Detection of Transient Signals With Unknown Localization

Francisco M. Garcia, *Member, IEEE,* and Isabel M. G. Lourtie, *Senior Member, IEEE*

***Abstract*—In the context of real-time detection of transient signals, a likelihood ratio (LR) test is evaluated at every sampling interval. Performing the LR tests at a lower rate reduces significantly the computational complexity of the detection algorithm. However, in general, this simplification also leads to a strong degradation of the detector performance. For example, with a small shift error, an arriving transient may be in quadrature with its model. This degradation is particularly noticeable when the signals to detect are deterministic and sampled at a frequency close to the Nyquist rate. This letter proposes a method to overcome this limitation by using locally stationary models of the signals to detect. The resulting detectors are robust to shift errors and computationally efficient.**

***Index Terms*—Bandpass signals, detection, local stationarity, nonstationary processes, real-time processing, transient signals.**

I. Introduction

DETECTION of transient signals is an issue of major concern in many applications such as wireless communications, sonar and radar. Recent works in this field include [1] and [2] where, respectively, the gap metric and order statistics are used to detect signals with unknown parameters, namely delays or unknown arrival times. In a similar line of thought, in this letter, we propose a method to develop detectors for transient signals with random amplitude that are robust to shift errors, and exhibit low computational complexity.

This work addresses the problem of detecting a bandpass transient signal with unknown localization. For example, consider a surveillance environment where a certain signal is expected to arrive at the receiver with unknown arrival time. The generalized likelihood ratio test (GLRT) is the classical processor to solve this problem. It consists of an estimation/detection scheme, where the likelihood ratio is evaluated continuously along the time axis and its maximum value compared to a threshold. In general, the observation process itself is filtered and sampled with a sampling interval T_s, and the likelihood ratio (LR) is evaluated at the same rate. The GLRT is computationally consuming because it requires high sampling frequencies to avoid performance degradation due to time-shift errors which are, at most, of length $T_s/2$.

To reduce the computational cost (number of operations per time unity) of the processor, it would be desirable to use sampling rates close to the Nyquist frequency of the signal to detect, and to evaluate the LR at every T_t time intervals, with $T_t = kT_s$ and $k > 1$.

When the signals to detect are Gaussian stationary processes, the corresponding optimal quadratic processors are insensitive to time shifts. This is not the case for nonstationary processes (see [3]). In this letter, we consider a particular class of nonstationary processes, where the signals are known up to a random amplitude. As shown later, the 2-D autocorrelation function (ACF) of such signal is not locally stationary (NLS). In this case, a small shift error can strongly degrade the processor performance since the signal arriving at the receiver may be in quadrature with its model. In order to reduce the performance loss due to shift errors, we use a locally stationary (LS), rather than NLS, signal model. This leads to a robust detector that allows larger LR time intervals, thus with smaller computational complexity.

Our framework is derived from second-order characteristics of stochastic signals, related to the sampling theorem [4] and to the local stationarity of nonstationary processes [3], [5]–[7]. It is shown in [3] and [7] that, due to the positive definiteness of the ACF of a $L^2(\mathbb{R})$ nonstationary process [8], both its 2-D power spectrum (2DPS) and its Wigner distribution (WD) map information regarding either the Nyquist frequency for sampling purposes and the existence (or not) of local stationarity of the process. In [3], we present a simple method to obtain a LS covariance matrix (which is a sampled version of the ACF) of a zero-mean second-order process from data. Here, the ACF of the signal has a single nonzero eigenvalue. We show that the corresponding LS ACF has 2 nonzero eigenvalues, and derive the corresponding eigenvectors as functions of the eigenvector of the signal to detect.

II. Locally Stationary Autocorrelation Functions

Let $x(t), t \in \mathbb{R}$, be a zero-mean $L^2(\mathbb{R})$ nonstationary stochastic process characterized by the autocorrelation function (ACF) $k(t_1, t_2)$, which can be expressed in terms of its eigenfunctions, $\phi_i(t)$, and eigenvalues, λ_i, by the Mercer-like expansion [7]

$$k(t_1, t_2) = \sum_{i=1}^{\infty} \lambda_i \phi_i(t_1) \phi_i(t_2).$$

Equivalently, the process $x(t)$ can be described either by the 2-D power spectrum (2DPS)

$$K(\omega_1, \omega_2) = \mathrm{FT}_{t_1} \left[\mathrm{FT}_{t_2} [k(t_1, t_2)](-\omega_2) \right] (\omega_1)$$

or by the Wigner distribution (WD)

$$S(t, \omega) = \mathrm{FT}_{\tau} \left[k \left(t + \frac{\tau}{2}, t - \frac{\tau}{2} \right) \right] (\omega)$$

where $\mathrm{FT}[\cdot](\omega)$ denotes the Fourier transform.

