

Short communication

A mode subspace approach for source localization in shallow water

S.M. Jesus

Saclant Undersea Research Centre, Viale San Bartolomeo 400, I-19026 La Spezia, Italy

Received 25 March 1991

Revised 29 August 1991 and 28 November 1991

Abstract. Range and depth source localization in shallow water amounts to the estimation of the normal mode structure of the acoustic field. The new technique presented in this paper uses the spectral decomposition of the sample covariance matrix in order to estimate the mode subspace spanned by the normal modes that are significantly excited by the source. Simulation results obtained using realistic environments show that the technique's performance is always better than or equal to that of the generalized maximum likelihood processor.

Zusammenfassung. Die Ortung der Entfernung und Tiefe einer Schallquelle in Flachwasser erfordert die Schätzung der Normalmodenstruktur des Schallfeldes. Diese Arbeit stellt ein neues Verfahren vor, das die Spektralzerlegung der empirischen Kovarianzmatrix benutzt, um den von denjenigen Normalmoden aufgespannten Unterraum zu schätzen, die durch die Schallquelle signifikant angeregt werden. Ergebnisse von Simulationen in wirklichkeitsgetreuen Umgebungen zeigen, daß die Leistungsfähigkeit des Verfahrens immer besser oder gleich gut wie die des verallgemeinerten maximum likelihood Verfahrens ist.

Résumé. La localisation en distance et profondeur d'une source sonore en eaux peu profondes revient à l'estimation de la structure des modes normaux du champ acoustique. Ce papier présente une nouvelle technique qui utilise la décomposition spectrale de la matrice de covariance pour l'estimation du sous-espace modal engendré par les modes normaux significativement excités par la source sonore. Les résultats obtenus par simulation dans des environnements réalistes montrent que la performance de la méthode est toujours supérieure ou égale à celle du processeur de maximum de vraisemblance généralisé.

Keywords. Normal-modes, matched-field processing, shallow water.

1. Introduction

Sound propagation in shallow water is dominated by waveguide effects, thus normal-mode theory is needed to predict the acoustic field in time

and space. In such environments, due to the strong sound wave-boundary interaction, a large amount of information concerning the source location can be obtained by observing the field along the vertical axis. As 'seen' by a vertical array of sensors, and from a modelling point of view, the normal-mode structure of the acoustic field appears as a set of non-plane coherent waves closely spaced in angle.

Correspondence to: S. Jesus, Saclant Undersea Research Centre, Viale San Bartolomeo 400, I-19026 La Spezia, Italy.

Range and depth source localization in shallow water amounts to the estimation of the normal-mode structure of the acoustic field.

Several approaches to this problem have been described. The matched-field processing approach [4] consists of passing the received acoustic pressure through a bank of matched filters followed by a multi-dimensional peak detector. An alternative technique uses the normal-mode dependence to solve a linear system of equations and directly estimate the amplitude and phase of each individual normal-mode impinging on the array [9]. The source location is then estimated by coherent processing of the estimated normal-mode amplitudes and phases. This technique, known as normal-mode matching, was shown to give, for some (known) array configurations, a significant improvement in detection performance when compared to classical matched-field processing [7]. However, for these two techniques and for the frequency \times water depth products of interest, the detection factor, defined as the difference in dB between the maximum and the highest sidelobe of the range/depth surface, remains limited to a few dBs. When using real data these few dBs often result in practical source misses due to model mismatch. Recently, a source localization method known as generalized minimum variance, capable of achieving high sidelobe suppression has been presented [2]. The idea is based on a generalization of the maximum likelihood processor [5] to non-plane waves.

The technique presented in this paper implicitly uses the structure of the normal-mode acoustic field and it is, therefore, primarily applicable to waveguide type propagation. The solution to the problem is obtained as the intersection between the replica acoustic vector continuum (for all possible source locations) and a vector subspace of known dimension spanned by the normal modes that are significantly excited by the acoustic source(s). Simulation results obtained using realistic environments show that the performance is always better than or equal to that of the generalized minimum variance processor.

