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CAN MAXIMUM LIKELIHOOD ESTIMATORS IMPROVE GENETIC ALGORITHM SEARCH IN GEOACOUSTIC INVERSION ?

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The principles for estimating seafloor parameters via matched-field processing (MFP) based techniques are now well known. In pure MFP, source localization is often seen as a range-depth estimation problem while the use of MFP on geoacoustic estimation generally involves a computationnaly intensive optimization procedure. In the last few years, most effort has been devoted to developing new or improving existing search procedures to solve the optimization problem and little, or no, attention has been given to the ensemble MFP-optimization treating it as a single technique. The question addressed in this paper is centered on the relation between the MFP parameter estimator technique, defining the objective function, and the search procedure used to optimize it. In particular, we are interested in questions like: can a faster search or more accurate estimate be achieved with a "peaky" surface instead of a flat and ambiguous surface ? Is the inversion process affected by cross-frequency estimators and model mismatch ? Does the search procedure need to be modified, and if yes how, to account for this "peaky" surface navigation ? This paper attempts to answer these and other related questions in the context of the June'97 geoacoustic inversion workshop data set.

1. Introduction

Geoacoustic seafloor properties are an essential requisite to properly predict acoustic propagation, especially in shallow water waveguides and/or at low frequencies. Traditional methods for measuring seafloor properties are costly and time-consuming. Therefore, in the last few years there has been a growing interest in providing solutions to the inverse problem consisting of determining seafloor properties from the measurement of the acoustic field in the water column. The advantages of this approach are obvious: there is no need for deploying equipment in the bottom, and a single inversion can cover a much larger area than measurements obtained from traditional local methods (coring, geophones, penetrometers, etc...).

The methods that have been proposed for geoacoustic characterization by inversion of the acoustic field may be classified into three groups: those based on perturbative techniques that essentially rely on the linearization of the relationships involving the geoacoustic model parameters,[1−4] those that attempt to directly solve the nonlinear problem by transforming it into the optimization of a given matched-field (MF) objective function [5-10] and finally those that attempt to interpolate the inverse map between the acoustic field and the acoustic parameters [11,12]. The approach discussed in this paper falls into the second group of matched-field (MF) methods. This group, of matched-field methods is based on two distinct, but closely related research fields: matched-field processing (MFP) and global search optimization. MFP can be viewed as a generalized beamformer in the parameter space that was originally developed in the acoustic source localization context[13]. In geoacoustic estimation, MFP just provides a coherent way of comparing the measured and predicted fields for obtaining a suitable objective function that the search procedure attempts to optimize. Due to the nonlinear and non-analytical form of this objective function, global search methods are needed that can escape from local extrema in a reasonable finite number of iterations. This is the reason why global search optimization has been attracting most of the attention while MFP or the MFP relation to global search has not. More specifically, the most often used objective function is the so called power correlator or Bartlett processor while genetic search and simulated annealing are the most common solutions to the optimization problem. The question that arises at this point is whether other MF processors (such as minimum variance or signal subspace based) can improve the performance of the global search optimization over that of the conventional power correlator. A subsidiary question is if the optimization procedure can (or should) be adapted to the particular MF processor being used.

One of the very basic characteristics of genetic search algorithms is their insensitivity to the shape of the objective function. That means that for an infinite number of iterations the solution is found regardless of the objective function, provided that the global maximum coincides with the true solution. That is, the estimator

$$\hat{\boldsymbol{\theta}} = \min_{\theta}\{F[\mathbf{y}, \mathbf{w}(\boldsymbol{\theta})]\} \tag{1}$$

obtained by minimization of a suitable cost function $F[\,]$ using genetic search algorithms, is independent from the choice of F. There are, however, two aspects to be taken into account in practice: one is that the number of iterations is never infinite so only the assymptotical behavior is of interest and the other is that in the statement above F is implicitly assumed to be a deterministic function of deterministic quantities, which is not often the case in real life situations. In that case, the functional F is often taken as a (sufficient) statistic of the parameter $\boldsymbol{\theta}$, given the measurement $\mathbf{y}$. These two aspects are of interest to geoacoustic estimation applications, and we prove through simulated examples drawn from the June'97 geoacoustic workshop test cases that the choice of a suitable objective function and a modified genetic algorithms (GA) procedure can provide better mean results than the standard MFP-GA approach.

