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Cooperative Path Following of Autonomous Vehicles with Model Predictive Control and Event Triggered Communications ⋆

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Abstract: This paper presents a solution to the problem of multiple vehicle cooperative path following (CPF) that takes explicitly into account the constraints on the vehicles inputs and the topology of the inter-vehicle communications network. The solution involves decoupling the original constrained CPF problem into two sub-problems: i) single vehicle constrained path following and ii) multi-agent system (MAS) coordination. The first is solved by adopting a sampled-data model predictive control (MPC) scheme, whereas the latter is tackled by using a distributed control law with an event triggered communication (ETC) mechanism. We show that this design methodology yields a stable closed-loop CPF system: the path following error for each vehicle is globally asymptotically stable (GAS) and the coordination errors between the vehicles are bounded. A simulation example consisting of three autonomous vehicles following a given 2D- desired formation illustrates the efficacy of the CPF strategy proposed.

Keywords: Cooperative Path following, Event Triggered Communications, MPC.

1. INTRODUCTION

Cooperative path following, an important class of multiple vehicle formation control, is defined as the problem of steering a group of vehicles along a set of spatial paths, at speeds that may be path dependent, while holding a feasible geometric pattern. Among a myriad of applications related to CPF, we point out the use of unmanned aerial vehicles (UAVs) for coastal monitoring (Kaminer et al. [2017], Klemas [2015]), autonomous marine vehicles (AMVs) for marine habitat mapping, and geo-technical survey (Abreu et al. [2016]).

Different approaches to the CPF problem have been proposed in the literature over the last decade, see for example (Aguiar and Pascoal [2007], Cao et al. [2017], Ghabcheloo et al. [2009]), and references therein. The key idea exploited in these publications is to decouple the CPF problem into a path following problem of a single vehicle and a problem of MAS coordination. The first problem is solved using classical nonlinear control methods such as Lyapunov based techniques, while the later is tackled by resorting to tools from network control theory for multi agent systems, see for example (Olfati-Saber et al. [2007]) for an introduction to consensus laws that are pervasive in the literature. Stability of the resulting closed-loop CPF system is typically examined by using cascade analysis tools (Aguiar and Pascoal [2007]). The main limitation of the results published in above literature is that the vehicles' inputs (speed, heading rate for example) are assumed to be unconstrained, which does not hold in realistic applications.

Due to its ability to handle explicitly input constraints, Model Predictive Control has recently been proposed as a key ingredient in the solution of CPF problems, see for example (Alessandretti and Aguiar [2017], Rucco et al. [2015]). Using a decoupling approach, the authors in (Rucco et al. [2015]) propose an MPC scheme to solve the path following problem while the coordination problem is solved by using a classical consensus law. In spite of its elegance, the approach in (Rucco et al. [2015]) has two important limitations. Firstly, the MPC scheme is designed for a linearized path following error system, which implies that stability of the resulting system is only guaranteed locally. In addition, with the coordination law used in (Rucco et al. [2015]), there is no guarantee that the total speed assigned to each vehicle, which is the summation of the nominal desired speed and the correction speed issued by the coordination law, satisfies the vehicle's speed constraint. In (Alessandretti and Aguiar [2017]), the authors address the CPF problem using a distributed MPC framework. However, the method requires that the speed of vehicles be allowed to be negative, a constraint that is practically impossible to meet for some classes of autonomous vehicles such as fixed-wing UAVs or AMVs.

Motivated by the above considerations, this paper proposes a solution to the CPF problem that takes explicitly into account the constraints on the vehicles inputs and the nature of the inter-vehicle communications network. We also derive an ETC mechanism for multiple vehicle cooperation. Compared with the results available in the literature, we take into account explicitly the practical constraint that the speeds of the vehicles should be bounded away from zero. In addition, the MPC scheme proposes for path following problem does not require a terminal set, yielding a global region of attraction for single vehicle path following. This strengthens the results in Alessandretti and Aguiar [2017], Rucco et al. [2015]). Finally, with the proposed ETC mechanism, inter-vehicle communications

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are reduced, making the scheme attractive for scalable networks with limited bandwidth.

2. INPUT-CONSTRAINED COOPERATIVE PATH FOLLOWING

In what follows, $\{\mathcal{I}\} = \{x_{\mathcal{I}}, y_{\mathcal{I}}\}$ denotes an inertial frame and $\{\mathcal{B}\} = \{x_{\mathcal{B}}, y_{\mathcal{B}}\}$ denotes a body frame attached to a vehicle. We consider a set of $N \geq 2$ homogeneous vehicles following a set of N spatial paths

$$\{\mathcal{P}^{[i]} : \gamma^{[i]} \rightarrow [\mathbf{p}_{\mathrm{d}}^{[i]}(\gamma^{[i]}), \psi_{\mathrm{d}}^{[i]}(\gamma^{[i]})]^{\mathrm{T}} \in \mathbb{R}^3, i \in \mathcal{N}\}, \quad (1)$$

where $\mathcal{N} := \{1, ..., N\}$ denotes the set of the vehicles or paths, $\gamma^{[i]}$ denotes the parameterizing variable of path i, $\mathbf{p}_{\mathrm{d}}^{[i]}(\gamma^{[i]}) = [x_{\mathrm{d}}^{[i]}(\gamma^{[i]}), y_{\mathrm{d}}^{[i]}(\gamma^{[i]})]^{\mathrm{T}}, i \in \mathcal{N}$ are the position vectors of points on the paths expressed in the inertial frame, and $\psi_{\mathrm{d}}^{[i]}(\gamma^{[i]}), i \in \mathcal{N}$ are the angles that the tangents of the paths at these points make with $x_{\mathcal{I}}$.
Let $\mathbf{p}^{[i]} = [x^{[i]}, y^{[i]}]^{\mathrm{T}}, i \in \mathcal{N}$ be the position vectors of the centers of mass of the vehicles expressed in the inertial frame. Assuming that the vehicles have zero sway speed, their kinematic models are given by

