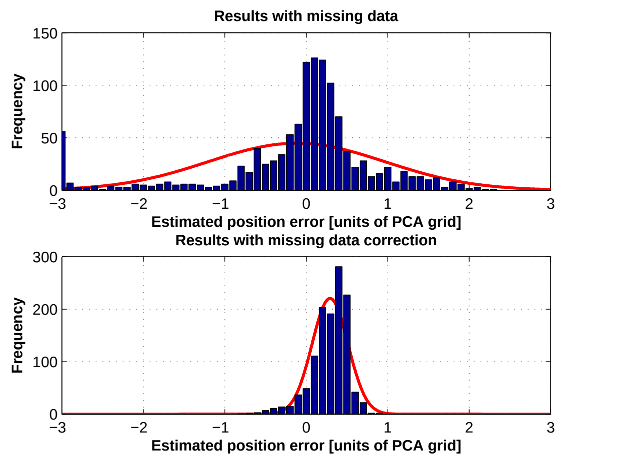

Fig. 7. Distribution of the estimated position error for both methods, considering a PCA grid with 0.1 m

odometry, through a KF, always provides a smoothly results and near of the real robot localization.

Finally, analyzing the histogram of the position error obtained by the PCA-based position sensor after the 10 performed tests (see Fig. 7), it is possible to see that the error of both methods is approximately Gaussian with a mean error close to zero.

Considering that the data to create the PCA eigenspace are acquired with 0.1 m of distance, it is possible to observe that all estimated position errors are less than the sampling distance of PCA eigenspace, while that considering signals with missing data, only about $68\ \%$ of results (1 standard deviation) are inside of this distance.

V. CONCEPT VALIDATION IN 2D LOCALIZATION

In order to solve the problem of 2D localization, a new PCA eigenspace is created with a set of captured depth images along a grid map with a distance of $0.3m$ (in x and y axis) in an area of $5m \times 4.5m$ (Fig. 8). The captured depth images are cropped with a circular mark allowing the rotation and comparison of captured depth images when the robot is in the same position, but with different attitude, during a mission. In order to compress the amount of data, the depth images are sampled with a compression ratio of $100:1$ and converted into

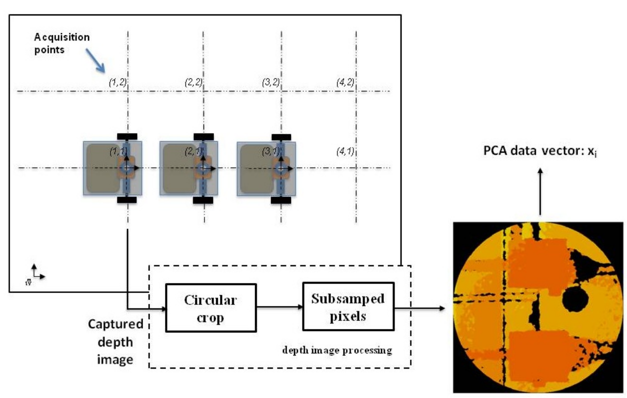

Fig. 8. Grid map and depth image processing to create a PCA eigenspace

a vector that will be added to PCA eigenspace. In [7], [5], the authors followed a similar approach using a RGB camera but the method revealed to be sensitive to illumination conditions.

During an experiment, it is possible to estimate the robot attitude and position, as well as the angular motion speed and the robot angular slippage, using only the signals obtained by the onboard sensors (Kinect, compass and encoders), through a self-localization sensor based in two KF and one PCA algorithm, with an architecture as detailed in Fig. 9.