This paper is organized as follows: in chapter 2 the background to the geoacoustic parameter estimation problem is presented; chapter 3 presents some test cases from the June'97 geoacoustic workshop and finally chapter 4 draws some conclusions.

2. Theoretical Background

2.1. *On MFP for geoacoustics*

The problem of estimating geoacoustic parameters from the acoustic field measured on an array of sensors can be stated as follows: let us assume that the received array field $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{x}(\boldsymbol{\theta}_T) + \boldsymbol{\epsilon} \tag{2}$$

where $\mathbf{x}(\boldsymbol{\theta}_T) = f(\boldsymbol{\theta}_T)$ is a nonlinear and non-analytical function of the parameter $\boldsymbol{\theta}_T$ and $\boldsymbol{\epsilon}$ is the noise component assumed white, Gaussian, zero mean and non correlated with the signal.

In the deterministic case, the classical estimator $\hat{\boldsymbol{\theta}}_T$ of $\boldsymbol{\theta}_T$ is given by

$$\hat{\boldsymbol{\theta}}_T = \max_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta})\} \tag{3}$$

where the functional $r(\boldsymbol{\theta})$ is the (finite length) power correlator detector,

$$r(\boldsymbol{\theta}) = |\mathbf{y}^H \mathbf{w}(\boldsymbol{\theta})|^2 \tag{4}$$

and where $\mathbf{w}$ is a replica of the nonlinear function f. The result is known to be given by the solution of

$$\frac{d\mathbf{w}(\boldsymbol{\theta})}{d\boldsymbol{\theta}} = \mathbf{0}. \tag{5}$$

providing an alternative for solving (3). This approach was adopted by Gerstoft [14], but has the disadvantage of only being possible when analytical forms for the derivatives in (5) are available. In general (5) has more than one solution and one has to resort to global methods for solving (3).

In the stochastic case, (4) is known to be a sufficient and minimal statistic of $\mathbf{x}$, and therefore of $\boldsymbol{\theta}$ [15]. Even if (3) provides an optimum estimator of $\boldsymbol{\theta}$ for both the deterministic and stochastic models, there is no constraint on the behaviour of $r(\boldsymbol{\theta})$ out of the look direction $\boldsymbol{\theta}_T$. In particular, in the stochastic case and in case of model mismatch, i.e, when $\mathbf{w}(\boldsymbol{\theta}_T) \neq \mathbf{x}(\boldsymbol{\theta})$, there might be other values than $\boldsymbol{\theta}_T$ for which (5) is satisfied.

The (approximate) maximum likelihood (ML) provides the correct framework to constrain the output variance to be minimum away from the look direction, *i.e.*, to minimize interferences from values of $\boldsymbol{\theta}$ different of $\boldsymbol{\theta}_T$. The deterministic and stochastic cases can be dealt with by assuming that both the modelling errors and the noise are modelled as white and normally distributed, thus

$$\mathbf{y} - \mathbf{w}(\boldsymbol{\theta}) = \boldsymbol{\epsilon}(\boldsymbol{\theta}) \tag{6}$$

where the noise/modelling errors $\boldsymbol{\epsilon}$ are $N(\mathbf{0}, \sigma_\epsilon^2 \mathbf{I})$. The classical approximate ML derivation gives [16]

$$\hat{\boldsymbol{\theta}}_T = \arg\ \max_{\boldsymbol{\theta}}\{\sum_{n=1}^{N} \frac{1}{\sigma_\epsilon^2} \parallel \mathbf{y}_n - \frac{\mathbf{w}(\boldsymbol{\theta})^H \mathbf{y}_n}{\parallel \mathbf{w}(\boldsymbol{\theta}) \parallel^2} \mathbf{w}(\boldsymbol{\theta}) \parallel^2\}^{-1}, \tag{7}$$

where index $n = 1, \ldots, N$ indicates successive observation time samples (if available).