$$\dot{x}^{[i]} = u^{[i]} \cos \psi^{[i]}, \ \dot{y}^{[i]} = u^{[i]} \sin \psi^{[i]}, \ \dot{\psi}^{[i]} = r^{[i]}, \quad (2)$$

where $u^{[i]}, \psi^{[i]}, r^{[i]}, i \in \mathcal{N}$ denote the speed, the yaw angle, and the yaw rate of vehicle i, respectively. Due to physical limitations of the vehicles, we consider that the speed and the heading rate are constrained, i.e. $(u^{[i]}, r^{[i]}) \in \mathbb{U}$, for all $i \in \mathcal{N}$, where $\mathbb{U}$ is referred as an input constraint set that is common for all vehicles and is defined explicitly as

$$\mathbb{U} := \{(u^{[i]}, r^{[i]}) : u_{\min} \leq u^{[i]} \leq u_{\max}, |r^{[i]}| \leq r_{\max}\}, \quad (3)$$

for all $i \in \mathcal{N}$. Here, $u_{\min} > 0$ and $u_{\max}$ are lower and upper bounds on the speed, respectively, and $r_{\max}$ is an upper bound on the heading rate.
In cooperative path following, vehicle i is assigned path i to follow. We consider a scenario where the fleet of vehicles are not only required to follow their assigned paths but also to converge to and maintain a desired geometric formation, while maneuvering with desired speed profiles compatible with the formation. To solve the constrained CPF problem, the methodology used in this paper decouples the constrained CPF problem into two sub-problems: a constrained path following problem aims at vehicle to converge to its assigned path and a constrained coordination problem that requires the vehicles to exchange information on their progression along the paths (given by the path parameters) and negotiate their speeds to reach the desired formation. Using this methodology, we shall show that to control a fleet of vehicles maneuvering with a desired formation, the vehicles need to exchange very limited information (a simple scalar path parameter that is used for coordination).

2.1 Input-constrained path following

In this subsection, we shall formulate the constrained path following problem for a single vehicle that requires the vehicle converge to a path while ensuring also that the speed of the path parameter tracks a desired speed profile. To this end, we exploit the concept of "tracking a virtual reference" introduced in (Lapierre et al. [2006]). In this section we consider a single vehicle. For the sake of simplicity we drop the superscript $[i]$ in the variables in equations (1),(2) and (3). Later, in subsequent sections, we will re-introduced the original notation when necessary.
Consider the path following problem for a single vehicle with the kinematics model given by (2), subject to the constraints on the inputs given by (3), following a path parameterized by the variable γ given by (1). Consider a Parallel Transport frame $\{\mathcal{F}\} = \{x_{\mathcal{F}}, y_{\mathcal{F}}\}$ whose origin is at a point S on the path and its axes defined as follows: $x_{\mathcal{F}}$ is aligned with the tangent to the path at that point and points in the direction of increasing path length and $y_{\mathcal{F}}$ is determined by rotating $x_{\mathcal{F}}$ 90 degree clock wise (see Fig. 1). In the set-up adopted for path-following, the Parallel Transport frame moves along the path in a manner to be determined and plays the role of a "virtual reference" for the position and heading angle that the vehicle must track to achieve good path following. Let

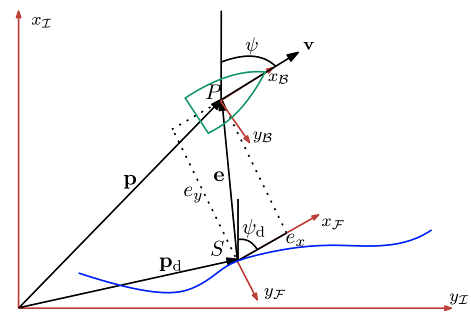

Fig. 1. Vehicle and reference frames. Velocity vector in the body frame $\mathbf{v} = [u, 0]$. P is the center of mass of the vehicle and S is a point on the path.

$\mathbf{e}_{\{\mathcal{I}\}} = \mathbf{p} - \mathbf{p}_{\mathrm{d}}(\gamma)$ be position error vector expressed in the inertial frame and $e_{\psi} = \psi - \psi_{\mathrm{d}}(\gamma)$ the orientation error between the path and the vehicle. We define $R_{\mathcal{I}}^{\mathcal{F}} : \psi_{\mathrm{d}} \rightarrow R_{\mathcal{I}}^{\mathcal{F}}(\psi_{\mathrm{d}}) := [\cos(\psi_{\mathrm{d}}), \sin(\psi_{\mathrm{d}}); -\sin(\psi_{\mathrm{d}}), \cos(\psi_{\mathrm{d}})]$ as the rotation matrix from the inertial frame to the Parallel Transport frame. Let $\mathbf{e}_{\{\mathcal{F}\}} = [e_x, e_y]^{\mathrm{T}}$ be the position error expressed in the Parallel Transport frame, computed as $\mathbf{e}_{\{\mathcal{F}\}} = R_{\mathcal{I}}^{\mathcal{F}} \mathbf{e}_{\{\mathcal{I}\}}$. Collectively, defining $\mathbf{x} = [e_x, e_y, e_{\psi}]^{\mathrm{T}}$ as the path following error vector and using the methodology exposed in Lapierre et al. [2006] for wheeled robots, the evolution of the path following error in the Parallel Transport frame is described by the dynamic equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} -g(\gamma)v(1 - \kappa(\gamma)e_y) + u\cos(e_{\psi}) \\ -\kappa(\gamma)g(\gamma)ve_x + u\sin(e_{\psi}) \\ r - \kappa(\gamma)g(\gamma)v \end{bmatrix}, \quad (4)$$