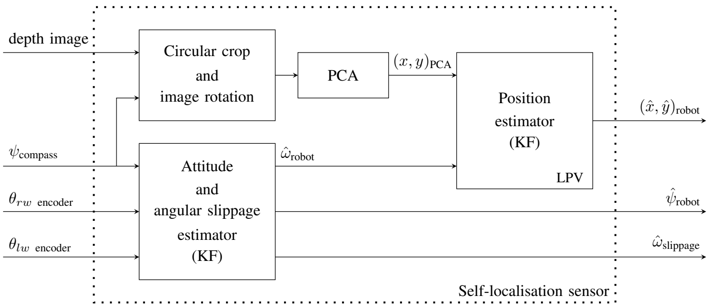

Fig. 9. Architecture of the self-localization sensor

The following notation is used in Fig. 9:

- $\psi_{compass}$ - orientation angle given by the compass;
- $\theta_{rwencoder}$ - angle given by the encoder of the right wheel;
- $\theta_{lwencoder}$ - angle given by the encoder of the left wheel;
- $(x, y)_{PCA}$ - coordinates given by the PCA sensor;
- $(\hat{x}, \hat{y})_{robot}$ - estimated robot coordinates in the world referential;
- $\hat{\omega}_{robot}$ - estimated angular speed;
- $\hat{\omega}_{slippage}$ - estimated differential slippage.

Detailing the architecture of the self-localization sensor presented in Fig. 9, the KF depicted on the left of the figure implements the attitude optimal estimator model that is responsible to estimate the mobile robot attitude and the angular slippage (see Appendix C). Once all acquired depth images for the PCA database are taken with the same orientation and compressed with a circular crop (Fig. 8), then during a mission, the acquired depth images must be rotated to zero degrees of attitude, using the compass angle, and compressed with the same circular crop. The position estimator (on the right of the figure) implements a Linear Parameter-Varying (LPV) model as a function of the estimated angular speed in a KF, fusing it with the position obtained by the PCA algorithm (see Appendix B).

Resorting to this architecture, it is possible to estimate the position, attitude and angular slippage of the mobile robot with a global stable error dynamic. For more details about this self-localization architecture see technical report [7].

A. Results for 2D localization

To test the mobile robot self-localization performance of the proposed approach in a environment (considering 2D localization), several tests are performed with the classical lawnmower type trajectory, combining both straight lines and

