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Single sensor source localization in a range-dependent environment

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Abstract— **Source localization with a single sensor explores the time spread of the received signal as it travels from the emitter to the receiver. In shallow water, and for ranges larger than a few times the water depth, the received signal typically exhibits a large number of closely spaced arrivals. However, not all the arrivals are equally important for estimating the source position since a number of them convey redundant information. Theoreticaly, identifying the non-redundant arrivals is feasible in a isovelocity range independent waveguide. In previous work, the number of non-redundant arrivals and the dimension of the data sample signal subspace have been related in a range-independent case. This paper addresses the problem of determining the number of significant arrivals for localizing a sound source over a range-dependent environment off the West coast of Portugal during the INTIMATE'96 sea trial.**

Keywords— **Source localization, subspace methods, shallow water, broadband signals.**

I. Introduction

Source localization with a single hydrophone is known to be a difficult problem in underwater acoustics, due to the reduced amount of spatial information. The lack of spatial information is to be compensated by the time spread of the emitted signal as it travels from the source to the receiver. Whether that time spread is sufficiently correlated to the medium of propagation to uniquely pinpoint the source position depends on a variety of factors such as the source range, water depth, sea bottom acoustic impedance, sea surface roughness, depths of source and receiver relative to the sound speed profile, etc... Previous work has shown that the correlation between the predicted and the estimated channel impulse responses was sufficient to track an acoustic source over various shallow water propagation environments [1],[2]. Alternatively, classical eigen analysis of the received time series allows to decompose the data set into two orthogonal subspaces that were used for source localization in a range independent environment [3]. A crucial step in time series analysis is to determine the order of the underlying signal model, that is to say, the dimension of the signal subspace. This paper attempts to shows how the signal subspace dimension can be interpreted in physical means by associating the identified eigenvectors with uncorrelated acoustic rays. In a range-dependent environment ray propagation is significantly altered and the number of eigenrays participating to the signal subspace should generally increase translating, in some sense, a higher degree of diversity and therefore an increased potential for localization. The real data that serves as illustration was obtained during the INTIMATE'96 experiment, off the west coast of Portugal, in a mild range-dependent environment (130 to 160 m water depth) for source ranges varying from 1 to 12 km. The emitted signal was a 300-800 Hz linear FM, with a 2 second duration.

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II. Background

A. Data model

A widely accepted model for the time series received at one acoustic sensor due to a sound source emitting a signal $s_0(t)$ at location $\theta_s = (r_s, z_s)$ is

$$y_n(t, \theta_s) = \sum_{m=1}^{M} a_{n,m}(\theta_s) s_0[t - \tau_m(\theta_s)] + \epsilon_n(t), \quad (1)$$

where $\epsilon_n(t)$ is the observation noise, assumed spatially and temporally white, zero-mean and uncorrelated with the signal and $a_{n,m}$ and τ_m are the replica amplitudes and time delays respectively. M is the number of signal replicas due to successive signal reflections between the source and the receiver. An implicit assumption in model (1) is that the M replicas observed at the receiver are stable within the data window, *i.e.*, that the variation in time delays $\tau_{n,m}$ with snapshot n is negligeable and therefore can be approximated by a single mean value τ_m.

A compact form for (1) is

$$\mathbf{y}_n(\theta_s) = \mathbf{S}[\boldsymbol{\tau}(\theta_s)]\mathbf{a}_n(\theta_s) + \boldsymbol{\epsilon}_n, \quad (2)$$

with the following matrix notations,

$$\mathbf{y}_n(\theta_s) = [y_n(1, \theta_s), y_n(2, \theta_s), \ldots, y_n(T, \theta_s)]^t, \quad \dim T \times 1 \quad (3a)$$

$$\boldsymbol{\tau}(\theta_s) = [\tau_1(\theta_s), \ldots, \tau_M(\theta_s)]^t, \quad \dim M \times 1 \quad (3b)$$

$$\mathbf{s}_0(\tau) = [s_0(-\tau), \ldots, s_0((T-1)\Delta t - \tau)]^t, \quad \dim T \times 1 \quad (3c)$$

$$\mathbf{S}[\boldsymbol{\tau}(\theta_s)] = [\mathbf{s}_0(\tau_1), \ldots, \mathbf{s}_0(\tau_M)], \quad \dim T \times M \quad (3d)$$