Manuscript received October 9, 2003; revised February 10, 2004. This work was supported by the FCT, POSI, and FEDER under Project POSI/32708/CPS/1999. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Marcelo G. S. Bruno.

The authors are with the ISR-Instituto de Sistemas e Robótica, IST-Instituto Superior Técnico, 1049-001 Lisboa, Portugal (e-mail: fmg@isr.ist.utl.pt; iml@isr.ist.utl.pt).

Digital Object Identifier 10.1109/LSP.2004.833448

1070-9908/04$20.00 © 2004 IEEE

Let us also assume that $x(t)$ is a real bandpass transient that has most of its energy lying in the interval $I = I^- \cup I^+$, where $I^+ = [\omega_{\min}; \omega_{\max}]$ and $I^- = [-\omega_{\max}; -\omega_{\min}]$. In other words, its 2DPS, $K(\omega_1, \omega_2)$, is approximately zero everywhere in the plane $\{\omega_1, \omega_2\}$, except in the following situations [3], [7]:

1) $(\omega_1, \omega_2) \in (I^+ \times I^+)$ or $(\omega_1, \omega_2) \in (I^- \times I^-)$ (regions in the first and third quadrants of the (ω_1, ω_2) plane).

2) $(\omega_1, \omega_2) \in (I^+ \times I^-)$ or $(\omega_1, \omega_2) \in (I^- \times I^+)$ (regions in the second and fourth quadrants of the (ω_1, ω_2) plane).

In general, the process $x(t)$ is NLS which means that region 2) referred to above does not fade.

However, as detailed in [3], a LS ACF of a bandpass process is easily obtained by eliminating in any of the equivalent representations of the process (ACF, 2DPS, or WD) the terms that are nonzero only when the process is NLS. For example, a LS ACF is obtained by i) making the second and fourth quadrants of $K(\omega_1, \omega_2)$ equal to zero, and ii) taking the double inverse Fourier transform of the resulting 2DPS. For the WD, local stationarity is characterized by the absence of cross-terms around frequency $\omega = 0$. This corresponds to a basic difference between the WD for locally stationary processes and deterministic signals, where cross-terms are always present unless the analytic signal, rather than the original signal itself, is transformed.

III. Transient Detection Robust to Shift Errors

The detection problem is formulated as a simple binary test. The observation process $r(t)$ is defined as

$$r(t) = \begin{cases} s_1(t-\tau) + n(t), & \text{under hypothesis } H_1 \\ n(t), & \text{under hypothesis } H_0 \end{cases}$$

where $s_1(t)$ is a Gaussian-distributed stochastic process, τ is an unknown delay term and $n(t)$ corresponds to a zero mean Gaussian distributed process with autocorrelation function $\sigma^2 \delta(t_1 - t_2)$, where $\delta(t)$ and σ^2 represent the Dirac delta function and the proportionality coefficient of the noise variance, respectively.

When $\tau = 0$, the optimal solution for the detection problem is the quadratic processor [9], defined by the LR test

$$\mathrm{LR} = \sum_{i=1}^{N} \frac{c_i^2 d_i}{\sigma^2 + d_i} \underset{\mathrm{H_0}}{\overset{\mathrm{H_1}}{\gtrless}} \eta \quad (1)$$

where N is the number of nonzero eigenvalues of the ACF of $s_1(t), c_i$ denotes the coefficient resulting from the internal product between the observation process and the i-th eigenfunction, and d_i and η correspond to the i-th eigenvalue and a comparison threshold, respectively. The value of η may be chosen using a Bayes or Neyman-Pearson criteria [9].