2.2. *On evolution algorithms*

The purpose of this chapter is to simply recall the basics of evolution algorithms. To simplify let us consider that each population evolution (iteration) can be divided in three steps:

1. calculate objective function (fitness in the GA jargon) and sort the sampled models
2. apply the roulette scheme: randomly select the parents of the new generation giving a higher probability to those individuals with a higher fitness.
3. genetic crossing and mutation of the parents for determining the children population, stop or goto 1.

Holland[17] has proved that algorithm convergence is mostly dependent on step 2, although the choice of other parameters controlling crossing and mutation may influence the rate of convergence. Crossing and mutation operators define the probability density function of location of the children population independently from the iteration number and parents location or fitness. One possibility for tightening the relation between the GA procedure and the objective function is to modify (in a statistical manner) the distance of reach defined by the crossing and mutation operators according to the location/fitness of each individual at each iteration. For example, higher probability of move could be given to areas of increasing fitness.

2.3. *Proposed MFP-modified GA optimization procedure*

Among other comments, in the last section it was mentioned that the choice of the objective function was irrelevant for the convergence of genetic algorithms. That insensitivity is due to the fact that GA keeps no memory of models of previous iterations. In fact the same point in the surface may be evaluated many times during the same search and this is inherent in the random nature of the GA search and cannot be avoided without compromising the convergence of the algorithm itself. At initialization and at each iteration, by crossover and mutation, randomness is maintained giving a constant probability of selection no matter where the genetic parents are situated in the space. In other words, and despite the genetic reality of humans, there is no guarantee that the children will be "genetically close" or improve the species (fitness) relative to their parents. Genetically close is a measure difficult to define and opens perspectives for further research in the area. In order to avoid such difficulties, let us first state the following fact: amongst the most interesting points in the parameter space are those that correspond to the extrema of the objective function. That ensemble of points is important because, if the ensemble is complete, the global extremum is one of them. Although depending on the shape of the objective function, it can be stated that the ensemble of extrema is only a reduced subset of the whole space, which also means that, if the GA strategy was applied only in the extrema subset it would have a substantial improvement in efficiency. The only problem is that the extrema subset has to be complete or asymptotically complete. The proposed MFP-modified GA procedure follows:

(i) start with a random population
(ii) select the extrema closest to the actual population.
(iii) evaluate the objective function at the extrema location and sort
(iv) apply the roulette scheme and select reproduction pond
(v) select children population by crossover and mutation
(vi) if number of iterations has been reached stop, otherwise go to step (ii)

Since the selection of the extrema is done directly on the objective function any appropriate iterative procedure allowing for estimation of local extrema can be used. In our application a steepest descent (gradient) algorithm was used. Its implementation was as follows: select parameter value at iteration $n+1$ according to

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + \alpha_n \mathbf{p}(\boldsymbol{\theta}_n) \tag{8}$$

where $\boldsymbol{\theta}_n$ is the parameter value at iteration n, starting at $\boldsymbol{\theta}_0$ =actual population model, $\mathbf{p}(\boldsymbol{\theta}_n)$ is the direction vector going in the opposite direction of the gradient of the objective function at position $\boldsymbol{\theta}_n$ and α_n is a convergence scalar also called step size. The choice of α_n is such as to optimize the objective function taken at point $\boldsymbol{\theta}_n + \alpha \mathbf{p}(\boldsymbol{\theta}_n)$[18].

For practical purposes the selection of extrema is only started after a few GA iterations, in order to decrease the number of points to iterate (the children population size is always less than the initial population size) and the extrema set is more likely to contain the global extremum.