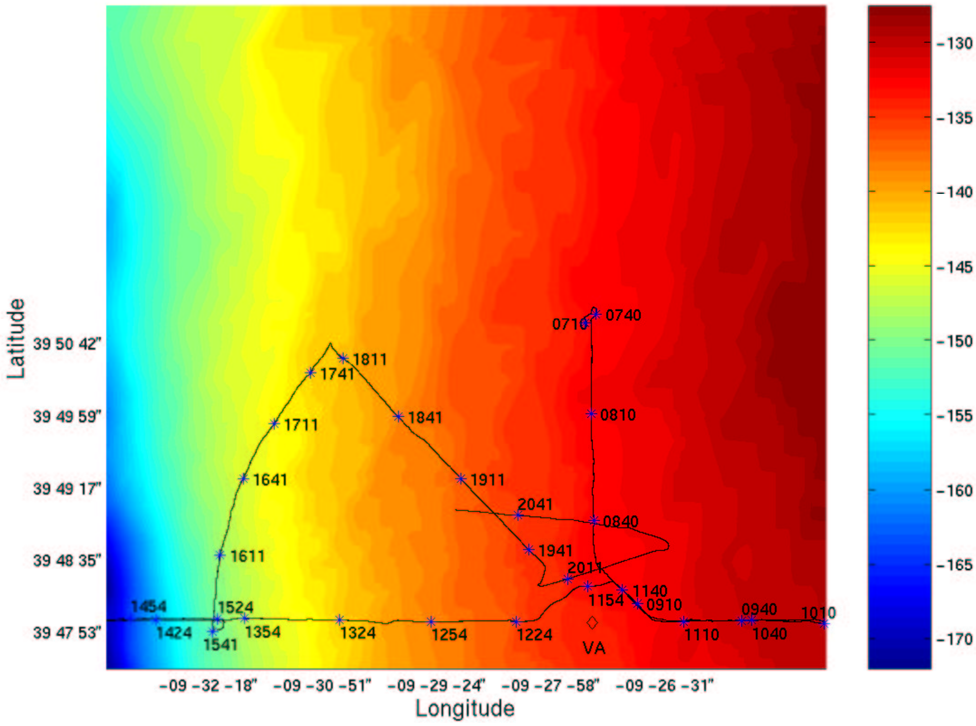

Fig. 1. INTIMATE'96: bathymetry map, source track and vertical array position (◇ VA) during Event II, on June 16, 1996.

and

$$\mathbf{a}_n(\theta_s) = [a_{n,1}(\theta_s), \ldots, a_{n,M}(\theta_s)]^t, \quad \dim M \times 1 \tag{3e}$$

where T is the number of time samples on each snapshot n.

B. Time delays and source localization

A classical objective in source localization as well as in travel-time tomography is to estimate the set of arrival times $\boldsymbol{\tau}$. Matching that set of arrivals with the model predicted values is the basis for the source localization process and of tomography inversion. A common procedure is to correlate the received time series with the source emitted signal to obtain the so-called pulse-compressed arrival pattern.That arrival pattern is an estimate of the channel impulse response that would be an exact image for a source signal with an infinite bandwith. It is well known that the maxima of the arrival pattern provide an optimum estimate of the arrival times $\tau_m; m = 1, \ldots, M$ in the maximum likelihood (ML) sens and under the assumption that the arrivals are uncorrelated, thus

$$\{\hat{\tau}_m^{\text{ML}}; m = 1, \ldots, M\} = \arg\{\max_{\tau} \sum_{n=1}^{N} \| \mathbf{y}_n^H \mathbf{s}_0(\tau) \|^2\}. \tag{4}$$

Assuming zero-mean random amplitudes, (2) becomes a linear random observation model and one may ressort to second order statistics for estimating $\boldsymbol{\tau}$. Decomposition of the data matrix $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N]$ allows for determining the principal components associated with the highest singular values that span the same subspace as the columns of matrix $\mathbf{S}$. Therefore an alternative estimator for the arrival times is given by

$$\{\hat{\tau}_m^{\text{SS}}; m = 1, \ldots, M\} = \arg\{\max_{\tau} \| \mathbf{U}_M^H \mathbf{s}_0(\tau) \|^2\}, \tag{5}$$

where the superscript ^{SS} denotes that the estimator is based on the signal subspace. A geometrical interpretation of (5) is that the arrival estimates are given by the intersection of the emitted signal continuum $\mathbf{s}_0(\tau)$, for all possible values of τ, and the subspace spanned by the columns of $\mathbf{U}_M$. As a matter of completeness one could as well determine the arrival estimates as the inverse of the projections onto the $\mathbf{U}_M$ orthogonal complement - the so-called noise subspace [4].