Signal Processing

2. Theory

2.1. The data model

The received signal is modelled as the solution of the wave equation at the receiver location for a narrow-band point source exciting a horizontally stratified, parallel waveguide. The normalized spatial dependence of the acoustic pressure measured at a vertical array of L sensors due to a source at location $\theta_T^T = (z_T, r_T)$, where superscript T stands for transpose and subscript T indicates the true source location, is commonly expressed as a linear combination of the waveguide normal-mode depth functions, i.e.,

$$p(\theta_T) = Ax(\theta_T), \quad (1)$$

where $p(\theta_T)$ is the L -dimensional array output signal vector, A is an $L \times M$ real matrix whose columns are the normal-mode depth functions expressed for all sensor depths $\{z_l; l=1, \dots, L\}$ and M is the number of modes supported by the waveguide. The M -dimensional complex vector $x(\theta_T)$ represents the model normal-mode structure for the true source parameter location θ_T defined as

$$x_m(\theta_T) = \frac{a_m(z_T)}{\sqrt{k_m}} e^{-\alpha_m r_T} e^{ik_m r_T}, \quad (2)$$

where α_m is the m th mode attenuation coefficient. The two sets $\{\alpha_m(z); m=1, \dots, M; 0 < z < H\}$ and $\{k_m; m=1, \dots, M\}$ are the mode depth functions and the corresponding mode horizontal wavenumbers characterizing the propagation channel of depth H . Note that (1)–(2) have been obtained by normalizing out the range dependence, a phase shift and an arbitrary constant. The received acoustic pressure consists of N time samples $\{y_n; n=1, \dots, N\}$ from a multivariate normal distributed random variable $Y, N(0, R_y)$, where the signal (1) is assumed to be corrupted by additive, uncorrelated and zero-mean Gaussian noise ϵ_n ; thus

$$y_n(\theta_T) = b_n Ax(\theta_T) + \epsilon_n, \quad (3)$$

where b_n is a random variable that represents the source amplitude. The vector $\mathbf{x}(\theta_T)$ may be considered as either deterministic with $\sum_{a=1}^N \mathbf{x}_a = \mathbf{0}$ and $\sum_{a=1}^N \mathbf{x}_a \mathbf{x}_a^H = \mathbf{R}_x$, or random, with $E\{\mathbf{x}\} = \mathbf{0}$ and $E\{\mathbf{x} \mathbf{x}^H\} = \mathbf{R}_x$. In either case, and even if \mathbf{A} is singular, \mathbf{Y} is distributed as defined above with $\mathbf{R}_y = \mathbf{A} \mathbf{R}_x \mathbf{A}^T + \mathbf{R}_\varepsilon$ [1]. The method presented in this paper applies equally well in both cases.

Source localization consists of the estimation of the source location parameter θ_T from the observation data set $\{\mathbf{y}_n; n=1, \dots, N\}$. A global estimation approach consists of the coherent processing of the received data directly in the sensor space. This is the approach taken in the conventional and minimum variance matched-field processors [4, 5]; neither of these approaches takes into account the specific structure of the data model. An attempt to use the normal-mode structure of the acoustic field was made in the normal-mode matching technique [7, 9]. This technique can be viewed as a conventional matched-field processor where the array structure is taken into account by reducing the dimension of the mode depth matrix \mathbf{A} such that $\mathbf{A}^T \mathbf{A} = \mathbf{I}$. Since it only depends on the array sensor depth, this is a completely deterministic procedure and it essentially differs from the technique presented in this paper where the sound field itself – by means of the power received through each normal-mode – is accounted for to set the rank of the data

cross-covariance matrix. The technique presented here brings together the ideas of noise suppression and a priori model structure knowledge in order to enhance source localization in waveguide types of propagation.

2.2. The mode subspace approach

An efficient way of solving (3) uses the mapping of the data vector into the subspace spanned by the columns of matrix \mathbf{A} : the mode subspace. The modal structure $\mathbf{x}(\theta_T)$ simply represents the coordinates of one point in the mode subspace. The intersection of the model replica acoustic vector continuum $\{\mathbf{p}(\theta); \theta \in \Theta\}$ and the mode subspace will give the solution $\mathbf{x}(\theta_T)$ and thus θ_T .

The implementation obviously requires an estimation of the mode subspace. The subspace spanned by the (M) eigenvectors of the sample cross-covariance matrix associated with the largest M eigenvalues is the maximum likelihood estimation of the required mode subspace. Therefore, if \mathbf{R}_y is the sample cross-covariance matrix with eigen decomposition $\mathbf{R}_y = \mathbf{E} \mathbf{\Lambda} \mathbf{E}^H$, the mode subspace is

$$\mathbf{E}_M = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_M], \quad (4)$$

with the corresponding eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$. Finding the intersection between the mod

