RANGE DEPENDENT TOMOGRAPHY OF INTERNAL TIDES WITH RELATIVE ARRIVALS

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1 INTRODUCTION

Travel time based inversion is a technique of Ocean Acoustic Tomography [1], which allows to estimate a field of sound speed perturbation, δc, by inverting a system of linear equations, relating travel time and δc, through the so-called observation matrix. Such matrix can be calculated by representing the waveguide as a layered system, with δc being estimated at each layer. For practical purposes, the parameterization of δc on a basis of orthogonal functions can be more advantageous. For instance, the inversion developed in [2] introduces an expansion of δc on a basis of two-dimensional plane waves, whose amplitudes are estimated in order to map the temporal evolution of the environmental field in a complex bathymetry, through the usage of several sources and a significant amount of receivers. Theoretical modes can be used also as a basis for the estimation of $\delta c(r, z)$ [3]. For a reduced amount of receivers the vertical structure of the field can be expanded on the modes, while the range dependence can be constrained using horizontal plane waves. Such choice of basis significantly reduces the number of parameters to be estimated, but its reliability depends on plane wave propagation along range, which is only a particular case.
Furthermore, travel time inversion requires the synchronization between the emitted and received signals, so the instant of emission can be taken as the origin of the time axis for the arrivals. Otherwise, the technique has to be modified in order to develop the inversion. In the discussion presented in [4] relative-time inversion is developed, by expanding δc on a basis of empirical modes, and optimizing the match between observed and modeled arrivals within the search space of mode amplitudes. Although the validity of such approach depends on the uniqueness of the solution, the corresponding estimates of depth-averaged temperatures provided a good agreement with independent observations. The approach discussed in this paper reconsiders the parameterization of δc on a basis of theoretical modes, without imposing particular analytical constraints to their amplitudes, except that their variation is sufficiently smooth along range. Thus, wave propagation is not restricted to plane waves. Further, the system of equations is rewritten in order to relate modal amplitudes to relative arrivals. The performance of this approach is discussed using environmental and acoustic data from the INTIMATE'96 experiment [5].

2 THEORETICAL BACKGROUND

2.1 Travel time inversion

Within the context of acoustic tomography a travel time delay can be approximated as

$$\Delta\tau_j = -\int_{\zeta_j} \frac{\delta c}{c_0^2}\, d\zeta \;, \qquad (1)$$

where ζ_j corresponds to the jth stable eigenray, c_0 represents the background sound speed field and $\delta c(r,z)$ corresponds to the perturbation from c_0, which can be expanded on a basis of theoretical modes $\Psi_m(z)$ as [5]:

$$\delta c(r,z) = \sum_{m=1}^{M} \alpha_m(r) \frac{dc_0}{dz} \Psi_m(z) \ . \tag{2}$$

Substituting Eq.(2) into Eq.(1) leads to the following expression:

$$\Delta\tau_j = -\int_{\zeta_j} \sum_{m=1}^{M} \alpha_m(r) \frac{1}{c_0^2} \frac{dc_0}{dz} \Psi_m(z)\, d\zeta \ . \tag{3}$$

Discretizing along range r with K intervals, between the source and the receiver ($r = 0$ and $r = R$, respectively), it follows that

$$\begin{aligned} \Delta\tau_j &= -\sum_{k=1}^{K} \sum_{m=1}^{M} \int_{\zeta_j(r_k)}^{\zeta_j(r_{k+1})} \alpha_m(r) \frac{1}{c_0^2} \frac{dc_0}{dz} \Psi_m(z)\, d\zeta \approx \\ &\approx -\sum_{k=1}^{K} \sum_{m=1}^{M} \alpha_{km} \int_{\zeta_j(r_k)}^{\zeta_j(r_{k+1})} \frac{1}{c_0^2} \frac{dc_0}{dz} \Psi_m(z)\, d\zeta = \mathbf{S}_j^{\mathsf{t}} \boldsymbol{\alpha} \ , \end{aligned} \tag{4}$$

where

$$\boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_k \\ \vdots \\ \boldsymbol{\alpha}_K \end{bmatrix} , \quad \boldsymbol{\alpha}_k = \begin{bmatrix} \alpha_{k1} \\ \alpha_{k2} \\ \vdots \\ \alpha_{km} \\ \vdots \\ \alpha_{kM} \end{bmatrix} , \quad \mathbf{S}_j = \begin{bmatrix} \mathbf{S}_{j1} \\ \mathbf{S}_{j2} \\ \vdots \\ \mathbf{S}_{jk} \\ \vdots \\ \mathbf{S}_{jK} \end{bmatrix} , \quad \mathbf{S}_{jk} = \begin{bmatrix} S_{jk1} \\ S_{jk2} \\ \vdots \\ S_{jkm} \\ \vdots \\ S_{jkM} \end{bmatrix} \tag{5}$$

and

$$S_{jkm} = -\int_{\zeta_j(r_k)}^{\zeta_j(r_{k+1})} \frac{1}{c_0^2} \frac{dc_0}{dz} \Psi_m(z)\, d\zeta \ . \tag{6}$$