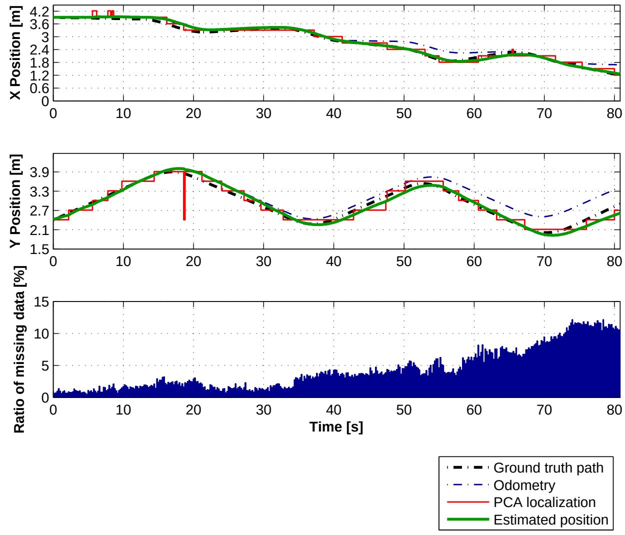

Fig. 10. Estimated position along time

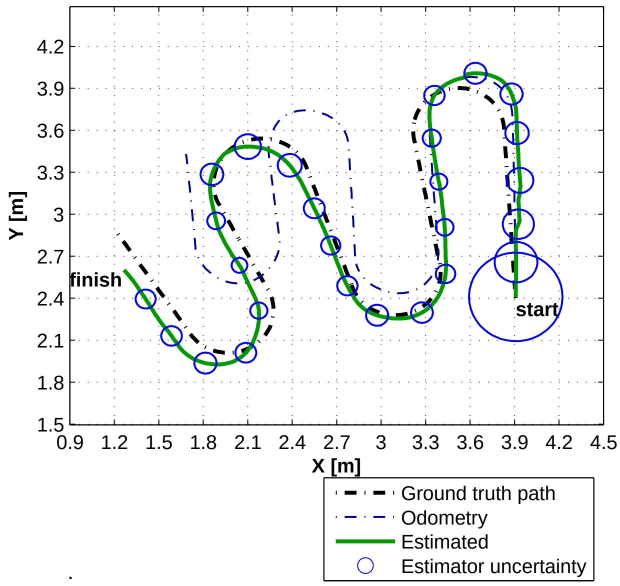

Fig. 11. Map with estimated position considering a ground truth path.

curves, with a $0.1 \text{ m} \cdot \text{s}^{-1}$ robot speed and 5 Hz of sampling frequency. During the robot motion the real mobile robot trajectory is measured allowing the comparison of the estimated position with the real one (ground truth test) and the corresponding position errors analyzed.

As it is possible to see in Fig. 10, that the position results obtained by the PCA algorithm is very close to the ground truth trajectory. Therefore, fusing the kinematic model of the robot with the position obtained by PCA in the KF allows estimating position values with a very good accuracy (see Appendix B for details).

Figure 11 shows the position estimated with the ground truth trajectory and the position obtained by the odometry. Comparing the results of the odometry with the estimated position it is possible to see an angular slippage in motion, that is increasing the difference between the estimated attitude

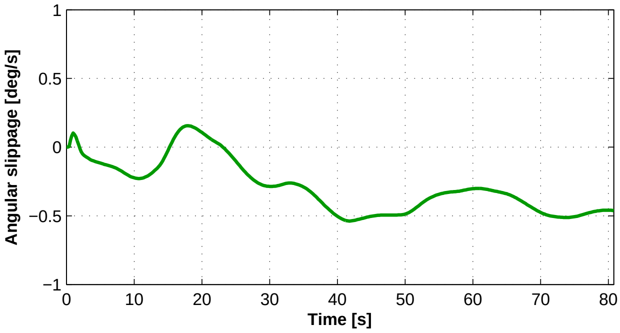

Fig. 12. Angular slippage estimated

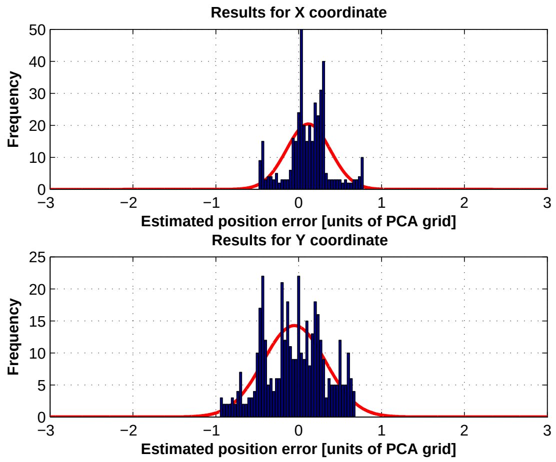

Fig. 13. Distribution of the estimated position error for both axis, considering a PCA grid with 0.3 m

and the one obtained by the odometry along time. This angular slippage is caused by systematic errors, such as uncertainties in the dimensions of the wheels, eccentric shaft problems, misalignment of the shafts, etc. It is possible to observe that in the initial part of the trajectory the estimator obtains a result close to the odometry. However, the localization system can approximate the estimated position with the ground truth trajectory.

Furthermore, analyzing Fig. 12, it is possible to observe the existence of an angular slippage of $-0.5\ \mathrm{rad} \cdot \mathrm{s}^{-1}$ (positive for slippage in clockwise direction), that is detected at $40\ s$ by the attitude estimator. Looking at Fig. 10 after $40\ s$ (instant which is detected angular slippage), the results of the position estimator are closer to the ground truth path than the odometry.