3. The Geoacoustic

3.1. *The simulation*

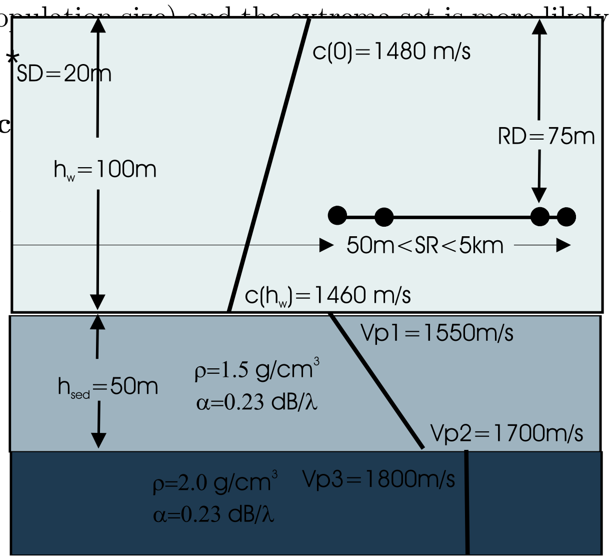

Fig. 1. Simulation canonical environment.

The canonical simulation environment, figure 1, corresponds to the horizontal 5 km long array configuration at 75 m depth of the June'97 geoacoustic inversion workshop. The procedure adopted by the workshop organization comittee for generating the data was based on the SAFARI numerical propagation code with an internal modification introducing a variation of the wavenumber integration interval. That variation therefore introduces a model mismatch with virtually every unmodified public domain existing propagation code. In order to qualitatively quantify the model mismatch, figure 2 shows the array received transmission loss at 200 Hz from the calibration data set and that generated by SNAP in the environmental conditions given in figure 1. In the sequel and in order to limitate model mismatch only the data between 1 and 5 km will be used.

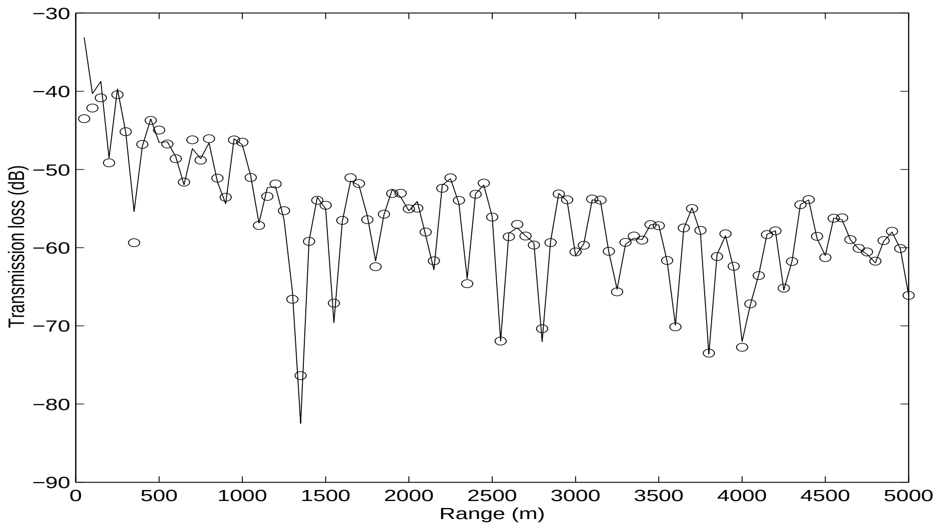

Fig. 2. Transmission loss at 200 hz: calibration test (solid) and obtained best fit (o).

3.2. *Calibration test*

For the purpose of data inversion, it is usefull to characterize the objective function at least in a 2D parameter space. Figure 3 compares the ambiguity surfaces obtained in the perfect SNAP/SNAP match case - (a) and (b) - with those obtained by te calibration data/SNAP mismatch case - (c) and (d) - for the variation of the compressional velocities at the water-sediment interface ($c_{sed}(h_w)$) and at the sediment-subbottom interface ($c_{sed}(h_w + h)$): for the conventional power correlator (PC) Eq. (3) - (a) and (c) - and for the approximate ML(aML) Eq. (7) - (b) and (d).