Furthermore, for a set of T travel time delays

$$\begin{aligned} \Delta\tau_1 &= \mathbf{S}_1^{\mathsf{t}} \boldsymbol{\alpha} \\ \Delta\tau_2 &= \mathbf{S}_2^{\mathsf{t}} \boldsymbol{\alpha} \\ &\cdots \\ \Delta\tau_{\mathsf{T}} &= \mathbf{S}_{\mathsf{T}}^{\mathsf{t}} \boldsymbol{\alpha} \end{aligned} \tag{7}$$

one can obtain the following system of equations:

$$\mathbf{y} = \mathbf{Q}^{\mathsf{t}} \boldsymbol{\alpha} + \mathbf{n} \ , \tag{8}$$

where

$$\mathbf{y} = \begin{bmatrix} \Delta\tau_1 \\ \Delta\tau_2 \\ \vdots \\ \Delta\tau_{\mathsf{T}} \end{bmatrix} , \quad \mathbf{Q} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_{\mathsf{T}} \end{bmatrix} , \tag{9}$$

and $\mathbf{n}$ stands for the contribution of noise from unknown sources. Therefore, the problem of estimating the field $\delta c(r,z)$ is reduced to the problem of estimating the modal amplitudes $\boldsymbol{\alpha}_k$, which are mapped into the vector $\boldsymbol{\alpha}$. Eq.(8) can be solved by calculating the generalized inverse of the observation matrix $\mathbf{Q}$ [6].

2.2 Inversion with relative arrivals

In the absence of synchronization the arrival τ_j becomes contaminated with an unknown time offset δt_c, so the perturbation in travel time for any two different arrivals i and j, at a single hydrophone, can be written as:

$$\Delta\tau_i = \mathbf{S}_i^{\mathsf{t}}\boldsymbol{\alpha} + n_i + \delta t_c \quad \text{and} \quad \Delta\tau_j = \mathbf{S}_j^{\mathsf{t}}\boldsymbol{\alpha} + n_j + \delta t_c \ . \tag{10}$$

Combining the two equations one can obtain the following relationship for the relative arrival $\tau_{j,i}$:

$$\tau_{j,i} = \mathbf{D}_{j,i}^{\mathsf{t}}\boldsymbol{\alpha} + N_{j,i} \ , \tag{11}$$

where $\tau_{j,i} = \Delta\tau_j - \Delta\tau_i$, $\mathbf{D}_{j,i} = \mathbf{S}_j - \mathbf{S}_i$ and $N_{j,i} = n_j - n_i$. Further, collecting all relative arrivals into a single vector $\mathbf{Y}$, one can introduce a non-synchronized travel time equivalent of Eq.(8):

$$\mathbf{Y} = \mathbf{D}^{\mathsf{t}}\boldsymbol{\alpha} + \mathbf{N} \ , \tag{12}$$

where

$$\mathbf{Y} = \begin{bmatrix} \tau_{2,1} \\ \tau_{3,1} \\ \vdots \\ \tau_{\mathsf{T},1} \\ \tau_{3,2} \\ \tau_{4,2} \\ \vdots \\ \tau_{\mathsf{T},\mathsf{T}-1} \end{bmatrix} = \begin{bmatrix} \Delta\tau_2 - \Delta\tau_1 \\ \Delta\tau_3 - \Delta\tau_1 \\ \vdots \\ \Delta\tau_{\mathsf{T}} - \Delta\tau_1 \\ \Delta\tau_3 - \Delta\tau_2 \\ \Delta\tau_4 - \Delta\tau_2 \\ \vdots \\ \Delta\tau_{\mathsf{T}} - \Delta\tau_{\mathsf{T}-1} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{S}_2 - \mathbf{S}_1 \\ \mathbf{S}_3 - \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_{\mathsf{T}} - \mathbf{S}_1 \\ \mathbf{S}_3 - \mathbf{S}_2 \\ \mathbf{S}_4 - \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_{\mathsf{T}} - \mathbf{S}_{\mathsf{T}-1} \end{bmatrix}$$
$$\text{and} \quad \mathbf{N} = \begin{bmatrix} N_{2,1} \\ N_{3,1} \\ \vdots \\ N_{\mathsf{T},1} \\ N_{3,2} \\ N_{4,2} \\ \vdots \\ N_{\mathsf{T},\mathsf{T}-1} \end{bmatrix} = \begin{bmatrix} n_2 - n_1 \\ n_3 - n_1 \\ \vdots \\ n_{\mathsf{T}} - n_1 \\ n_3 - n_2 \\ n_4 - n_2 \\ \vdots \\ n_{\mathsf{T}} - n_{\mathsf{T}-1} \end{bmatrix} . \tag{13}$$