Finally, analyzing the histograms of Fig. 13 it is possible to conclude that the statistical distribution of the estimated position errors is approximately Gaussian with a mean close to zero. Moreover, comparing the variation of the distribution with the distance of the grid map acquired to create the PCA eigenspace, it is possible to see that, the proposed self-localization system is able to estimate the position with an error less than the distance between the acquired depth images, as happened with the localization in 1D.

VI. CONCLUSIONS

The existence of missing data in image is sometimes inevitable and it can induce a positioning system to an erroneous localization. In this paper an extension of a PCA methodology aiming to avoid the negative impact of missing data in signals is developed and experimentally validated. The proposed localization system is based only on a Kinect sensor installed onboard, looking upwards to the ceiling, where the depth sensor often provides signals with missing data, caused by IR beams that not were reflected.

All tests were successfully performed, allowing to conclude that the proposed approach can be useful in a number of mobile robotics applications where the existence of missing data is inevitable and causes a localization systems performance degradation. Moreover, the proposed method allows to validate the application of the Kinect depth sensor, in a mobile robot localization system based on a extension of a classical PCA algorithm to operating in unstructured environments. The propose localization system is optimal and globally stable, under the Gaussian approaches resorting to classical Kalman filtering techniques.

The method was successfully validated in a self-localization system, using only onboard sensors and estimates the position with a global stable error dynamics.

In the future, the proposed localization method will be implemented in a path following control approach, where the self-localization system will be integrated in a control close loop. Later, in order to increase the self-knowledge about the place, the proposed PCA algorithm will be updated to create a dynamic PCA database. This development will allow an architecture able to perform different tasks like obstacle avoidance, robot-human interaction, rescue activities or integration in a multi-robots platform for collaborative work.

APPENDIX

For the sake of readability of this paper, this appendix shows the models and estimators used for the 1D and 2D validation experiments. These models and estimators may be found, with more detail, in [7].

A. Model for position estimation in 1D localization

The mobile robot kinematic model that describes the movement in a straight line (1D) is

$$\dot{x} = u + b + \mu_1 \quad (5)$$

$$\dot{b} = 0 + \mu_2 \quad (6)$$

considering the following assumptions:

- the slippage velocity is constant or slowly varying (i.e. $\dot{b} = 0$);
- the noise in the actuation (motors are in closed loop) and the slippage velocity are assumed as zero-mean uncorrelated white Gaussian noise, $\mu_i \sim N(0, \sigma_i^2)$.

Expressing the model dynamics in a state-space system with $\mathbf{x} = [x\ b]^T$,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad (7)$$

$$y = [1\ 0]\,\mathbf{x} + \gamma \quad (8)$$

The output of this system y is the positioning sensor measurement described Section III. Since the position estimator is processed in a digital processor, the discrete model is obtained assuming that the vehicle velocity u is constant (zero order hold assumption) between two consecutive processing times, resulting

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} T \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} T & T^2/2 \\ 0 & T \end{bmatrix} \mu(k) \tag{9}$$

$$y(k) = [1\ 0]\,\mathbf{x}(k) + \gamma(k) \tag{10}$$

The design of a linear time-invariant Kalman filter for the underlying model described above is by now classic and the reader is referred to [10].

B. Model for position estimation in 2D localization

The classic differential drive mobile robot model is given by

$$\dot{x} = u\cos\theta \tag{11}$$
$$\dot{y} = u\sin\theta \tag{12}$$
$$\dot{\theta} = \omega \tag{13}$$

where u is the common mode speed, x and y are the robot coordinates in the world referential, θ is the orientation angle of the robot in the world referential and ω is the angular speed.