When τ is unknown, the classical processor is the GLRT. In this letter, we consider $s_1(t) = As(t)$, where $s(t)$ is a known bandpass transient, and A is a zero-mean Gaussian distributed random amplitude with unit variance (for simplicity). Its ACF

$$k_1(t_1, t_2) = \lambda \phi(t_1) \phi(t_2) \quad (2)$$

has a single nonzero eigenvalue λ and corresponding eigenfunction $\phi(t)$, given by

$$\lambda = \int_{-\infty}^{+\infty} s^2(t)\, dt, \quad \phi(t) = \frac{s(t)}{\sqrt{\lambda}}. \quad (3)$$

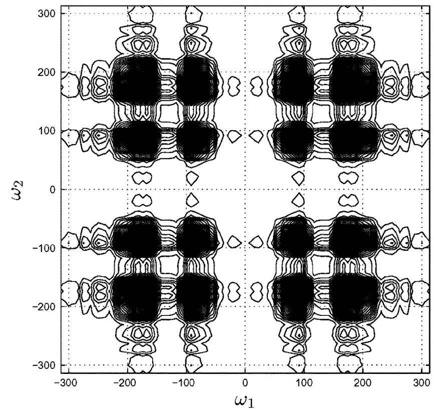

Fig. 1. Absolute value of the 2DPS of a nonlocally stationary autocorrelation function.

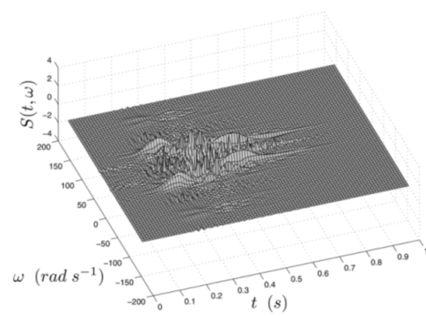

Fig. 2. Wigner distribution of the nonlocally stationary autocorrelation function.

In Figs. 1 and 2, we represent, respectively, a typical 2DPS and WD of such process. Note that the nonzero terms on the second and fourth quadrants of the 2DPS give rise to the cross-terms around $\omega = 0$ in the WD. As stated before, the classical receiver (GLRT) for this process is sensitive to shift errors, thus requiring the evaluation of the LRs at a high rate, resulting computationally complex.

If, in (1), the coefficients c_i are computed based on a LS ACF, the corresponding receiver is robust to shift errors. To obtain a LS ACF $k_2(t_1, t_2)$ from $k_1(t_1, t_2)$, we eliminate the nonzero terms in both the second and fourth quadrants of the 2DPS, $K_1(\omega_1, \omega_2)$, and keep unchanged its first and third quadrants. For the NLS process $k_1(t_1, t_2)$ given by (2), the correspondent LS ACF

$$k_2(t_1, t_2) = \sum_{i=1}^{2} \alpha_i \psi_i(t_1) \psi_i(t_2)$$

has two nonzero eigenvalues $\alpha_i, i = 1, 2$, and eigenfunctions $\psi_i(t)$, that satisfy, in the frequency domain, the following relationships:

$$K_2(\omega, \omega) = \sum_{i=1}^{2} \alpha_i |\Psi_i(\omega)|^2 = \lambda |\Phi(\omega)|^2 = K_1(\omega, \omega)$$

where $\Psi_i(\omega)$ and $\Phi(\omega)$ are, respectively, the Fourier transforms of $\psi_i(t)$ and $\phi(t)$, and

$$K_2(\omega, -\omega) = \sum_{i=1}^{2} \alpha_i \Psi_i^2(\omega) = 0.$$

The solution to the previous equations satisfies

$$\alpha_1 = \alpha_2 = \frac{\lambda}{2}, \quad |\Psi_1(\omega)| = |\Psi_2(\omega)| = |\Phi(\omega)|,$$

and

$$\Psi_2(\omega) = j\text{sign}(\omega)\Psi_1(\omega) \tag{4}$$

where $\text{sign}(\omega)$ stands for the signum function ($+1$ for $\omega > 0$, and -1 for $\omega < 0$). From the Parseval relationship, for any functions $\psi_1(t)$ and $\psi_2(t)$ related by (4), we have

$$\begin{aligned}\int_{-\infty}^{+\infty} \psi_1(t)\psi_2(t)\,dt &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Psi_1(\omega)\Psi_2(-\omega)\,d\omega \\ &= \frac{j}{2\pi}\int_{-\infty}^{+\infty} |\Psi_1(\omega)|\text{sign}(-\omega)\,d\omega \\ &= 0.\end{aligned}$$

