Joint Localization of Underwater Vehicle Formations Based on Range Difference Measurements

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***Abstract*—This work addresses the problem of determining the positions of a set of nodes (underwater vehicles, in the present context) through two-way time-of-flight measurements (equivalently, ranges) and interpacket delays (equivalently, range differences). The approach is based on reconstruction and factorization of Euclidean Distance Matrices (EDM), which provides a solid framework to deal with noisy measurements and partial loss of data, while retaining a compact mathematical formulation. The paper discusses techniques for gathering range and range difference data (interrogation schemes) and assesses the performance of EDM completion algorithms to deal with incomplete measurements or outliers.**

I. Introduction

Localization in formations of Autonomous Underwater Vehicles (AUVs) is an essential task for precise navigation and to properly georeference any collected data. The present work is developed in the scope of EU FP7 project MORPH [1], where small heterogeneous teams of AUVs are expected to perform collaborative survey tasks in a tightly coupled way, taking advantage of the combined sensor package of the ensemble to form a so-called MORPH supra-vehicle. Precise navigation systems are not expected to be installed in all of the agents, and as such inter-node, relative-navigation capabilities will be used, with measurements provided as a by-product of acoustic modem transactions.

A common way to estimate the distance between two underwater assets is through the measurement of acoustic time of flight, with useful side information provided by simpler sensors such as depth cells or altimeters. These measured ranges are currently the predominant means to determine (relative) positions in underwater systems, given the unavailability of GPS signals. Commercial underwater modems from Evologics are used in MORPH, which already have built-in functionality for precise timing of packet arrival times to infer propagation delays, and hence ranges, or range differences.

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Different types of range-based localization problems may be formulated, depending on the available prior knowledge [2], [3]. The network localization problem of interest here aims at finding the positions of a *constellation* of nodes from (a subset of) pairwise range or range difference measurements given only a few (possibly none) anchor positions [4]. The shared-medium context of wireless networks, including underwater acoustic networks, makes this problem even richer, as it becomes possible for a node to eavesdrop on the channel and thereby acquire information about pairwise distances of other nodes that may help to estimate the full constellation geometry.

In a fully connected network configuration, envisaged for project MORPH and assumed in this paper, the matrix of pairwise distances might be built with the following *interrogation scheme*: (*i*) Each node sends a request packet to query its neighbours (all other nodes); (*ii*) The neighbours sequentially reply, and distances to the originating node are determined through Round-Trip Times (RTTs); (*iii*) The node broadcasts the set ot RTTs (or a subset thereof) to make them available to other nodes; (*iv*) When all nodes have completed the query/broadcast phase the set of relevant pairwise distances is known across the network.

The number of transmissions required by the above scheme is relatively large[1], which constitutes a problem in underwater networks due to the scarce available bandwidth. An alternative approach is pursued here where measured time differences between packet arrival times provide information on the range differences between propagation paths. Figure 1 illustrates the concept: Node i transmits a packet that is received at several nodes, including nodes k and j. Prompted by that event, node k sends a reply packet back to node i that is also received at node j. Assuming that node j duly compensates for known packet durations and processing delays, the time difference between the instants when the original transmission from node i and the reply packet from node k are received is proportional

[1]The procedure may be streamlined by removing redundant information.

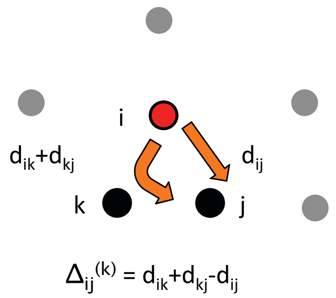


Fig. 1. Time differences of arrival between packets received at node j are proportional to the differences in path lengths $i \rightarrow k \rightarrow j$ and $i \rightarrow j$.

to $d_{ik} + d_{kj} - d_{ij}$, where d_{ik} denotes the pairwise distance between nodes i and k, and similarly for the other terms.