Assuming a gaussian distribution of the components of $\mathbf{Y}$ the weighted inverse matrix of $\mathbf{D}^{\mathsf{t}}$, $\mathbf{D}^i$, can be written as [6]:

$$\mathbf{D}^i = \mathbf{P}^{-1}\mathbf{D}\mathbf{R}_Y \ , \tag{14}$$

where $\mathbf{P} = \mathbf{D}\mathbf{R}_Y\mathbf{D}^{\mathrm{t}}$, $\mathbf{R}_Y = \sigma_\alpha^2\mathbf{D}\mathbf{D}^{\mathrm{t}} + \mathbf{R}_N$, σ_α^2 represents the variance of $\boldsymbol{\alpha}$, and $\mathbf{R}_N$ corresponds to the covariance matrix of $\mathbf{N}$. For white uncorrelated noise $\mathbf{n}$, with a variance σ_n^2, $\mathbf{R}_N$ corresponds to

$$\mathbf{R}_N = \sigma_n^2 \begin{pmatrix} 2 & 1 & 1 & \dots & 1 & 1 & 0 \\ 1 & 2 & 1 & \dots & 1 & 0 & 1 \\ 1 & 1 & 2 & \dots & 0 & 1 & 1 \\ \vdots & & & & & & \\ 0 & 1 & 1 & \dots & 1 & 1 & 2 \end{pmatrix} . \tag{15}$$

Thus, the vector of modal amplitudes can be estimated as

$$\boldsymbol{\alpha}_Y^{\#} = \mathbf{D}^i\mathbf{Y} . \tag{16}$$

The multi-hydrophone case, for N hydrophones, can be handled by concatenating the systems of equations for the corresponding set of vectors $\mathbf{Y}_n$ and matrices $\mathbf{D}_n$ ($n = 1, 2, \dots, \mathsf{N}$) [3].

3 APPLICATION TO REAL DATA

Acoustic and environmental data from the INTIMATE'96 represented an important effort, directed to the acoustic mapping of internal tides. Acoustic data, acquired with a set of three hydrophones (at average depths of 35, 105 and 115 m), exhibit an excellent resolution of independent arrivals, although combines the lack of synchronization between the emitted and received signals, with a reduced amount of receiving hydrophones. Source depth was 90 m. Among the processed acoustic data it was chosen a particular set of $\mathsf{T} = 71$ arrivals, which could be validated with environmental data, acquired simultaneously at the position of the source and at the hydrophones (see Fig.1). The environmental variation of the field suggested the value $\sigma_\alpha = 0.5$ m, while preliminary simulations indicated that $\sigma_n = 1{\times}10^{-3}$ s. The choice of the number of modes, M, and the discretization in range, K, can not be done independently of T. In principle, a large value of M corresponds to a more complete basis for the expansion of δc on the set of theoretical modes. However, modes of higher orders can be very difficult to resolve from acoustic data. On the other side, a low value of K can degrade the approximation used in Eq.(4). In this sense it was decided that a reasonable compromise was to consider $M = 5$ and $K = 30$. In order to control the presence of noise only the smallest $M \times K$ relative arrivals were used in the estimation of $\boldsymbol{\alpha}$. An additional problem in the estimation was the fact that the matrix $\mathbf{P}$ was nearly singular, so the corresponding estimates of δc calculated with $\mathbf{P}^{-1}$ were unrealistic. In order to deal with this issue it was decided to calculate the inverse of $\mathbf{P}$ through a Singular Value Decomposition, using the smallest amount of singular values that provided a realistically bounded field of δc (i.e. $|\delta c| < 2$ m/s).

The corresponding estimates of $\delta c(r, z)$ at the position of the acoustic source and at the receiving hydrophones (i.e., estimates of δc in the first and last interval, respectively) are shown in Fig.2. The estimate at the acoustic source exhibits a good agreement in both phase and amplitude, while the estimate at the receiving hydrophones has been slightly overestimated.

4 CONCLUSIONS

The method presented in this paper represents a flexible approach to the general case of inversion, whether one deals with complex variations of the environment and/or the bathymetry, and when the emitted and received signals are not synchronized. Furthermore, when applied to real data from the INTIMATE'96 experiment, the method provided and accurate amplitude and phase estimate of the sound speed field at the position of the acoustic source, while the phase was correctly estimated at the position of the receiving hydrophones, with the amplitude being slightly overestimated. Such overestimation is considered to be a direct consequence of using a reduced amount of receivers. It is believed that a further improvement of the method can be achieved by understanding better the statistical properties of relative arrivals, so the presence of noise can be reduced in a more efficient way, rather than using an small set of relative arrivals.