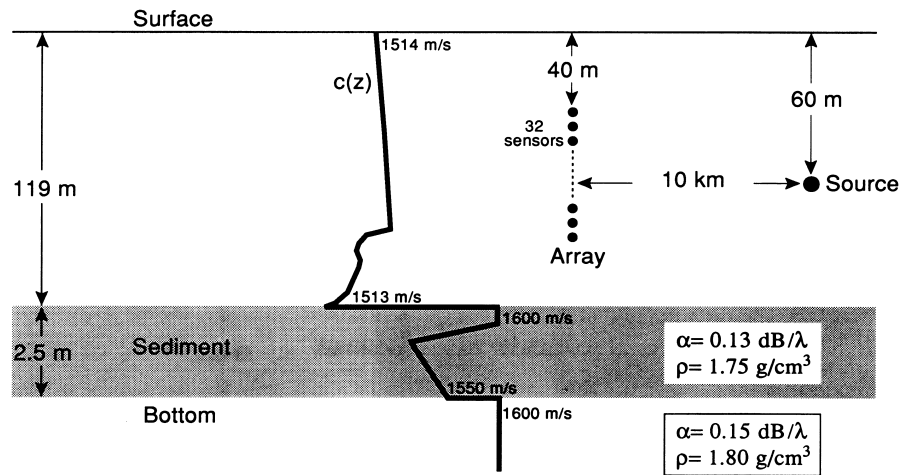


Fig. 1. Environmental/source/receiver scenario used for simulation.

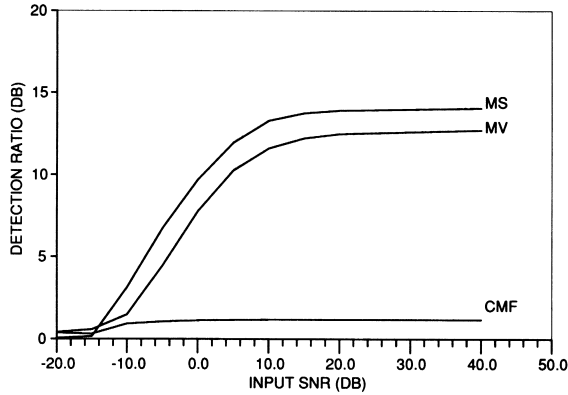


Fig. 2. Detection ratio versus input signal to noise ratio for mode subspace technique (MS), minimum variance (MV) and conventional matched-field processor (CMF).

subspace and the replica vector continuum $\{p(\theta); \theta \in \Theta\}$ amounts to the minimization of the length of $d = p(\theta) - p^\perp(\theta)$, where

$$p^\perp(\theta) = E_M E_M^H p(\theta) \quad (5)$$

is the orthogonal projection of the replica vector $p(\theta)$ onto the mode subspace. Obviously, d belongs to the mode subspace orthogonal complement E_{L-M} and the problem is solved by the minimization of the square distance

$$d^2(\theta) = |E_{L-M} E_{L-M}^H p(\theta)|^2. \quad (6)$$

The final estimate $\hat{\theta}_T$ is given by the coordinates of the maximum of the multi-dimensional

surface obtained by plotting the functional $\{[d^2(\theta)]^{-1}; \theta \in \Theta\}$.

3. Simulation performance example

The system/environment scenario used for simulation is shown in Fig. 1. The parameters shown were chosen to correspond to the environmental parameters which existed during a sea test experiment which was conducted north of Elba Island (Italy) in November/December 1989. The receiver is a 32 element 2 m spaced vertical array spanning the water column from 40 to 102 m depth. The sound source is located 10 km away from the receiver and its depth is 60 m. The source signal is a continuous wave tone at 323.7 Hz. Different signal-to-noise ratio (SNR) sequences were simulated by varying the source power $\sigma_b^2 = E[b_n^2]$ according to the definition

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \frac{\sigma_b^2 |p(\theta_T)|^2}{\sigma_\epsilon^2}. \quad (7)$$

Figure 2 shows the detection factor (difference in dB between the maximum and the highest sidelobe in the range/depth ambiguity surface) as a function of SNR for the three different techniques: conventional matched-field (CMF), minimum variance (MV) and mode subspace (MS). The number of