where $g(\gamma) := \|\mathbf{p}_{\mathrm{d}}'(\gamma)\|, \kappa(\gamma) = \frac{x_{\mathrm{d}}'(\gamma)y_{\mathrm{d}}''(\gamma) - x_{\mathrm{d}}''(\gamma)y_{\mathrm{d}}'(\gamma)}{\|\mathbf{p}_{\mathrm{d}}'(\gamma)\|^3}$ and $v = \dot{\gamma}$ is refereed as the speed of the "virtual reference" that gives an extra degree of freedom in the process of designing path following controllers; $\kappa(\gamma)$, by definition, is the curvature along the path and $\mathbf{u} := [u, v, r]^{\mathrm{T}}$ is the input vector of the path following error system.
Notice that we have introduced a new input v to control the evolution of the path parameter γ that plays the role of a "virtual reference" for the vehicle to follow. Later, for the purpose of designing an input constrained path following controller, v should lie in a constraint set $\mathbb{U}_v$, defined explicitly as

$$\mathbb{U}_v := \{v : |v| \leq v_{\max}\}, \quad (5)$$

where $v_{\max}$ will be specified.
We are now in a position to state the following constrained path following problem.

Problem 1 - [Constrained Path Following]. *Given a spatial path $\mathcal{P}$ parameterized by γ, a desired positive and bounded speed profile $v_{\rm d}(\gamma)$ along the path, and the constraint sets for the vehicle's inputs and the speed of the "virtual reference" given by* (3), (5) *respectively, derive a feedback control law for $(u, r) \in \mathbb{U}$ and $v \in \mathbb{U}_v$ to drive the path following error $\mathbf{x}$ with the dynamics described in* (4) *asymptotically to zero, while ensuring also asymptotically that v tracks the desired speed profile $v_{\rm d}(\gamma)$, that is, $v(t) - v_{\rm d}(\gamma(t)) \to 0$ as $t \to \infty$.*

It is important to notice that since the path parameter γ is not necessarily the arc-length, in general v is not the speed of the "virtual reference" in the inertial frame. In fact, the speed of the "virtual reference" in the inertial frame that the vehicle needs to track is $g(\gamma)v$.

2.2 Cooperative path following

In what follows, we assume that the paths that the vehicles must follow are appropriately parameterized to ensure that a given formation is reached when the path parameters, also called coordination states, are equal. For example, to make a number of vehicles follow an equal number of concentric circumferences and be aligned radially along their radii, it suffices to parametrize these paths in terms of their normalized lengths, that is, $\gamma^{[i]} = s^{[i]}/2\pi$, where $s^{[i]}$ is the curvilinear abscissa along the path. Clearly, the vehicles are coordinated and maneuver with a desired normalized path dependent speed $v_{\rm d}(\cdot)$ if $\gamma^{[i]}(t) = \gamma^{[j]}(t)$ and $\dot{\gamma}^{[i]}(t) = \dot{\gamma}^{[j]}(t) = v_{\rm d}(\gamma^{[i]})$ for all $i, j \in \mathcal{N}$. See (Ghabcheloo et al. [2009]) for an introduction to these concepts.
Suppose now that each vehicle has equipped with a path following controller (to be designed later using an MPC scheme) that solve **Problem 1**, that is, the controller makes the vehicle converge to and follow the path $(\mathbf{x}^{[i]}) = \mathbf{0}, i \in \mathcal{N})$. Assume for the time being that the vehicles maneuver independently and do not attempt to coordinate their motions. In this situation, the path parameter of each vehicle evolves according to the dynamics

$$\dot{\gamma}^{[i]} = v^{[i]} = v_{\rm d}(\gamma^{[i]}), \qquad i \in \mathcal{N}. \tag{6}$$

From (4), the vehicles maneuver with nominal speeds

$$u^{[i]} = g^{[i]}(\gamma^{[i]}) v_{\rm d}(\gamma^{[i]}), \qquad i \in \mathcal{N}. \tag{7}$$

The desired formation can be achieved by adjusting the speeds of vehicles about the nominal speeds in (7) such that all vehicles reach agreement in the coordination states (the path parameters) and maneuver with the common normalized desired speed $v_{\rm d}(\cdot)$.
The coordination problem now is to design a distributed control law for the adjusted speed of vehicles such that all path parameters reach consensus, i.e. $\gamma^{[i]}(t) = \gamma^{[j]}(t)$ for all $i, j \in \mathcal{N}$ as $t \to \infty$. Let $g^{[i]}(\gamma^{[i]}) v_{\rm c}^{[i]}$ be the corrected speed for vehicle i, where $v_{\rm c}^{[i]}$ is a new input for the coordination. The dynamics of the path parameters in (6) are now extended as

$$\dot{\gamma}^{[i]} = v_{\rm d}(\gamma^{[i]}) + v_{\rm c}^{[i]}, \qquad i \in \mathcal{N}. \tag{8}$$