References

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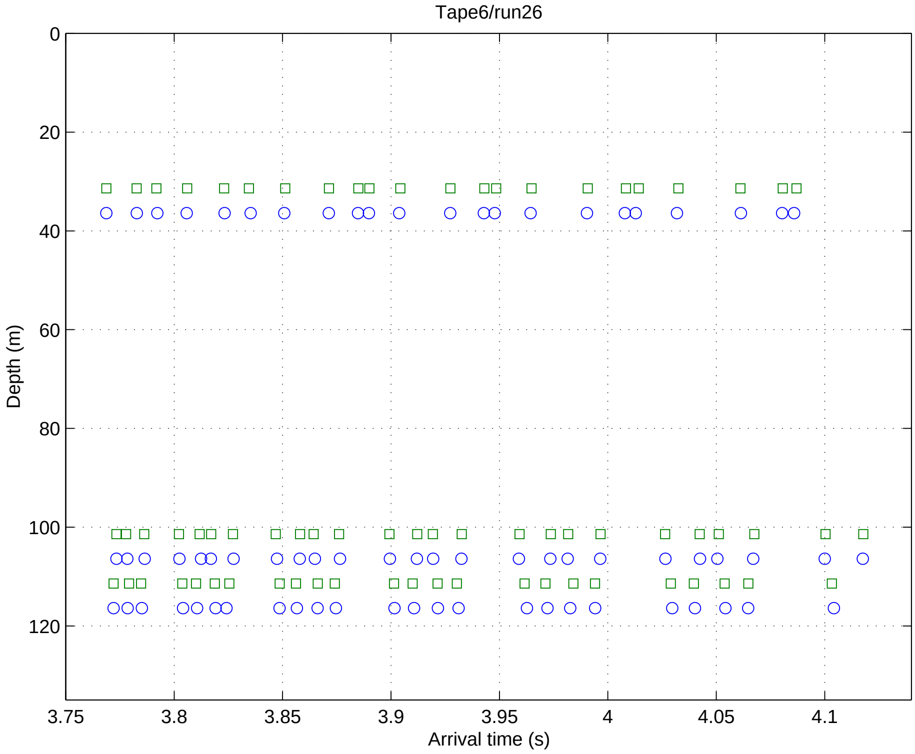

Figure 1: Stable arrivals aligned along receiver depths: circles correspond to real arrivals, while squares correspond to model arrivals. Real acoustic data is not synchronized.

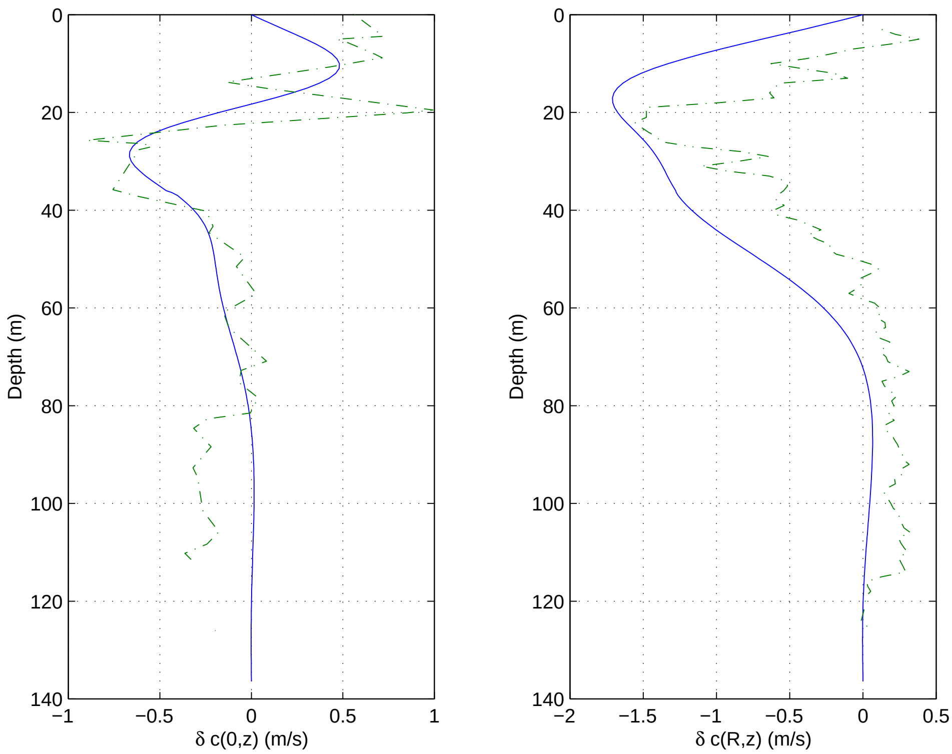

Figure 2: Extrapolated profiles near the acoustic source and the receiving hydrophones.