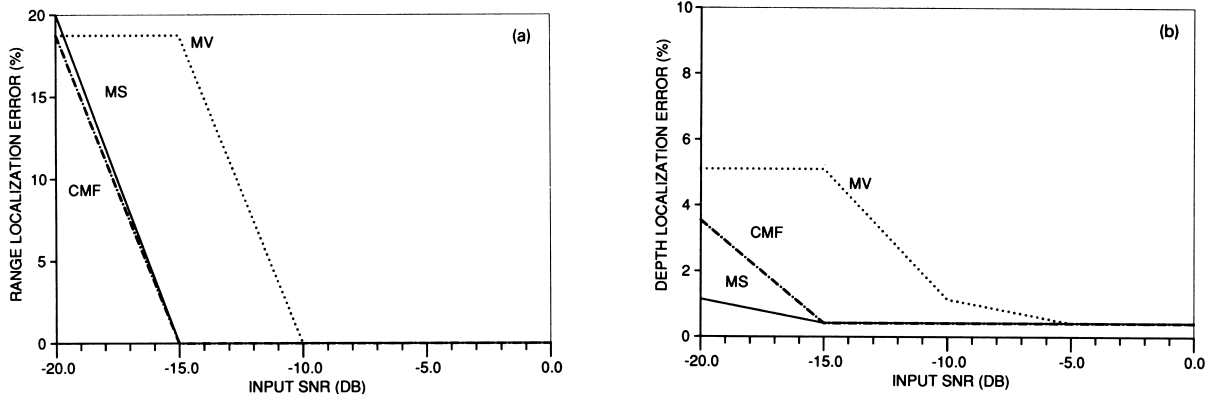


Fig. 3. Localization error versus input signal to noise ratio for the mode subspace (MS) technique, minimum variance (MV) and conventional matched-field (CMF) processor for range (a) and for depth (b).