It can be noted that: i) in the perfect match case, the two objective functions gave very different results: very broad peak and high sidelobes within 0.5 dB of the main peak for the PC - fig. 3(a) - and a well defined peak with a peak to sidelobe rejection higher than 25 dB for the aML - fig. 3(b); ii) using the calibration data - figure 3 (c) and (d) for the PC and aML respectively - the difference between the two processors is smaller than in the perfect match case, and the aML shows a visible sidelobe structure. Such behavior of the aML was expected in presence of model mismatch. Results might be different in presence of noise where the aML has a higher noise rejection ratio out of the look direction than the

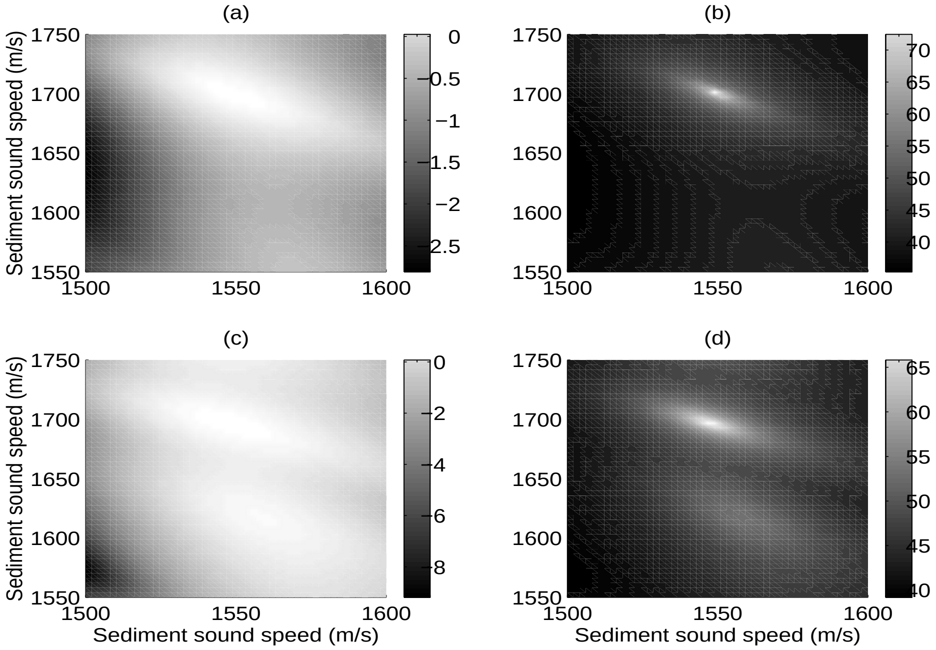

Fig. 3. Objective function for the compressional velocities at the water-sediment interface ($c_{sed}(h_w)$) and at the sediment-subbottom interface ($c_{sed}(h_w + h)$) at 100 Hz: power correlator without mismatch (a) and with mismatch (c), approximate ML without mismatch (b) and with mismatch (d).

PC and therefore a higher chance of keeping the global extremum at the true position (at least in absence of mismatch).

In order to assess the relative performance of the proposed MFP-modified GA procedure it has been tested on the calibration data (Table 1), using both the PC and the aML objective functions with the non-modified and MFP-modified GA versions. The results are expressed in terms of mean and standard estimation errors of the normalized percentile * over a test set of 20 uncorrelated runs.

From Table 1 one can draw the following conclusions: i) there is a constant improvement going from left to right in the mean error values of the first four parameters, ii) going from non-gradient to the gradient versions only the mean error decreases and not its variance and iii) the last two parameters are not observable with this system configuration and/or at this frequency.