At this stage, it is clear that the coordination problem is reduced to finding $v_{\rm c}^{[i]}, i \in \mathcal{N}$ such that the total speed of each vehicle still satisfies (3), that is,

$$u_{\rm min} \le g^{[i]}(\gamma^{[i]})(v_{\rm d}(\gamma^{[i]}) + v_{\rm c}^{[i]}) \le u_{\rm max}, i \in \mathcal{N}, \tag{9}$$

and the path parameters are driven to reach consensus. To solve the consensus problem, each vehicle needs to exchange the path parameters (coordination states) with other vehicles. In this work, we consider that each vehicle is capable of communicating bidirectionally with a set of its neighboring vehicles. Let $\mathcal{G}$ be the bidirectional graph induced by the interconnection network of the vehicles and $\mathcal{N}^{[i]}$ the set of neighboring vehicles of vehicle i. We are now in position to formulate the coordination problem as follows.
Problem 2 - [Constrained Coordination]. *Given the dynamics of the coordinated states in* (8), *and the graph $\mathcal{G}$, derive a distributed control law for $v_{\rm c}^{[i]}(\gamma^{[i]}, \gamma^{[j]}), j \in \mathcal{N}^{[i]}$, subject to the constraint* (9), *such that $(\gamma^{[i]}(t) - \gamma^{[j]}(t)), \forall i, j \in \mathcal{N}$ and $v_{\rm c}^{[i]}(t)$ converge to zero as $t \to \infty$.*

3. CONTROLLER DESIGN

Based on the idea of decoupling the constrained CPF problem into the constrained path following problem and the coordination problem, we propose a CPF control system that for each vehicle has the architecture depicted in Fig. 2. The objective of the Coordination block is to compute the correction speed $v_{\rm c}^{[i]}$. Once the correction speed has been computed, the MPC controller is used to make the vehicle converge to and follow its assigned path. In other words, the MPC controller is used to stabilize (4) for each vehicle. We start by defining a set of conditions

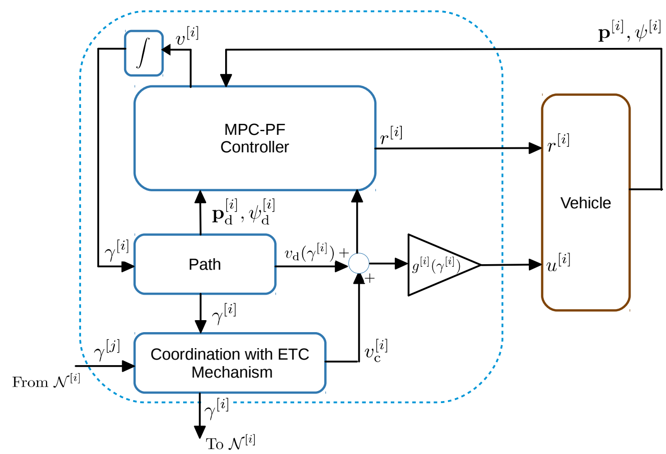

Fig. 2. CPF control system for vehicle $i^{\rm th}$ with event triggered communications.

under which the constrained CPF problem is solvable.
Condition 1.

C1.1 Condition on the linear speed:
$u_{\rm min} < g^{[i]}(\gamma^{[i]}) v_{\rm d}(\gamma^{[i]}) < u_{\rm max}, \forall \gamma^{[i]}$ *and* $i \in \mathcal{N}$.

C1.2 Condition on the turning rate:
$|\kappa^{[i]}(\gamma^{[i]}) g^{[i]}(\gamma^{[i]}) v_{\rm d}(\gamma^{[i]})| < r_{\rm max}, \forall \gamma^{[i]}$ *and* $i \in \mathcal{N}$.

Remark 1. Obviously, the above conditions are necessary to ensure the paths are followable. In C1.1, the term $g^{[i]}(\gamma^{[i]}) v_{\rm d}(\gamma^{[i]})$ are the desired linear speeds in the inertial frames for the vehicles to track. In C1.2, the left hand side of the inequality are the desired heading rates.

In the following subsection we propose a distributed control law with an event triggered communications mechanism to update the correction speed $v_{\rm c}^{[i]}, i \in \mathcal{N}$.

3.1 Distributed controller for the coordination problem

A. Continuous communications

For clarity of presentation of the concepts involved, we start by assuming that the communications are continuous so that the path parameters of neighboring vehicles are always available. The following lemma states a result with continuous communication.

Lemma 1. (Continuous communications).
Consider **Problem 2**. *Let Condition 1 hold for all* $i \in \mathcal{N}$ *and let* $\mathcal{G}$ *be an undirected and connected graph. Then, the distributed control law for* $v_{\rm c}^{[i]}$ *given by*

$$v_{\rm c}^{[i]} = -k_{\rm c}^{[i]} \tanh(\textstyle\sum_{j\in\mathcal{N}^{[i]}} z^{[i]} - z^{[j]}),\ i \in \mathcal{N}, \tag{10}$$

where $z^{[i]}$ *is given by*

$$z^{[i]} := \int_0^{\gamma^{[i]}} \frac{1}{v_{\rm d}(\gamma)}{\rm d}\gamma, \qquad i \in \mathcal{N}, \tag{11}$$

and $k_{\rm c}^{[i]}$ *are positive gains satisfying the condition*

$$\begin{aligned} &0 < k_{\rm c}^{[i]} \le c^{[i]}, \\ &c^{[i]} := \frac{1}{g_{\max}^{[i]}} \min\{u_{\max} - g_{\max}^{[i]} v_{\rm dmax},\ g_{\min}^{[i]} v_{\rm dmin} - u_{\min}\}, \end{aligned} \tag{12}$$

for all $i \in \mathcal{N}$ *drives the path parameters to reach consensus asymptotically.*

The proof is omitted due to the space limitations.