However, the classic non-linear model for differential drive mobile robots can be rewritten for a new state variables, becoming in a Linear Parameter Varying (LPV) model. Thus, differentiating: (11)–(13):

$$\ddot{x} = -u\,\omega\sin\theta = -\omega\dot{y} \tag{14}$$
$$\ddot{y} = u\,\omega\cos\theta = \omega\dot{x} \tag{15}$$
$$\ddot{\theta} = \dot{\omega} \tag{16}$$

and choosing as state vector $\mathbf{x} = [x\ \dot{x}\ y\ \dot{y}]^T$, a new LPV model for differential drive mobile robot is obtained:

$$\dot{\mathbf{x}} = \overbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & 0 & 1 \\ 0 & \omega & 0 & 0 \end{bmatrix}}^{\mathbf{A}} \mathbf{x} \tag{17}$$

$$\dot{\theta} = \omega \tag{18}$$

Considering the LPV model (17)–(18) and assuming that ω is constant between two sampling times (zero order hold assumption), the follow discrete model can be obtained (see [7] for more details):

$$\begin{aligned}\mathbf{x}(k+1) = {} & \overbrace{\begin{bmatrix} 1 & \frac{\sin\omega T}{\omega} & 0 & \frac{1}{\omega} + \frac{\cos\omega T}{\omega} \\ 0 & \cos\omega T & 0 & -\sin\omega T \\ 0 & \frac{1}{\omega} - \frac{\cos\omega T}{\omega} & 1 & \frac{\sin\omega T}{\omega} \\ 0 & \sin\omega T & 0 & \cos\omega T \end{bmatrix}}^{\mathbf{A}(\omega)} \mathbf{x}(k) + \\ & + \underbrace{\begin{bmatrix} T & \frac{1-\cos\omega T}{\omega^2} & 0 & -\frac{\omega T-\sin\omega T}{\omega^2} \\ 0 & \frac{\sin\omega T}{\omega} & 0 & -\frac{1-\cos\omega T}{\omega} \\ 0 & \frac{\omega T-\sin\omega T}{\omega^2} & T & \frac{1-\cos\omega T}{\omega^2} \\ 0 & \frac{1-\cos\omega T}{\omega} & 0 & \frac{\sin\omega T}{\omega} \end{bmatrix}}_{\mathbf{G}(\omega)} \mathbf{v}(k)\end{aligned} \tag{19}$$

$$\mathbf{y}(k) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{C}} \mathbf{x}(k) \tag{20}$$

Finally, in order to estimate the mobile robot position, the Linear Parameter Varying (LPV) model (19–20) is fused with the position obtained by the PCA-based position sensor, through the KF presented in Fig. 14, where and $x(k)$ and $y(k)$ are the position obtained by the PCA sensor in instant k and $\hat{x}(k)$ and $\hat{y}(k)$ are the estimated position in the same instant.

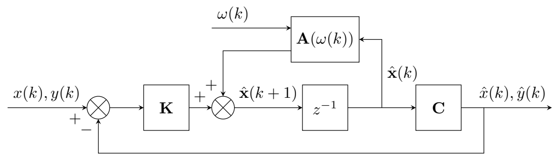

Fig. 14. Block diagram of the position estimator

C. Model for attitude and angular slippage estimation in 2D localization

The model that describes the angular motion of the differential drive mobile robot is

$$\dot{\psi} = \omega + s \tag{21}$$
$$\dot{s} = 0 \tag{22}$$

where ω is the angular speed, ψ is the attitude of the robot and s is the angular slippage in differential motion.

Considering the state vector $\boldsymbol{\theta} = [\psi \quad s]^T$, the kinematic model in state space can be defined by:

$$\dot{\boldsymbol{\theta}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \boldsymbol{\theta} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega \tag{23}$$

Assuming that signals processing is performed by a digital processor, ω and ψ are constant between sampling times (zero order hold assumption), allowing to obtain the discrete model of attitude:

$$\boldsymbol{\theta}(k+1) = \overbrace{\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}}^{\mathbf{A}} \boldsymbol{\theta}(k) + \overbrace{\begin{bmatrix} T \\ 0 \end{bmatrix}}^{\mathbf{B}} \omega(k) \tag{24}$$

$$y(k) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{C}} \boldsymbol{\theta}(k) \tag{25}$$