Due to the nature of sequential interrogation schemes, any given node will overhear multiple interdependent packet transmissions from other nodes, from which it may locally compute differences in arrival times that are obtained from sums/differences of several unknown pairwise distances in the constellation. Sharing these measurements will eventually allow any node to gather enough linear equations and invert the system to obtain the ranges. That is the essence of the differential approaches considered here.

II. Interrogation Schemes

The goal of interrogation schemes is to convey pairwise ranges to nodes so that all coordinates in the constellation can be jointly computed through EDM completion and factorization. Depending on the context, the constellation may be reconstructed at a single location, or simultaneously at multiple nodes. The former naturally gives rise to *master/slave* schemes, whereas for the latter *symmetric schemes* such as the one described below are more appropriate.

Figure 2 illustrates a master/slave interrogation scheme for a network of 3 nodes, used in the MORPH'12 sea trial[2], that allows node 1 to determine pairwise distances [2]. Initially, it broadcasts a request that is first answered by node 2. The distance d_{12} is determined from the RTT. When node 3 replies d_{13} is similarly obtained from the RTT, but the packet additionally includes in the payload the locally measured difference in arrival times between the original request from node 1 and the reply from node 2. As argued in connection with Figure 1, that interval is proportional to $d_{12} + d_{23} - d_{13}$, from which node 1 determines d_{23} with the help of the 2 measured RTTs.

A. Symmetric Interrogation

When all nodes must regenerate the constellation, master/slave interrogation does not take full advantage of packets flowing through the shared transmission medium. In fact, in one interrogation cycle slave nodes may gather much useful spatial information for their own localization by eavesdropping

[2]In practice, packets are scheduled using TDMA to avoid collisions.

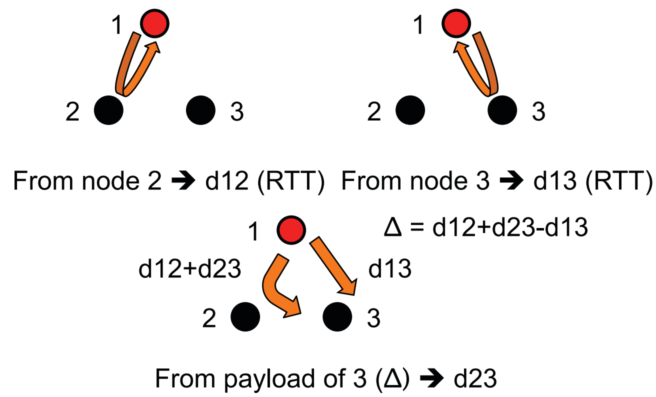


Fig. 2. Master/slave interrogation: Determining pairwise distances at node 1 in a network of 3 nodes.

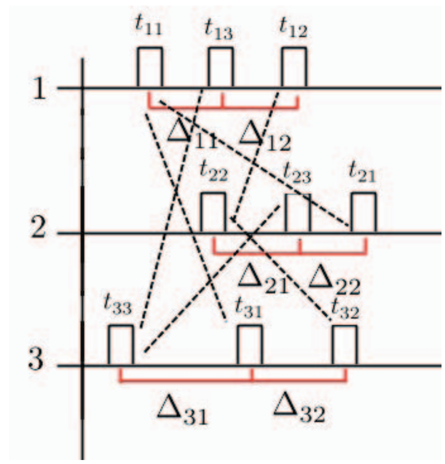


Fig. 3. Symmetric Interrogation Scheme. Example for a 3 node network.

on packets, but this is not exploited. A more effective alternative is proposed here to reduce the amount of data transmitted and still determine all the distances. This method does not establish a hierarchy in each cycle, it is symmetric in the sense that all the nodes play the same role.

For a network of N nodes each cycle starts with a round of queries followed by a second round where each one of the nodes sends $N-1$ time differences between arrivals. Figure 3 illustrates the process for a 3 node network. As in master/slave schemes there is no need for clock synchronization.