Corollary 1. Consider **Problem 2** *and let all conditions stated in Lemma 1 hold. Further, assume that the speed profile is constant, i.e* $v_{\rm d}(\gamma^{[i]}) = c > 0$ *for all* $i \in \mathcal{N}$. *Then, the distributed control law for* $v_{\rm c}^{[i]}$ *given by*

$$v_{\rm c}^{[i]} = -k_{\rm c}^{[i]} \tanh(\textstyle\sum_{j\in\mathcal{N}^{[i]}} \gamma^{[i]} - \gamma^{[j]}),\ i \in \mathcal{N}, \tag{13}$$

where $k_{\rm c}^{[i]}$ *satisfies* (12) *drive all path parameters to reach consensus asymptotically.*

B. Event triggered communications

In the proposed ETC mechanism, instead of using the true neighboring states ($\gamma^{[j]}, j \in \mathcal{N}^{[i]}$), the control law (10) uses their estimates. Let $\hat{\gamma}^{[ij]}$ be an estimate of $\gamma^{[j]}$ computed by agent i (as will be shown later). The event triggered distributed control law that we propose is given by

$$v_{\rm c}^{[i]} = -k_{\rm c}^{[i]} \tanh\left(\textstyle\sum_{j\in\mathcal{N}^{[i]}} (z^{[i]} - \hat{z}^{[ij]})\right), i \in \mathcal{N}, \tag{14}$$

where

$$\hat{z}^{[ij]} := \int_0^{\hat{\gamma}^{[ij]}} \frac{1}{v_{\rm d}(\gamma)}{\rm d}\gamma, \tag{15}$$

and $k_{\rm c}^{[i]}$ satisfy condition (12).

We now introduce a triggering condition for each vehicle to send its state to its neighboring vehicles. Let $\hat{\gamma}^{[i]}$ be the estimate of $\gamma^{[i]}$ at vehicle i and define $\tilde{\gamma}^{[i]} = \gamma^{[i]} - \hat{\gamma}^{[i]}$ as the estimation error. Further, let $\epsilon > 0$ be so-called the trigger threshold. We propose an ETC mechanism as follows: if at time $t_k^{[i]}$ the estimation error is greater than the threshold, i.e. $|\tilde{\gamma}^{[i]}(t_k^{[i]})| > \epsilon$, then vehicle i sends the current value $(\gamma^{[i]}(t_k^{[i]}))$ to its neighboring vehicle $j, j \in \mathcal{N}^{[i]}$ and resets the estimator $\hat{\gamma}^{[i]}(t_k^{[i]}) = \gamma^{[i]}(t_k^{[i]})$. Once vehicle i receives $\gamma^{[j]}(t_k^{[j]})$ from vehicle $j, j \in \mathcal{N}^{[i]}$, the estimator of $\hat{\gamma}^{[ij]}$, which estimates $\gamma^{[j]}$ at vehicle i, is also reset. Assuming that communication delay is negligible, we have $\hat{\gamma}^{[ij]}(t_k^{[j]}) = \gamma^{[j]}(t_k^{[j]})$ and $\hat{\gamma}^{[ji]}(t_k^{[i]}) = \gamma^{[i]}(t_k^{[i]})$. The dynamics for the estimators are proposed as follows:

$$\begin{aligned} &\dot{\hat{\gamma}}^{[i]} = v_{\rm d}(\hat{\gamma}^{[i]}), \hat{\gamma}^{[i]}(t_k^{[i]}) = \gamma^{[i]}(t_k^{[i]}), i \in \mathcal{N} \\ &\dot{\hat{\gamma}}^{[ij]} = v_{\rm d}(\hat{\gamma}^{[ij]}), \hat{\gamma}^{[ij]}(t_k^{[j]}) = \gamma^{[j]}(t_k^{[j]}), i \in \mathcal{N}, j \in \mathcal{N}^{[i]} \end{aligned} \tag{16}$$

Because $v_{\rm d}(\cdot)$ is common for all vehicles, $\hat{\gamma}^{[ji]}(t) = \hat{\gamma}^{[i]}(t)$ for all $i \in \mathcal{N}, j \in \mathcal{N}^{[i]}$. The intuitive idea behind the proposed event triggered mechanism is that $\tilde{\gamma}^{[i]}$ measures the error of the estimate that vehicle j has of vehicle i, based on which vehicle i decides when to broadcast to the neighboring vehicles $j, j \in \mathcal{N}^{[i]}$.

At this point, let $\mathbf{z} = [z^{[1]}, z^{[2]}, ..., z^{[N]}]^{\rm T} \in \mathbb{R}^N$ and $\bar{z} = \frac{1}{N}\sum_{i=1}^N z^{[i]}$ be the average of $\mathbf{z}$. We define the coordination error vector $\boldsymbol{\xi} = \mathbf{z} - \bar{z}\mathbf{1}$ where $\mathbf{1} = [1]_{N\times 1}$. The following lemma states a result for the coordination problem with the proposed ETC mechanism.

Lemma 2. (Event triggered communications).
Consider **Problem 2** *and let conditions stated in Lemma 1 hold. Further, let the coordination system be driven by the proposed ETC mechanism and the distributed control law given by* (14). *Then, the coordination error vector* $\boldsymbol{\xi}$ *is ultimately bounded with the ultimate bound* b_1 *given by*

$$b_1 = \sqrt{\frac{\lambda_N}{\lambda_2}} \frac{N_{\max} N \sqrt{N}}{\lambda_2 v_{\rm dmin}} \epsilon, \tag{17}$$

where $N_{\max} = \max_{i\in\mathcal{N}} |\mathcal{N}^{[i]}|$ *and,* λ_2 *and* λ_N *are the second smallest and the largest eigenvalues of the Laplacian matrix* L *of the graph* $\mathcal{G}$.