Source location can be readily deduced from the above estimators as the sum of the arrival amplitudes for the model predicted arrival set for each tentative source location. Two estimators will be compared in this paper, one based in (4)

$$\hat{\theta}_{\text{ML}} = \arg\{\max_{\boldsymbol{\tau}(\theta)} \frac{1}{NM} \sum_{n=1}^{N} \sum_{m=1}^{M} \| \frac{\mathbf{s}_0[\tau_m(\theta)]^H \mathbf{y}_n}{a_m(\theta)} \|^2\}, \quad \theta \in \Theta. \tag{6}$$

and one based in (5)

$$\hat{\theta}_{\text{SS}} = \arg\{\max_{\boldsymbol{\tau}(\theta)} \sum_{m=1}^{M} \| \frac{\mathbf{U}_M^H \mathbf{s}_0[\tau_m(\theta)]}{a_m(\theta)} \|^2\}, \qquad \theta \in \Theta. \tag{7}$$

Estimators (6) and (7) are very similar to those used in [4] for a range-independent data set. In the presente work normalization by the predicted amplitude $a_m(\theta)$ of each arrival was introduced to account for a 12 km wide range-dependent search interval for. Without that normalization the source location estimate was biased for the most energetic (first) arrivals and the source was consistently located at the beginning of the search interval.

C. Redundant arrivals and subspace dimension

In (5) and (7) $\mathbf{U}_M$ is a matrix whose columns are the vectors associated with the M highest singular vectors of data matrix $\mathbf{Y}$. In practice, and in presence of noise, a number of low-amplitude arrivals may be undetectable at the receiver and one problem is that of estimating the dimension of the data underlying signal subspace. In this paper, we used the minimum description length (MDL), which is a likelihood-based criterium proposed by Wax et all. [5] for estimating signal subspace dimension in a linear observation Gaussian model. Recent work has shown that, for a range-dependent isovelocity propagation channel and a generic geometry with source and receiver at diferent depths, the non-redundant arrivals were included in a single quadruplet[6]. Under some mild approximations this result could be generalized to non-isovelocity channels and compared to the results obtained in a range-independent event of the INTIMATE'96 data set where the MDL estimated signal subspace dimension was found to be in average equal to four [4].

Similar theoretical analysis is cubersome, if not impossible, for a range-dependent environment. However, it is well known, that in such environment, the quadruplet strucuture of the arrival pattern is destroyed, and therefore a higher number of non-redundant arrivals may be expected, possibly leading to a higher discrimination and therefore a better potential for source localization.

III. The Intimate'96 range-dependent data set

The Intimate'96 sea trial took place off the west coast of Portugal during June 1996. Results obtained in that data set have been reported elsewhere so the reader is referred to Demoulin *et al.* [7] for a complete description of the area and environmental conditions of the sea trial. The results presented here address the data gathered during Event II, from 07:10 to 20:41 of June 16, 1996. The bathymetry map, the source track and vertical receiving array position are shown in Fig. 1. The portion of the track until approximately 12:00 is nearly range-independent therefore we will concentrate in the remaining portion when the source ship goes off to the west until 14:30 and then in a arc-shaped track to the NE and finally back to the vertical array (VA). Water depth at source location during Event II is shown in Fig. 2.

During the same period of time CTD's were continuously made at the VA location and XBT's have been performed at the source position. Fig. 3 shows the sound speed profile calculated from XBT48 (14:03) that was used to initialize ray model TRIMAIN [8] to compute the predicted arrival times and amplitudes for each tentative source range. The bottom was characterized by a 1750 m/s compressional speed sandy layer with a density of 1.9 g/cm^3 and a com-

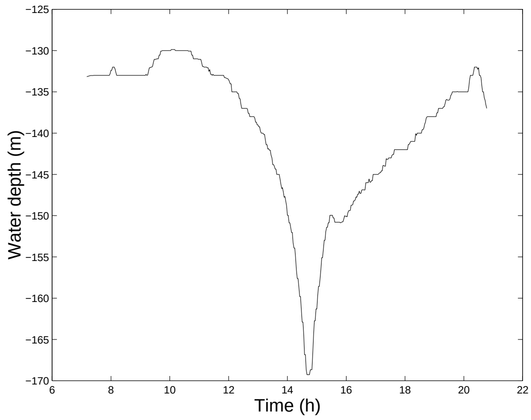

Fig. 2. Water depth at the source during Event II.

pressional attenuation of 0.8 dB/λ [4]. Bottom impedance characterization was found to be important for predicting late arrival amplitudes. Since these amplitudes were used for data balancing in (6) and (7) their estimation its was critical for source localization over this environment.

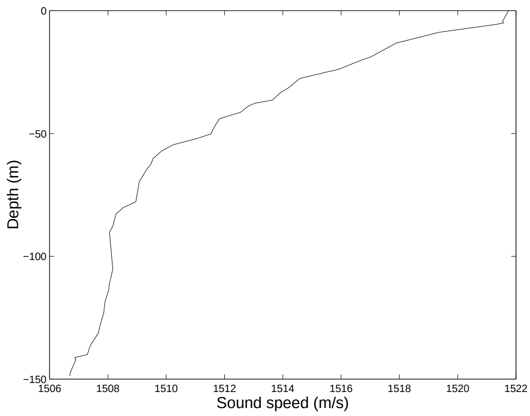

Fig. 3. Sound speed profile from XBT 48 (14:03).