Let Δ be the vector grouping the time intervals and x the vector of unknown variables, containing not only the pairwise distances divided by the sound speed but also the instants at which each one of the nodes has sent its query (t_{ii}). Vectors Δ and x are related through a matrix A such that $Ax = \Delta$. A has dimensions $\left(N^2 - N, \frac{N^2+N}{2}\right)$ and therefore the system is overdetermined when all the Δ entries are known for networks with more than 3 nodes. In that case, x results from $x = A^+\Delta$, where $(\cdot)^+$ denotes the pseudoinverse. Note that one of the variables t_{ii} must be assumed as the origin of the time axis, causing x to lose a variable and A to lose a column. Typically, each node will consider the moment at which the query was sent (t_{ii}) as the origin of the time axis. A becomes a full rank matrix when enough entries are known in Δ .

When some of the interpacket delays are unknown (e.g., due to deliberate suppression to save bandwidth, or caused by packet decoding errors), the rows of A corresponding to the unknown Δ entries are eliminated. If due to the row eliminations, A becomes rank deficient, then for some of the variables in x the solution will not be unique.

III. Framework Reconstruction

In this work, *framework reconstruction* denotes the process of regenerating the constellation from range data based on the properties of EDM. Consider a matrix E, where each entry E_{ij} is the squared distance between nodes i and j. It represents a valid spatial constellation if and only if E is an EDM. It thus obeys the following set of rules [5], [6]

$$E \in \mathbb{EDM} : E_{ii} = 0, \quad E_{ij} \geq 0 \quad \mathbf{J}E\mathbf{J} \preceq 0 \quad i,j = 1,\ldots,N, \tag{1}$$

where $E \in \mathbb{EDM}$ means that matrix E lies in the EDM cone, $\mathbf{J}$ is the projector on the orthogonal complement of $\mathbf{1}$ (the vector of ones) given by $\mathbf{J} = \mathbf{I} - \frac{1}{N}\mathbf{1}\mathbf{1}^T$, and $\mathbf{I}$ is the identity matrix as usual.

Let $\tilde{A}$ be the matrix whose columns hold all node coordinates, translated so that the centroid of the constellation lies at the origin. Then the Gram matrix $\tilde{G} = \tilde{A}^T\tilde{A}$, can be obtained from $\tilde{G} = -(1/2)\,\mathbf{J}^T E\mathbf{J}$ [5], and $\tilde{A}$ regenerated from it, up to an arbitrary unitary matrix, by factorization using the Singular Value Decomposition (SVD). The rank of the coordinate matrix returned by the SVD may be higher than the embedding dimension of the problem, in which case a common procedure is to truncate it to the appropriate rank. To achieve global positioning, the remaining rotation, reflection, and translation ambiguities must be removed with the help of anchor nodes. Briefly (see also [2]), residual rotation and reflection are eliminated by solving an orthogonal Procrustes problem. The solution is the rotation matrix Q which closely maps the subset of $\tilde{A}$ corresponding to the anchor nodes with the true anchor positions. Obviously, both coordinate matrices must be translated so that their centroids lie at the origin. Finally, the constellation can be translated to the correct location using the centroid of the true anchor coordinates.

A. EDM Completion via Semidefinite Relaxations

The ranging process may yield a noisy and incomplete set of pairwise distances. In that case, before framework reconstruction can be applied, the closest EDM to the given set of measurements must be found; a problem also known as nearest EDM completion (EDMCP) or, in particular, low-dimensional EDMCP if the rank of the solution is constrained. Let D be the *pre-distance matrix* with zero diagonal entries and with some nonnegative elements equal to the *squares* of the available observed ranges. Formally,

$$\begin{aligned} \underset{E}{\text{minimize}} \quad & \|W \odot (E - D)\|_F^2 \\ \text{subject to} \quad & E \in \mathbb{EDM} \\ & \operatorname{rank}(\mathbf{J}E\mathbf{J}) = r \end{aligned} \tag{2}$$

where W is a mask matrix with zeros in the entries corresponding to the free elements of D and ones elsewhere. As usual, $\odot$ denotes the Hadamard (element-wise) matrix product and $\|\cdot\|_F$ denotes the Frobenius norm. The rank constraint ensures that the underlying constellation lies in the appropriate ambient space $\mathbb{R}^r$, with $r = 2$ (2D) or $r = 3$ (3D). As low-dimensional EDMCP is known to be an NP-hard problem [6], [7] alternatives must be used to tackle (2). One popular strategy is to turn to convex relaxations which can be solved efficiently, but may not solve the original problem. The three convex formulations introduced next drop the rank constraint, leading to semidefinite programs (SDP) that can be handled by standard convex optimization solvers.