3.2 MPC for the constrained path following

Section 3.1 provided an algorithm that computes the corrective speed $v_{\rm c}^{[i]}$ in order to achieve coordination. We now introduce a sampled-data MPC scheme that solves the constrained path following problem where the total speeds assigned to vehicles are given by

$$u^{[i]} = (v_{\rm d}(\gamma^{[i]}) + v_{\rm c}^{[i]}) g^{[i]}(\gamma^{[i]}), i \in \mathcal{N}. \tag{18}$$

It can be easily seen that if $v_{\rm c}^{[i]}$ is given by (14) and the coordination gain satisfies (12), the reference speed for $u^{[i]}$ given by (18) satisfies its constraint, i.e. $u_{\min} \le u^{[i]} \le u_{\max}$ for all $i \in \mathcal{N}$. Replacing the vehicle speed u in (4) by (18) for vehicle i, we obtain a new equation for the path following error system of vehicle i, given by

$$\dot{\mathbf{x}}^{[i]} = \mathbf{f}^{[i]}(\mathbf{x}^{[i]}, \mathbf{u}^{[i]}) = \begin{bmatrix} g^{[i]}(\gamma^{[i]})\left(-v^{[i]}(1 - \kappa^{[i]}(\gamma^{[i]})e_y^{[i]}) + (v_{\rm d}(\gamma^{[i]}) + v_{\rm c}^{[i]})\cos(e_\psi^{[i]})\right) \\ g^{[i]}(\gamma^{[i]})\left(-\kappa^{[i]}(\gamma^{[i]})v^{[i]}e_x^{[i]} + (v_{\rm d}^{[i]} + v_{\rm c}^{[i]})\sin(e_\psi^{[i]})\right) \\ r^{[i]} - \kappa^{[i]}(\gamma^{[i]})g^{[i]}(\gamma^{[i]})v^{[i]} \end{bmatrix} \tag{19}$$

Since the speed $u^{[i]}$ has been assigned by (18), the input that needs to compute for the path following error system (19) is $\mathbf{u}^{[i]} = (v^{[i]}, r^{[i]})$. It follows from (3) and (5) that $\mathbf{u}^{[i]}$ is constrained to the set

$$\mathbb{U}_{\rm pf} := \{(v^{[i]}, r^{[i]}) : |v^{[i]}| \le v_{\max}^{[i]} \text{ and } |r^{[i]}| \le r_{\max}\}. \tag{20}$$

We are now in a position to design an MPC scheme to stabilize the path following error system (19) to zero subject to the input constraint set $\mathbb{U}_{\rm pf}$ defined by (20).

We define a finite horizon open loop optimal control problem $\mathcal{OPC}(\cdot)$ that the sampled-data MPC solves every sampling time as follows:

Definition 1. $\mathcal{OCP}(t, \mathbf{x}^{[i]}(t), \gamma^{[i]}(t), v_c^{[i]}(t), T_p)$

$$\min_{\bar{\mathbf{u}}^{[i]}(\cdot)} J^{[i]}\left(\mathbf{x}^{[i]}(t), \gamma^{[i]}(t), v_c^{[i]}(t), \bar{\mathbf{u}}^{[i]}(\cdot)\right), \tag{21}$$

with

$$J^{[i]}(\cdot) := \int_t^{t+T_p} l^{[i]}\left(\bar{\mathbf{x}}^{[i]}(\tau), \gamma^{[i]}(\tau), v_c^{[i]}(\tau), \bar{\mathbf{u}}^{[i]}(\tau)\right) d\tau$$

subject to

$$\dot{\bar{\mathbf{x}}}^{[i]}(\tau) = \mathbf{f}^{[i]}\left(\bar{\mathbf{x}}^{[i]}(\tau), \bar{\mathbf{u}}^{[i]}(\tau)\right), \quad \tau \in [t, t+T_p]. \tag{22a}$$

$$\bar{\mathbf{x}}^{[i]}(t) = \mathbf{x}^{[i]}(t). \tag{22b}$$

$$\bar{v}_c^{[i]}(\tau) = -k_c^{[i]} \tanh\left(\textstyle\sum_{j\in\mathcal{N}^{[i]}} \bar{z}^{[i]}(\tau) - \bar{\hat{z}}^{[ij]}(\tau)\right). \tag{22c}$$

$$\dot{\bar{\gamma}}^{[i]}(\tau) = v^{[i]}(\tau), \tau \in [t, t+T_p], \bar{\gamma}^{[i]}(t) = \gamma^{[i]}(t). \tag{22d}$$

$$\dot{\bar{\hat{\gamma}}}^{[ij]}(\tau) = v_d(\hat{\gamma}^{[ij]}(\tau)), \tau \in [t, t+T_p]. \tag{22e}$$

$$\bar{\hat{\gamma}}^{[ij]}(t) = \hat{\gamma}^{[ij]}(t), \quad j \in \mathcal{N}^{[i]}. \tag{22f}$$

$$\bar{\mathbf{u}}^{[i]}(\tau) \in \mathbb{U}_{pf}, \quad \tau \in [t, t+T_p]. \tag{22g}$$

$$\frac{\partial V}{\partial \mathbf{x}^{[i]}} \mathbf{f}^{[i]}(\mathbf{x}^{[i]}(t), \bar{\mathbf{u}}^{[i]}(t)) \le \frac{\partial V}{\partial \mathbf{x}^{[i]}} \mathbf{f}^{[i]}(\mathbf{x}^{[i]}(t), \mathbf{u}_n(\mathbf{x}^{[i]}(t))). \tag{22h}$$