3.3. *The single sediment and half space case*

The single sediment (SD) case provides three blind tests (A,B and C) for the estimation of the same six parameters of Table 1. The search bounds are fixed but the granularity is open to the author. The results obtained are summarized in Tables 2 and 3 for the band 50 to 150 Hz with a frequency step of 5 Hz. Incoherent summation was performed for the

*normalized percentile defined as $100 \times \frac{|\hat{x}-x|}{x}$

Table 1. Calibration case: normalized percentile mean and standard deviation of inversion estimation errors with the power correlator(pc), power correlator with gradient (pcg), approximate maximum likelihood(ml) and approximate maximum likelihood with gradient(mlg) (ν and σ respectively denote mean and standard deviation).

	pc		pcg		ml		mlg	
(in %)	ν	σ	ν	σ	ν	σ	ν	σ
$c_{sed}(h_w)$	0.4	0.41	0.28	0.33	0.21	0.15	0.19	0.15
$c_{sed}(h_w + h)$	4.3	1.9	3.8	2.0	0.45	0.36	0.37	0.23
c_{hsp}	2.1	2.0	1.0	2.0	0.66	0.81	0.7	1
ρ_{sed}	6.3	5.6	3.3	2.8	3.4	3.2	2.8	2.3
ρ_{hsp}	6.8	2.9	6.7	3.0	7.0	3.3	7.0	2.8
h_{sed}	25	11	30	13.8	23	12	23	15

cross-frequency integration.

Table 2. Single sediment cases A and B in the band 50-150 hz: inversion estimation results with gradient power correlator (pcg) and approximate maximum likelihood with gradient(mlg).

		case A				case B			
		pcg		mlg		pcg		mlg	
	units	ν	σ	ν	σ	ν	σ	ν	σ
$c_{sed}(h_w)$	m/s	1593	1.5	1594	1.6	1595	1.2	1595	1.1
$c_{sed}(h_w + h)$	m/s	1743	4.4	1744	4.1	1688	3.8	1689	3.0
c_{hsp}	m/s	1671	9.3	1674	8.8	1720	5.5	1720	5.7
ρ_{sed}	g/cm^3	1.85	0.009	1.84	0.004	1.64	0.016	1.65	0.01
ρ_{hsp}	g/cm^3	1.74	0.12	1.75	0.12	1.71	0.1	1.72	0.1
h_{sed}	m	28	13	27	12	29	11	26	12

From tables 2 and 3 it can concluded that both estimators gave approximately the same results except for the sediment thickness that, from the high standard deviations values obtained, can not be inverted with this data. Apart one or two exceptions, the standard deviations provided by the aML are always smaller or equal than those obtained with the PC. Note that case A showed an inversion of the density gradient from the sediment to the half space denoting a non-physical situation. Amazingly, this density inversion was accompained by an inversion on the compressional velocities in the same media. The results at single frequencies (not shown) are in many cases consistent with the broadband results with, however, always higher standard deviations.

4. Conclusion

This paper has addressed the problem of adapting GA search to the choice of the objective function. In particular, the proposed combination of an approximate maximum likelihood based MFP estimator and a gradient modified GA procedure clearly outperformed, at least in the calibration data set and despite the model mismatch present in the data, the conven-

Table 3. Single sediment case C in the band 50-150 hz: inversion estimation results with gradient power correlator (pcg) and approximate maximum likelihood with gradient(mlg).

		case C			
		pcg		mlg	
	units	ν	σ	ν	σ
$c_{sed}(h_w)$	m/s	1524	1.5	1524	1.5
$c_{sed}(h_w+h)$	m/s	1677	5.1	1677	4.9
c_{hsp}	m/s	1656	21	1655	10
ρ_{sed}	g/cm^3	1.47	0.02	1.48	0.02
ρ_{hsp}	g/cm^3	1.65	0.05	1.68	0.07
h_{sed}	m	27	9	30	11

tional MFP estimator-GA procedure combination. In the single sediment case, the results provided by the two methods in the frequency band 50 - 150 hz were very similar, however, the standard deviation for the aML-modified GA combination was slightly smaller. This study suggests that the generic framework provided by the GA random search can and should be adapted to the problem at hand and in particular to the objective function - thus to the measured data. This adaptation can be made without loss of the convergence properties of the GA. Further research along these ideas is being pursued and in particular by modifying the probability of movement of the crossing and mutation operators according to the slope of the objective function at each iteration.

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