Thus, $\psi_1(t)$ and $\psi_2(t)$ are orthonormal. Moreover, and since the eigenfunctions of both processes have the same absolute value, we may choose

$$\Psi_1(\omega) = \Phi(\omega) \quad \forall\omega \tag{5}$$

which guarantees that $\psi_1(t)$ and $\psi_2(t)$, given by (4)–(5), are unitary, and represent the eigenfunctions of process $k_2(t_1, t_2)$. Finally, we note that

$$\begin{aligned}K_2(\omega_1,\omega_2) &= \sum_{i=1}^{2} \alpha_i \Psi_i(\omega_1)\Psi_i(\omega_2) \\ &= \frac{\lambda}{2}[1 - \text{sign}(\omega_1)\text{sign}(-\omega_2)]\Phi(\omega_1)\Phi(-\omega_2) \\ &= \frac{1}{2}[1 - \text{sign}(\omega_1)\text{sign}(-\omega_2)]K_1(\omega_1,\omega_2).\end{aligned}$$

This expression clearly shows that $K_2(\omega_1, \omega_2)$ vanishes when the signs of ω_1 and ω_2 are opposite, that is, on the second and fourth quadrants of the (ω_1, ω_2) plane. Consequently, $k_2(t_1, t_2)$ represent the ACF of a LS process. The corresponding 2DPS and WD are represented, respectively, in Figs. 3 and 4 and clearly show that the process is locally stationary: both the terms of the second and fourth quadrant of the 2DPS and the cross-terms of the WD at frequency $\omega = 0$ vanish.

As mentioned above, the processor based on the LR test (1) is robust to shift errors if the coefficients c_i are computed based on a LS ACF. For example, in the particular case where the signal to detect is a narrowband process centered at frequency ω_0 and assuming a large SNR ($\sigma^2 \to 0$), it is easily shown that, under hypothesis H_1, in the NLS situation, we have $E[\text{LR}_{\text{NLS}} \,|\, H_1] \simeq \lambda\cos^2(\omega_0\tau)$ whereas, in the LS case, $E[\text{LR}_{\text{LS}} \,|\, H_1] \simeq \lambda$, where $E[\cdot]$ stands for the expected value. Therefore, for $\tau \simeq k2\pi/\omega_0, k \in \mathbb{Z}$, we have $E[\text{LR}_{\text{NLS}} \,|\, H_1] \simeq 0$ and the detector performance strongly degrades. On the contrary, the expected value of the LR using the LS model is approximately constant and independent of τ.

IV. Case Study

This experiment compares the robustness to shift errors and the computational complexity of the quadratic detectors based either on the LS or on the NLS ACFs. Several detection simulations, each of which with 100 000 Monte-Carlo runs, were conducted as follows: 1) the signal to detect is a compressed air shot sound recorded in an underwater environment; 2) for each Monte-Carlo run, the (known) signal is multiplied by a random amplitude with unitary variance; 3) a shift error corresponding to N_e sampling intervals is applied to the signal, where N_e is a continuous uniform random variable taking values in the interval $[-N_{\text{lim}}; N_{\text{lim}}]$; 4) the signal is corrupted by adding white noise with variance 0.005; and 5) for each processor, the LR tests (1) were computed. For each value of N_{lim}, the threshold η was chosen such that the desired probabilities of false alarm were matched. The classical NLS processor uses the single eigenfunction $\phi(t)$ defined in (3) ($N = 1$), whereas the LS processor uses the eigenfunctions $\psi_1(t)$ and $\psi_2(t)(N = 2)$, obtained from $\phi(t)$ as described in the previous section.

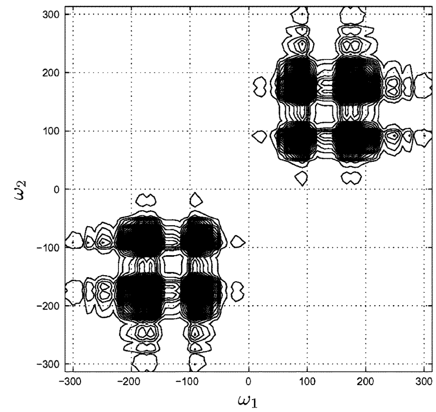

Fig. 3. Absolute value of the 2DPS of the locally stationary autocorrelation function.