In the constraint equations (22), the variables with bar denote predicted variables, to distinguish them from the real variables without a bar. Specifically, $\bar{\mathbf{x}}^{[i]}(\tau)$ is the predicted trajectory of the path following error which is computed using the dynamic model (19) and the initial conditions (22b); $\bar{\gamma}^{[i]}(\tau)$ is the predicted value of the path parameter $\gamma^{[i]}$ driven by the path following input $\bar{\mathbf{u}}^{[i]}(\tau)$; and $\bar{\hat{\gamma}}^{[ij]}$ is the prediction of the path parameter of neighboring vehicle $j, j \in \mathcal{N}^{[i]}$ using the estimator (16) over the prediction horizon T_p. The constraint (22h) is referred as a *stability constraint* to guarantee stability. This constraint is constructed based on a global Lyapunov function $V : \mathbb{R}^3 \to \mathbb{R}_{\ge 0}$ and its associated stabilizing constrained control law $\mathbf{u}_n : \mathbb{R}^3 \to \mathbb{U}_{pf}$. This setup is inspired by (de la Pena and Christofides [2008]) to improve the performance of path following. Finally, $l^{[i]} : \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}_{\ge 0}$ is the stage cost of the final horizon cost $J^{[i]}$.

In state feedback sampled-data MPC, the optimal control problem $\mathcal{OCP}(\cdot)$ is repeatedly solved at every discrete sampling instant $t_i = i\delta$, $i \in \mathbb{N}_+$, where δ is the sampling interval. Let $\bar{\mathbf{u}}^{[i]*}(\tau)$ be the optimal solution of the optimal control problem $\mathcal{OCP}(\cdot)$. The MPC control law $\mathbf{u}_{mpc}^{[i]}(\cdot)$ is then defined as

$$\mathbf{u}_{mpc}^{[i]}(t) = \bar{\mathbf{u}}^{[i]*}(t) \text{ for } t \in [t_i, t_i + \delta]. \tag{23}$$

Before proceeding to the main result for the path following problem, we make the following assumptions.

Assumption 1.

A1.1 The stage cost $l^{[i]}(\cdot)$ is continuous, and $l^{[i]}(\cdot) = 0$ when $\mathbf{x}^{[i]} = \mathbf{0}$ and $\mathbf{u}_a^{[i]} := [-v^{[i]} + (v_d(\gamma^{[i]}) + v_c^{[i]}) \cos e_\psi^{[i]}, r^{[i]} - \kappa^{[i]}(\gamma^{[i]}) g^{[i]}(\gamma^{[i]}) v^{[i]}]^T = \mathbf{0}$.

A1.2 Given the path following error dynamics in (19), there exist a global Lyapunov function $V : \mathbb{R}^3 \to \mathbb{R}_{\ge 0}$ such that $V(\mathbf{x}^{[i]}) = 0$ only for $\mathbf{x}^{[i]} = \mathbf{0}$, and an associated nonlinear feedback control law $\mathbf{u}_n : \mathbb{R}^3 \to \mathbb{U}_{pf}$ that satisfies $\frac{\partial V}{\partial \mathbf{x}^{[i]}} \mathbf{f}(\mathbf{x}^{[i]}, \mathbf{u}_n(\mathbf{x}^{[i]})) \le 0$ for all $\mathbf{x}^{[i]}$. Further, $\mathbf{u}_n(\mathbf{x}^{[i]})$ stabilizes (19).

Remark 2. Assumption A1.1 *is motivated from the fact that* $\mathbf{u}_a^{[i]} \to \mathbf{0}$ *once the path following error is stabilized.*

We now state an important result for the constrained path following problem with the proposed MPC scheme.

Lemma 3. (Path following with MPC). *Consider the path following error system* (19) *subject to the input constraint set* $\mathbb{U}_{pf}$ *given by* (20)*, controlled by the proposed MPC scheme, and let Assumption 1 hold true. Then, the path following error system is GAS.*

The proof is omitted due to the space limitations.

The most important requirement in the proposed MPC scheme is the existence of a global stabilizing control law $\mathbf{u}_n(\cdot)$ and an associated Lyapunov function $V(\cdot)$ that satisfies Assumption A1.2. It can be shown that the control law in following lemma satisfies the assumption.

Lemma 4. (Global Constrained Nonlinear PF Controller). *Consider the path following error system* (19) *and* v_{max} *in* (20) *is chosen such that*

$$v_{dmax} + k_c < v_{max} < r_{max} / \max(|\kappa(\gamma) g(\gamma)|). \tag{24}$$

Then, the global Lyapunov based control law given by

$$\mathbf{u}_L(\mathbf{x}) = \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} \frac{1}{g(\gamma)}\left(u \cos(e_\psi) + k_1 \tanh(e_x)\right) \\ -\frac{k_3 e_y u \sin(e_\psi)}{(1+e_x^2+e_y^2) e_\psi} - k_2 \tanh(e_\psi) + \kappa(\gamma) g(\gamma) v \end{bmatrix}, \tag{25}$$

where $k_1, k_2, k_3 \in \mathbb{R}_{>0}$ *are tuning parameters that satisfy*

$$\begin{aligned} &0 < k_1 \le v_{max} g_{min} - u_{max}, \\ &0.5 k_3 u_{max} + k_2 \le r_{max} - \max(|\kappa(\gamma) g(\gamma)|) v_{max} \end{aligned} \tag{26}$$

renders the origin of the path following error system GAS. Furthermore, the Lyapunov function associated with the control law (25) *given by*

$$V_L(\mathbf{x}) = \frac{k_3}{2} \log(1 + e_x^2 + e_y^2) + \frac{1}{2} e_\psi^2, \tag{27}$$

satisfies Assumption A1.2.