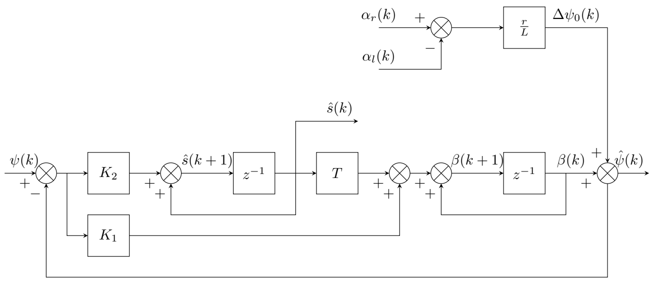

Fig. 15. Block diagram of the attitude and angular slippage estimator

Applying a KF to the discrete model (24–25), following the steps described in [7], the optimal attitude and angular slippage estimator presented in Fig. 15 is obtained, where $\psi(k)$ is the angle of compass captured in instant k, r is the radius of the wheels, l is the distance between wheels, $\alpha_r(k)$ and $\alpha_l(k)$ are the lengths of the paths of left and right wheels (that can be read directly from the encoders onboard) and $\hat{\psi}(k)$ and $\hat{s}(k)$ are the estimated attitude and angular slippage of the robot, respectively.

Finally, the angular speed of the robot applied in LPV (20) is obtained through a numerical difference of the estimated attitude of the robot:

$$\hat{\omega}(k) = \frac{\hat{\psi}(k) - \hat{\psi}(k-1)}{T} \qquad (26)$$

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REFERENCES

[1] Sergio Almansa-Valverde, Jos Carlos Castillo, and Antonio Fernndez-Caballero. Mobile robot map building from time-of-flight camera. *Expert Systems with Applications*, 39(10):8835 – 8843, 2012.

[2] Matej Artač, Matjaž Jogan, and Aleš Leonardis. Mobile robot localization using an incremental eigenspace model. In *IEEE International Conference on Robotics and Automation*, pages 1025–1030, Washington, DC, USA, May 2002.

[3] T. Bailey and H. Durrant-Whyte. Simultaneous localization and mapping (SLAM): part II. *Robotics Automation Magazine, IEEE*, 13(3):108 – 117, September 2006.

[4] C. Cardeira and J. Sá da Costa. A low cost mobile robot for engineering education. In *Industrial Electronics Society, 2005. IECON 2005. 31st Annual Conference of IEEE*, pages 2162–2167, Raleigh, USA, 2005.

[5] F. Carreira, C. Christo, D. Valério, M. Ramalho, C. Cardeira, J. M. F. Calado, and P. Oliveira. 2d pca-based localization for mobile robots in unstructured environments. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2012)*, pages 3767–3868, Vilamoura, Portugal, October 2012.

[6] F. Carreira, C. Christo, D. Valério, M. Ramalho, C. Cardeira, J. M. F. Calado, and P. Oliveira. Experimental validation of a PCA-based localization system for mobile robots in unstructured environments. In *Robotica - 12th International Conference on Autonomous Robot Systems and Competitions*, pages 69–74, Guimarães, Portugal, April 2012.

[7] F. Carreira, C. Christo, D. Valério, M. Ramalho, C. Cardeira, J. M. F. Calado, and P. Oliveira. Experimental validation of a PCA-based localization system for mobile robots in unstructured environments. IDMEC/CSI Internal Report, http://www1.dem.ist.utl.pt/cardeira/Mesh/Ca12b.pdf, June 2012.

[8] H. Durrant-Whyte and T. Bailey. Simultaneous localization and mapping: part I. *Robotics Automation Magazine, IEEE*, 13(2):99 –110, June 2006.