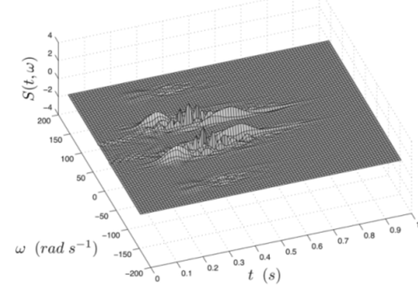

Fig. 4. Wigner distribution of the locally stationary autocorrelation function.

For integer values of $2N_{\text{lim}}$, the simulation study described in the previous paragraph accesses the performance of the detectors when the LRs are evaluated sequentially at every $T_t = 2N_{\text{lim}}T_s$ time intervals. The detection results are presented in Figs. 5 and 6, where we plot the probability of detection (P_D) as a function of N_{lim}, for probabilities of false alarm (P_{Fa}) of 0.001 and 0.1, respectively. As expected, the NLS processor outperforms the LS receiver for small values of N_{lim}. Remark that $N_{\text{lim}} = 0$ denotes the case where the receiver knows exactly the transient arrival time, for which the NLS detector is optimal. Moreover, the classical situation where the LR tests are performed at every sampling interval T_s corresponds to $N_{\text{lim}} = 0.5$, in which the performance of both processors is approximatively identical. For larger values of N_{lim}, the performance of the NLS processor degrades dramatically, whereas the LS re-

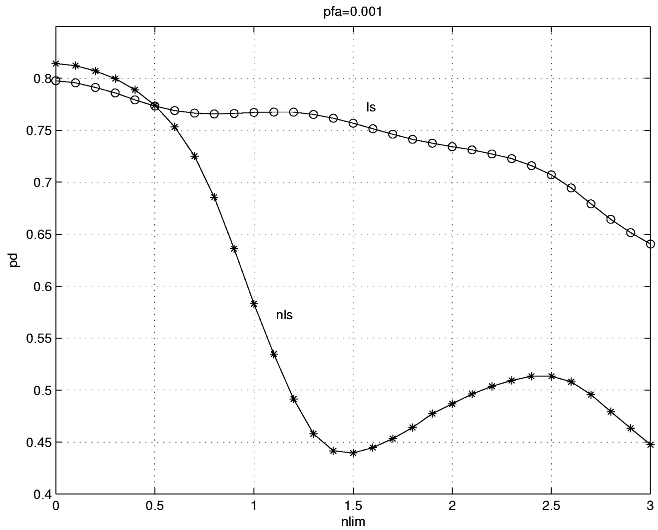

Fig. 5. Probability of detection versus $N_{\text{lim}}(\text{pfa} = 0.001)$.

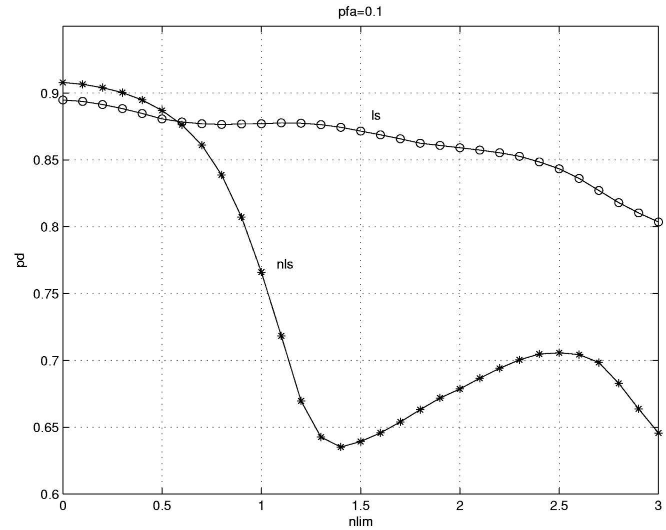

Fig. 6. Probability of detection versus $N_{\text{lim}}(\text{pfa} = 0.1)$.

ceiver exhibits a much smaller performance loss. For example, from $N_{\text{lim}} = 0.5$ to $N_{\text{lim}} = 1.5$, the NLS processor P_D loss reaches 0.25 in both situations. On the contrary, the variation observed on the P_D of the LS receiver is smaller than 0.02. These experiments demonstrate the robustness to shift errors of the proposed detector when compared to the classical solution.