Remark 3. For the sake of simplicity, we dropped the subscript $[i]$ *in equations* (24)–(27).

3.3 Overall closed-loop CPF system

We now state the main result of this paper.

Theorem 1. Consider the complete closed-loop CPF system composed by

- *a formation of N vehicles, whose motion is described by* (2) *guided along the paths given by* (1) *satisfying Condition 1, with i = 1, . . . , N. Each vehicle uses the proposed MPC scheme for path following.*
- *the distributed control law for the coordination problem given* (14).
- *the ETC mechanism given in subsection 3.1 with the event triggered threshold* ϵ.

Further, suppose that communication delays are negligible. Then, the overall closed-loop CPF system is globally stable. Specifically, the path following error of each vehicle is GAS while the coordination error vector $\boldsymbol{\xi}$ *is bounded asymptotically with the bound* b_1 *given by* (17).

The proof is omitted due to the space limitations.

4. SIMULATION RESULTS

We consider a fleet of three autonomous vehicles that are required to converge to and move along three circumferences. The vehicles are also required to be aligned radially along the radii of the circumferences. The graph induced by the network of the vehicles has the Laplacian matrix L=[1 -1 0; -1 2 -1;0 -1 1]. The vehicles are homogeneous and their speeds and heading rates are constrained by the constraint set (3) with the bounds set to $u_{\min} = 0.2\text{m}\,\text{s}^{-1}, u_{\max} = 2\text{m}\,\text{s}^{-1}, r_{\max} = 0.2\text{rad}\,\text{s}^{-1}$ The desired formation is formed by three circumferences that are parameterized as follows $\mathcal{P}^{[i]} : \mathbf{p}_{\text{d}}^{[i]}(\gamma^{[i]}) = [a^{[i]}\cos(\gamma^{[i]}), a^{[i]}\sin(\gamma^{[i]})]^{\text{T}}$, where $a^{[1]} = 30\text{m}, a^{[2]} = 35\text{m}, a^{[3]} = 40\text{m}$. The normalized desired speed of the formation is $v_{\text{d}} = 0.02\text{rad}\,\text{s}^{-1}$. The event triggered communication threshold is set to $\epsilon = 0.01$. The stage cost is defined as $l^{[i]}(\cdot) = \mathbf{x}^{[i]}(\tau)^{\text{T}} Q\mathbf{x}^{[i]}(\tau) + \mathbf{u}_{\text{a}}^{[i]}(\tau)^{\text{T}} R\mathbf{u}_{\text{a}}^{[i]}(\tau)$, where Q=diag(1, 1, 2) and R=diag(2, 10). The sampling time is set to $\delta = 0.2$s and the prediction horizon is set to $T_{\text{p}} = 2$s. To solve the finite optimal control problem $\mathcal{OCP}(\cdot)$, we used the open source optimization framework in (Casadi Andersson [2013]).

As a benchmark, we ran simulations with two CPF control strategies. The first approach (MPC-CPF) uses the proposed MPC scheme as the path following controller while in the second approach (L-CPF) the Lyapunov-based path following controller given in Lemma 4 is used. The MPC-CPF strategy also uses the controller and Lyapunov function given in Lemma 4 to construct the constraint (22h).
The performance of the two CPF strategies is illustrated in Fig. 3. It can be seen clearly from the figure that both CPF controllers drive the vehicles to converge to the paths and make the path parameters coordinated. However, with the MPC-CPF, the vehicles converge to the desired paths faster and coordination is achieved earlier. This can be explained due to the constraint (22h). In what concerns communications, after the path parameters reach consensus the vehicles no longer need to communicate. The example illustrates well the potential savings in communications.

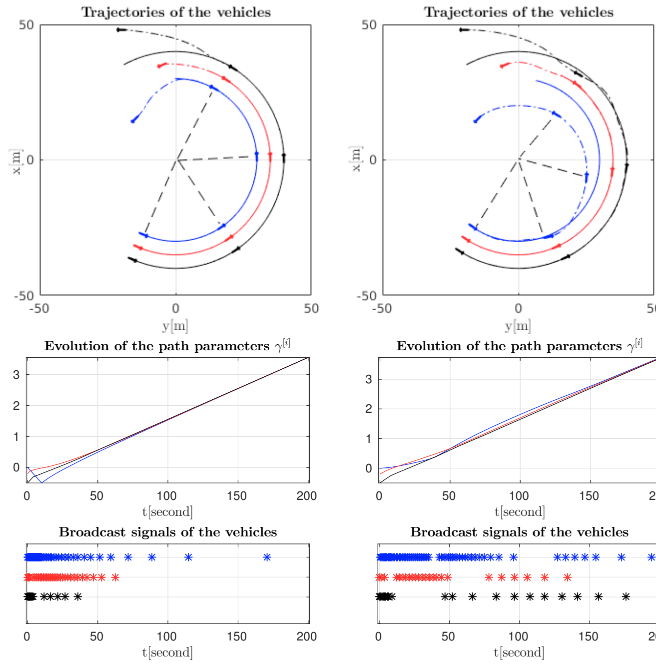

Fig. 3. The performance of the MPC-CPF (left) and the L-CPF (right) strategies with the ETC mechanism. Blue is with $i = 1$, Red is with $i = 2$ and Black is with $i = 3$.

5. CONCLUSIONS

We proposed a solution to the constrained CPF problem that exploits the tools of Model Predictive Control, distributed control, and event triggered communications. The main contribution of this work lies in the fact the proposed strategy is not only capable of explicitly handling practical constraints on vehicles' inputs and on the topology of the communications network, but also saves communication bandwidth. Further, we shown that the overall closed-loop CPF system is globally stable.

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