[9] Yasuaki Fukutani, Tomoyuki Takahashi, Masahiro Iwahashi, Tetsuya Kimura, Samsudin Siti Salbiah, and Norrima Binti Mokhtar. Robot vision network based on ceiling map sharing. In *11th IEEE International Workshop on Advanced Motion Control (AMC 2010)*, pages 164–169, Nagaoka, Niigata, March 2010.

[10] A. Gelb. *Applied optimal estimation*. MIT Press, 1974.

[11] Arturo Gil, Oscar Mozos, Monica Ballesta, and Oscar Reinoso. A comparative evaluation of interest point detectors and local descriptors for visual slam. *Machine Vision and Applications*, 21:905–920, 2010.

[12] A.S. Huang, A. Bachrach, P. Henry, M. Krainin, D. Maturana, D. Fox, and N. Roy. Visual odometry and mapping for autonomous flight using an rgb-d camera. In *15th International Symposium on Robotics Research (ISRR)*, Flagstaff, Arizona, USA, August 2011.

[13] W.T. Huang, C.L. Tsai, and H.Y. Lin. Mobile robot localization using ceiling landmarks and images captured from an rgb-d camera. In *IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM 2012)*, pages 855–860, Kachsiung, Taiwan, July 2012.

[14] I. Jolliffe. *Principal Component Analysis*. Springer-Verlag, 2002.

[15] Ben Kröse, Roland Bunschoten, Stephan Ten Hagen, Bas Terwijn, and Nikos Vlassis. Household robots look and learn: environment modeling and localization from an omnidirectional vision system. *IEEE Robotics & Automation Magazine*, 11:45–52, December 2004.

[16] Bor-Woei Kuo, Hsun-Hao Chang, Yung-Chang Chen, and Shi-Yu Huang. A light-and-fast slam algorithm for robots in indoor environments using line segment map. *Journal of Robotics*, 2011:12, 2011.

[17] U. Larsson, J. Forsberg, and A. Wernersson. Mobile robot localization: integrating measurements from a time-of-flight laser. *IEEE Transactions on Industrial Electronics*, 43(3):422–431, June 1996.

[18] Martin Lauer, Sascha Lange, and Martin Riedmiller. Calculating the perfect match: an efficient and accurate approach for robot self-localization. In *Robocup 2005: Robot soccer world cup IX*, pages 142–153. Springer, 2006.

[19] P. Oliveira. Interpolation of signals with missing data using pca. In *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2006)*, volume 3, page III, Toulouse, France, May 2006.

[20] P. Oliveira and L. Gomes. Interpolation of signals with missing data using principal component analysis. *Multidimensional Syst. Signal Process.*, 21(1):25–43, March 2010.

[21] Paulo Oliveira. MMAE terrain reference navigation for underwater vehicles using PCA. *International Journal of Control*, 80(7):1008–1017, July 2007.

[22] Davide Scaramuzza, Friedrich Fraundorfer, and Roland Siegwart. Real-time monocular visual odometry for on-road vehicles with 1-point ransac. In *IEEE International Conference on Robotics and Automation (ICRA2009)*, pages 4293–4299, Kobe, Japan, May 2009.

[23] Wolfram Burgard Sebastian Thrun and Dieter Fox. *Probabilistic Robotics*. Intelligent Robotics and Autonomous Agents. MIT Press, August 2005.

[24] C. Siagian and L. Itti. Biologically inspired mobile robot vision localization. *IEEE Transactions on Robotics*, 25(4):861–873, August 2009.

[25] J. Stowers, M. Hayes, and A. Bainbridge-Smith. Altitude control of a quadrotor helicopter using depth map from microsoft kinect sensor. In *IEEE International Conference on Mechatronics (ICM 2011)*, pages 358–362, Istanbul, Turkish, April 2011. IEEE.

[26] De Xu, Liwei Han, Min Tan, and You Fu Li. Ceiling-based visual positioning for an indoor mobile robot with monocular vision. *IEEE Transactions on Industrial Electronics*, 56(5):1617–1628, May 2009.