The robustness to shift errors exhibited by the LS processor enables the reduction of its computational complexity (CC). The CC is the number of operations (sums and multiplications) required by a processor per time unity. The costlier part of the LR evaluation corresponds to the internal products between the observation process and the eigenvectors. The NLS and LS processors have, respectively, 1 and 2 nonzero eigenvalues. Therefore, for a single LR computation, the LS processor performs approximately twice the number of operations of the NLS receiver's. This means that the LS processor CC is smaller than the NLS's only if it achieves a similar performance while computing the LRs at a rate at least half than the NLS processor's rate.

In Figs. 5 and 6, we observe that the performance of the NLS processor with a LR test interval equal to the sampling interval $(N_{\text{lim}} = 0.5)$ is only slightly better than the LS processor, when the LR tests are performed at every 2 sampling intervals $(N_{\text{lim}} = 1)$. In this case, the performance of both receivers is identical for the same CC. However, while the performance of the NLS processor degrades strongly for larger LR evaluation time intervals, the LS receiver still presents a small performance loss for $N_{\text{lim}} = 2$. For example, from Fig. 6, we see that the NLS processor achieves $P_D \simeq 0.885$ for $N_{\text{lim}} = 0.5$, while for the LS processor, $P_D \simeq 0.86$ for $N_{\text{lim}} = 2$. Thus, we obtain a gain of 50% in the CC at a small performance cost.

As a final remark, we note that the sampling frequency used in the experiment is larger (about 2.5 times) than the Nyquist frequency. When the sampling frequency gets closer to the Nyquist rate, which is the case in some applications where a low computational cost is mandatory, the probability of detection obtained for the NLS processor fades even faster than in the shown situation. For critical sampling rates, the performance of the LS processor with $N_{\text{lim}} = 1$ may, in general, be better than the NLS processor's with $N_{\text{lim}} = 0.5$ (same CC for both detectors).

V. Summary

In this letter, we develop a detector for transient signals that are known up to a random amplitude, which is based on a locally stationary (LS) description of the autocorrelation function (ACF) of the signal to detect. The analytical relationship between the eigenfunctions of the ACF and those of its LS estimate is also derived. The proposed detector is robust to shift errors between the signal model and a transient present in the observation process, and therefore is suited for real-time processing. Its performance/computational complexity improvement over the classical detector is illustrated by a real-data case study.

Acknowledgment

The authors would like to thank the reviewers and Prof. P. Aguiar from ISR/IST for the comments and suggestions that greatly improved the technical content of this manuscript.

References

[1] C. He and J. M. F. Moura, "Robust detection with the gap metric," *IEEE Trans. Signal Processing*, vol. 45, pp. 1591–1604, June 1997.

[2] E. Fishler and H. Messer, "Detection and parameter estimation of a transient signal using order statistics," *IEEE Trans. Signal Processing*, vol. 48, pp. 1455–1458, May 2000.

[3] F. M. Garcia and I. M. G. Lourtie, "Estimation of locally stationary covariance matrices from data," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Hong Kong, Apr. 2003.

[4] F. M. Garcia, I. M. G. Lourtie, and J. Buescu, "$L^2(\mathbb{R})$ nonstationary processes and the sampling theorem," *IEEE Signal Processing Lett.*, vol. 8, no. 4, pp. 117–119, Apr. 2001.

[5] W. Martin and P. Flandrin, "Wigner-Ville spectral analysis of nonstationary processes," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 1461–1470, Dec. 1985.

[6] S. G. Mallat, G. Papanicolau, and Z. Zhang, "Adaptive covariance estimation of locally stationary processes," *Ann. Statist.*, vol. 26, 1998.

[7] F. M. Garcia, I. M. G. Lourtie, and J. Buescu, "Local stationarity of $L^2(\mathbb{R})$ processes," in *IEEE Int. Conf. Acoust., Speech, Signal Processing*, Orlando, FL, May 2002.

[8] E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems*. New York: Springer-Verlag, 1985.

[9] H. L. Van Trees, *Detection, Estimation and Modulation Theory, Parts I and III*. New York: Wiley, 1968/1971.