The first one, termed EDM with squared ranges (EDM-SR) is a classical approach that simply drops the rank constraint. The second one is EDM with plain ranges (EDM-R), which was proposed in [4]. It can be written as

$$\begin{aligned} \underset{E,T}{\text{minimize}} \quad & \sum_{i,j} \left(E_{ij} - 2T_{ij}\sqrt{D_{ij}} \right) \\ \text{subject to} \quad & T_{ij}^2 \leq E_{ij} \\ & E \in \mathbb{EDM} \end{aligned} \tag{3}$$

where T_{ij} is an epigraph variable that ideally should equal $\sqrt{E_{ij}}$. This formulation doubles the number of variables relative to EDM-SR, but in [2] it was found to have more stable numerical behavior enabling the use of a faster convex solver.

EDM with Plain Ranges and Lower-Bounded Epigraph (EDM-RLB): The inequality constraint in (3) is equivalent to $-\sqrt{E_{ij}} \leq T_{ij} \leq \sqrt{E_{ij}}$, whereas one would like to have the (non-convex) equality constraint $T_{ij} = \sqrt{E_{ij}}$. To better approximate this the inequality in (3) is modified as $E_{ij}/\sqrt{E_{\max}} \leq T_{ij} \leq \sqrt{E_{ij}}$, where $\sqrt{E_{\max}}$ denotes the maximum foreseeable value for ranges[3].

B. Numerical Results

The performance of the three formulations presented above was evaluated in simulation against other simpler methods such as the LS approach of [3] and a raw data approach that applies framework reconstruction directly on the complete pre-distance matrix D (which might not have the properties of an EDM).

The convex optimization problems were solved using CVX [8], [9] with the standard SDPT3 solver. Only *solved* runs were accounted for, that is, results from CVX other than *solved* were ignored. The simulations consider a planar constellation with 10 nodes randomly placed on a unit square area with randomly chosen anchors. The level of noise in each run is controlled by the average noise power defined as $\overline{\sigma}^2 = 1/P\sum_{i=1}^{P}\sigma_i^2$ [10], where σ_i^2 is the noise covariance for the distance i and $P = \frac{N(N-1)}{2}$ is the number of pairs in the network. It is assumed that σ_i^2 is related to the distances according to the path loss law, so that σ_i^2/d_i^2 is a constant ratio. The performance criterion is the Root Mean-Square Error (RMSE)

[3]Zero could be used as a lower bound when ranges may be arbitrarily large.

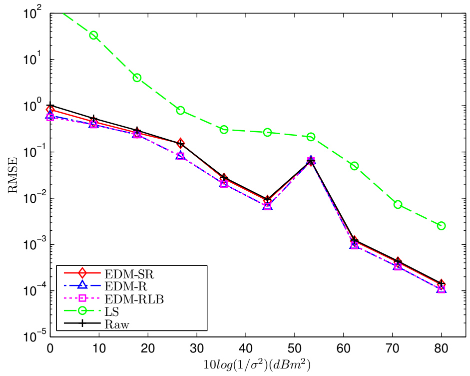


Fig. 4. RMSE over 100 Monte Carlo runs for a complete set of observed distances.

of the estimated positions over a maximum of 100 Monte Carlo runs.

The first experiment considers a scenario where the set of observed distances is complete but noisy, the goal being to evaluate the ability of each formulation to deal with noisy measurements. The number of anchor nodes is fixed, 3 out of 10 nodes, and the constellation changes randomly between runs. Results are shown in Figure 4. Since the experiments consider a square of unit area, a RMSE near 1 means that framework reconstruction is useless. As expected, the LS and raw data approaches exhibit lower accuracy than convex optimization methods. Although the performance gap between the raw method and alternative convex approaches is relatively small, note that direct reconstruction from the set of observed ranges can only be performed when all pairwise ranges are available, thus making it an infeasible option for the general case. The estimates for EDM-RLB are better than for EDM-SR and EDM-R, suggesting the use of this formulation over the others, at least in the absence of missing ranges.