The VA was composed of 4 hydrophones at nominal depths of 35, 70, 105 and 115 m. The emitted signal was an 300-800 Hz LFM sweep with 2 second duration and a repetition rate of 8 seconds. The source measured transfer function was used to filter the signal s_0 used for pulse compression at the receiver. Fig. 4 shows the source range estimation results at nominal depths obtained using only the hydrophone located at 115 m depth with both the ML and the SS methods (equations (6) and (7) respectively).

It can be remarked that despite the use of a range-independent ray model the source range (at correct depth) was correctly estimated at all times except for the largest ranges (approx. for r $>$ 9 km) corresponding to the strongest water depth variation of the run (Fig. 2). Attempts with range-dependent ray-tracing profiles were unsuccessful at the present time.

The source range and depth along track are shown in Fig. 5, where it can be seen that an evolution of the source towards or away from the VA contributes to an increase or a decrease of the dimensionality of the signal subspace

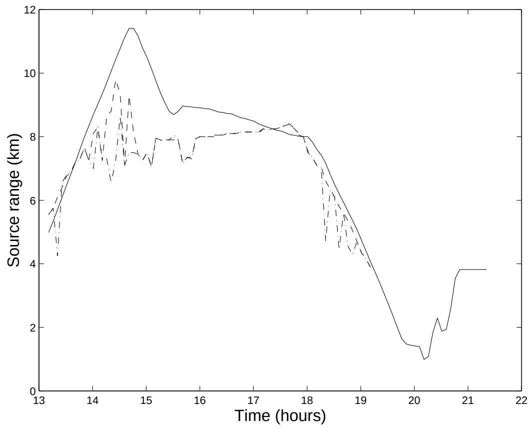

Fig. 4. Source range estimation during the range-dependent part of event II: GPS measured (—), ML estimated (-.-) and SS estimated (- - -).

that varies from a mean value of 4 at constant ranges and a value of up to 15 at ranges of less than, say, 5-6 km.

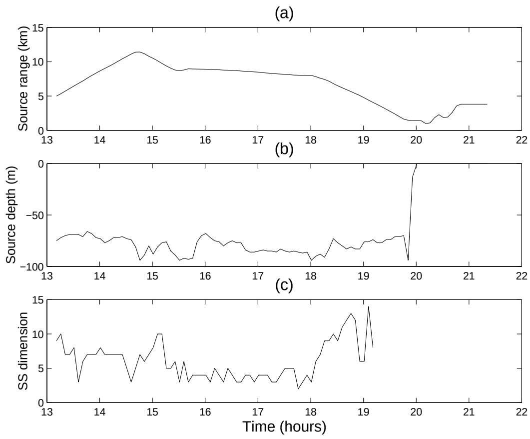

Fig. 5. INTIMATE'96: source range relative to vertical array (a), source depth (b) and estimated signal subspace dimension with MDL criteria (c), during the range-dependent part of event II, on June 16, 1996.

These results are in desagreement with the expectations. What is noted here is that somehow its the source range variation that has an impact on the dimensionality of the signal subspace and therefore seems to contribute to decorrelate the signal arrivals. It can be also noted that during these portions of the run (approximately 2.5 hours, from 13:00 to 14:00 and from 17:30 to 19:00) the SS method gave very accurate source range estimations and outperformed the ML estimator. Various tests keeping a constant subspace dimension throughout the run destroyed the localization.

IV. Conclusion

Source localization results in the INTIMATE'96 range-dependent data set have been reported by Porter et al. [2] using a direct correlation between the predicted and the estimated arrival paterns. To some extent these results are superior to those present in the present paper, specially at longer ranges. In this paper the goal was to understand the role of the uncorrelation between arrivals in a source tracking run over a range-dependent environment. In particular, from the analysis of over 6 hours of source range tracking in a 130 - 160 m water depth range-dependent environment, it was found that the number of independent arrivals varies significantly with the source range either outwards or towards the receiving array. At a constant range of 8 km, over an arc-shaped track, the number of uncorrelated arrivals defaults to approx. 4 or 5, despite the range-dependent nature of the propagation line. The source range estimation results given by the ML and the SS estimators are equivalent, except for the portions with stronger range variation where a correct tracking of the subspace dimension gave some advantage to the SS method. In conclusion one may ask what could be the advantages of using the SS method over the ML or the correlation based method of Porter et al. At this point only two advantages can be pointed out: one is theoretical and deals with the analysis of the principal components of the data and the understanding of their connection with the physical behaviour of ray-propagation. The other is practical and relates to the higher resolution of the SS method when compared to the ML method in cases of limited source signal bandwith (see simulated examples in [4]). Unfortunately there are also some drawbacks which are: a higher computational burden and a lower accuracy when compared to the direct correlation method of Porter et al..

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