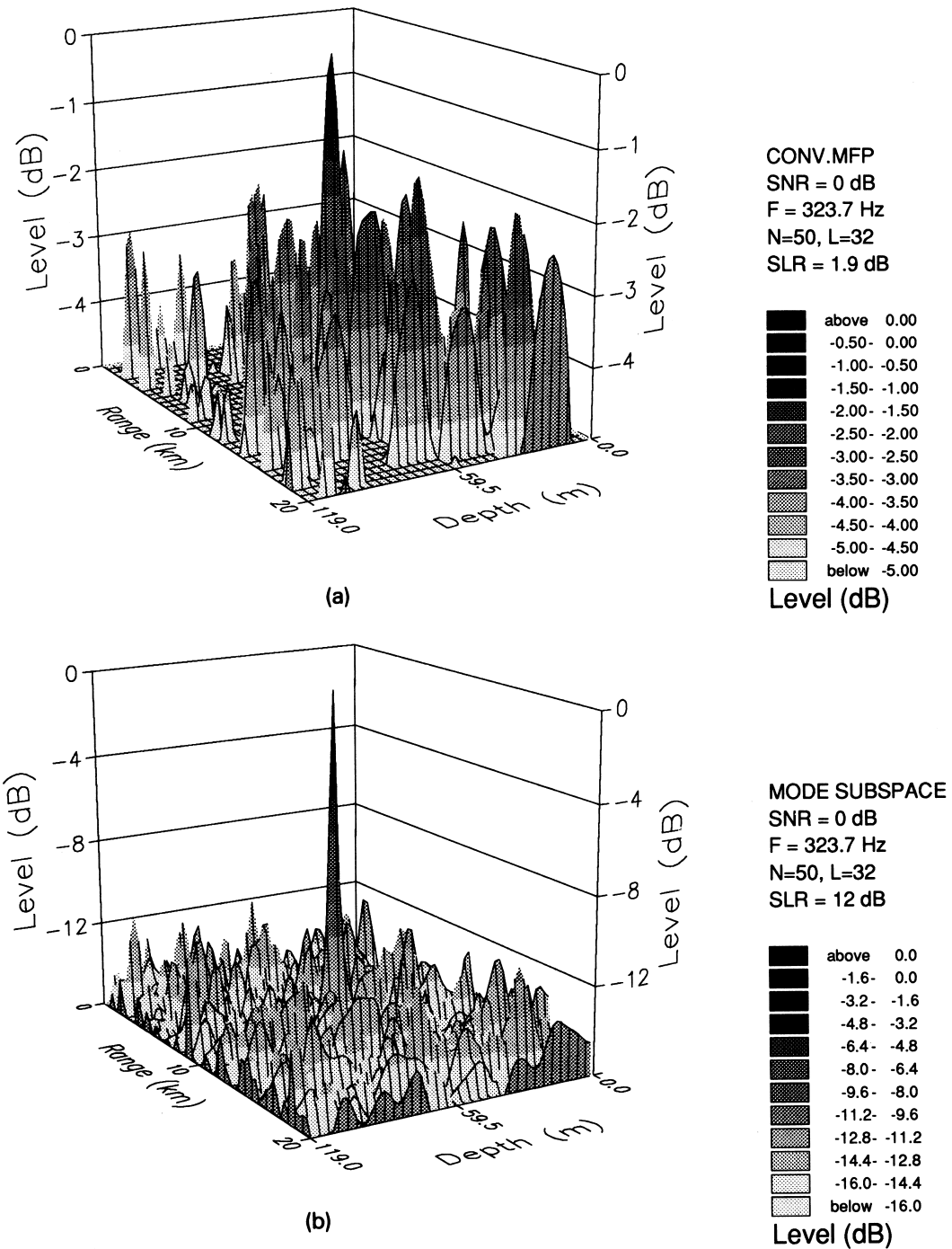


Fig. 4. Range/depth ambiguity surfaces for conventional matched-field (a) and mode subspace (b). Frequency is 323.7 Hz SNR=0 dB.

time snapshots was $N=50$. The performance of the MV and MS techniques clearly stand 9 dB and 11 dB, respectively, above that of the conventional matched field (CMF). The MS approach gave an improvement of almost 2 dB over that of the MV processor for almost all the SNRs. This result has to be linked to the source localization errors, shown in Fig. 3(a) for range and Fig. 3(b) for depth, where the best performance was achieved by the MS and CMF processors, while the MV technique gave the poorest result. An example of the range/depth ambiguity surface obtained for an SNR of 0 dB is shown in Fig. 4(a) for CMF and in Fig. 4(b) for the MS technique.

4. Conclusion

Range and depth localization of acoustic sources in complex propagation environments is possible by direct estimation of the vector subspace spanned by the normal modes that are significantly excited by the source: the mode subspace. The mode subspace approach is closely related with the MUSIC algorithm commonly used in spatial array processing for directions-of-arrival (DOA) estimation [3, 8]. In fact, in both cases the assumed data model has structure (3), but the quantities to be estimated are different: the matrix A in the DOA estimation case, and $x(\theta_T)$ in the source localization case. Also, and most importantly, the dimension of the mode subspace is known a priori (equal to the number of modes supported by the waveguide) in the mode subspace approach, while it has to be estimated in DOA estimation problems.

It should be emphasized that the important question presented in this paper is neither the MUSIC algorithm itself nor its performance in simulated data, which has been shown to be always better than or equal to that of the generalized minimum variance processor, but to show that when a MUSIC based processor is applied to matched-field processing in shallow water, a criterion based

on the number of modes effectively supported by the waveguide can be used to estimate the dimension of the mode (or signal) subspace. This is absolutely necessary in practical applications where one is dealing with non-white noise and in (interesting) cases such as short data records ($N=1$) and coherent multipath situations where known signal subspace dimension estimation criteria fail to perform. Note that the simulation results shown here were obtained without model mismatch, thus they may be optimistic with respect to achievable results with real data, where model mismatch is likely to occur. Nonetheless preliminary results showing the successful application of the technique to real data have been obtained [6].

References

- [1] T.W. Anderson, *An Introduction to Multivariate Statistical Analysis*, 2nd Edition, Wiley, New York, 1984, p. 552.
- [2] A.B. Baggeroer, W.A. Kuperman and H. Schmidt, "Matched field processing: Source localization in correlated noise as an optimum parameter estimation problem", *J. Acoust. Soc. Amer.*, Vol. 83, 1988, pp. 571–587.
- [3] G. Bienvenu, "Influence of the spatial coherence of the background noise on high resolution passive methods", *IEEE Trans. Acoust. Speech Signal Process.*, ICASSP, Washington, DC, April 1979, pp. 306–309.
- [4] H.P. Bucker, "Use of calculated sound fields and matched-field detection to locate sound sources in shallow water", *J. Acoust. Soc. Amer.*, Vol. 59, 1976, pp. 368–373.
- [5] J. Capon, "High-resolution frequency-wavenumber spectrum analysis", *Proc. IEEE*, Vol. 57, 1969, pp. 1408–1418.
- [6] S.M. Jesus, "A new method for range and depth source localization in shallow water", *J. Acoust. Soc. Amer.*, Vol. 88, 1990, S28:2UW10.
- [7] S.M. Jesus and R.M. Hamson, "Source localization: Matched field vs. matched mode. Synthetic and real data performance analysis", *J. Acoust. Soc. Amer. Suppl. 1*, Vol. 85, 1989, S17.
- [8] R.O. Schmidt, A signal subspace approach to multiple emitter location and spectral estimation, PhD. Dissertation, Stanford University, 1982.
- [9] T.C. Yang, "A method of range and depth estimation by modal decomposition", *J. Acoust. Soc. Amer.*, Vol. 82, 1987, pp. 1736–1745.