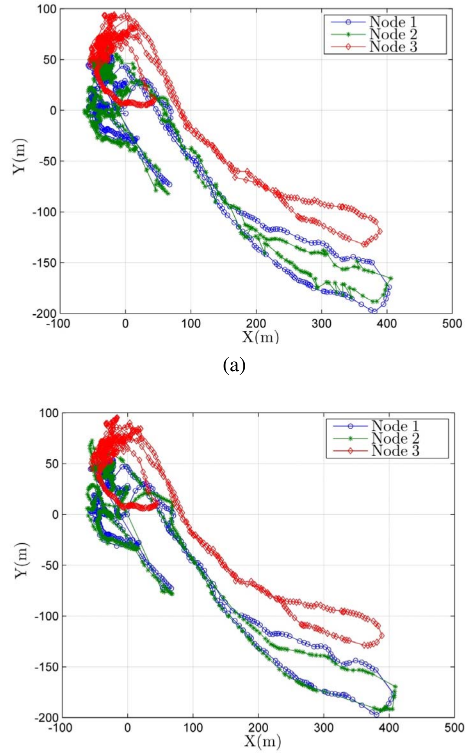
Results for reconstruction with missing interpacket delays are omitted in the interest of space. For 5 missing delays (out of 45) results are similar to those shown for EDM-based methods in Figure 4, with a (small) performance advantage of plain range techniques (EDM-R and EDM-RLB). The degradation in RMSE remains moderate for 10 missing ranges, and becomes severe when 20 distances are suppressed.

Results for the MORPH'12 Sea Trial: The trial was conducted in Faial, Azores, in July 2012. Details about the experimental setup, which comprises a network of 3 nodes towed from 2 ships[4] are given in [2]. The master/slave interrogation scheme of Figure 2 is used to obtain pairwise ranges, with the role of master cycling between nodes every 5 seconds. Figure 5a shows reconstructed trajectories obtained with EDM-RLB completion, which agree with GPS data (Figure 5b)[5].

To assemble the estimated relative positions of nodes into actual tracks the following procedure was used. After factorization of the Gram matrix to obtain normalized node coordinates (see Section III) the rotation ambiguity was resolved through a Procrustes problem using the GPS positions of nodes 1 and 3. The reflection ambiguity was tackled assuming that the GPS positioning error of node 2 was sufficiently low so that it could be used to decide whether the constellation should be flipped or not about the 1–3 axis. Finally, the points were translated so that the midpoint of the 1–3 axis coincided with the mean of their GPS coordinates.

[4]Nodes 1 and 2 are suspended at a distance of about 10 m from two sides of a ship, whereas node 3 is towed by a dinghy 10–50 m from that ship.

[5]Differences to [2] are due to a different EDM completion algorithm and a modified constellation alignment procedure (described next) to overcome GPS errors in one of the nodes.



(a)

(b)

Fig. 5. The MORPH'12 experiment in, Faial, Azores (a) Regenerated node tracks after framework reconstruction. (b) GPS position estimates.

IV. Conclusion

This paper proposed techniques for efficiently gathering spatial cues on the pairwise distances between nodes (vehicles) in an underwater acoustic network, and to derive actual coordinates by EDM completion and factorization. The performance of these methods was assessed through simulation and using data from the MORPH'12 sea trial, with encouraging results.

Several useful extensions and enhancements of this work have been developed, but are not included in the paper. These include convex formulations for jointly determining the pre-distance matrix and the EDM from interpacket delays. Also, techniques have been developed to lower the rank of the Gram matrix from which node coordinates are obtained, either by modifying the cost function for EDM completion or by taking advantage of existing degrees of freedom in the pre-distance matrix, and thus improve the factorization step.

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