Set-consensus for Multi-Agent Systems

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Set-Consensus using Set-Valued Observers

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Outline

1. Introduction
2. Problem Statement
3. Proposed Solution
4. Main Properties
5. Simulation Results
6. Concluding Remarks
Motivation

- Distributed Sensing - Each node computes estimates and need to synchronize them before aggregating them.

- Robot Coordination - Fleet of robots wishes to agree on direction/speed or rendezvous point.

- Asynchronous Algorithms - Nodes acting independently changes the number of considered time steps as seen by each individual nodes.
Set Consensus

- A group of $n$ nodes is trying to achieve consensus.
- Nodes have neither sensing nor self-localization capabilities.
- A tower uses a directional antenna to transmit to the nodes their position and velocity.
- Two main issues: measurements are corrupted by noise and taken at different time instants.
Motivating Example

- Consider the case of two vehicles given in the figure.
- Node 1 receives the last measurement at time instant $k - 4$.
- Due to sensor noise or disturbances, node 1 has only access to estimates.
- Then, the decision might result in a collision!
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Problem Outline

- Take \( n \) nodes, where each node \( i \) has dynamics of the form

\[
x_i(k+1) = A_i(k)x_i(k) + B_i(k)u_i(k) + E_i(k)d_i(k)
\]

- \( u_i(k) \) is the actuation signal and \( d_i(k) \) possible disturbances.

Set-Consensus Problem

How to achieve position or velocity consensus when instead of knowing \( x_i(k) \) only a set \( X_i(k) \) is known such that \( x_i(k) \in X_i(k) \).
Problem Model

- Each agent $i$ has a system of the form

$$x_i(k+1) = \left( A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell(k) A_\ell \right) x_i(k) + B_i(k) u_i(k) + E_i d_i(k)$$

- Each $S_i$ is a Linear Parameter-Varying (LPV) system
- $n_\Delta$ number of uncertainties
- $\Delta_\ell(k)$ are scalar uncertainties with $|\Delta_\ell(k)| \leq 1$
- $A_\ell$ are constant matrices
Proposed Solution

Broadcast Solution using Position

- Use Set-Valued Observers (SVOs) [1] to update the received $X_j(k - k_j)$ for each of the neighbors $j$;
- Compute the weighted average [2] of the updated $X_j(k)$;
- Compute the velocity vector to drive $X_i(k)$ to $X_{avg}(k)$.

Unicast Solution using Estimation

- Node $i$ receives sets $X_j(k - k_j)$ from a subset of its neighbors;
- Set $X_i(k)$ will include the concatenation of the updated $X_j(k)$ and disturbance terms to account for each node $j$ actuation;
- The velocity vector will take into account the estimated position and velocity of the neighbors.
SVOs

Given the previous set $X(k)$:

- Using SVOs, the algorithm predicts $\tilde{X}(k + 1)$ using the dynamics;
- Then, the set is intersected with the measurement set $Y(k + 1)$. 

\[ X(k) \rightarrow \tilde{X}(k + 1) \rightarrow X(k + 1) \]
Algorithm

- Node $i$ computes:

\[ X_i(k + 1) = \alpha X_i(k) + (1 - \alpha) \frac{1}{|N_i|} \sum_{j \in N_i} X_j(k) \]

- Velocity vector $v$ can be found through:

\[ v = \arg \min_{x,y} \max_{x,y} (|| (v + x) - y ||) \]

subject to

\[ x \in X_i(k) \]
\[ y \in X_i(k + 1), \]
Properties

- Nodes position converge to a ball of radius equal to the maximum uncertainty in the measurement sets;

- For the case of unicast communication and using estimates, uncertainty is higher as there are added disturbances and dynamics uncertainties in the update of the estimates;

- Convergence for a single cluster depends on the allocation of transmissions by the various directions.
Simulation Results (1/2)

Setup: 200-node network randomly distributed over a $50m \times 50m$ square and round-robin service using an offset to cover 10 partitions of the terrain.

- In a typical run nodes converge to a smaller number of clusters (5 in the example).
- Nodes *aligned* themselves along the partitions.
- Figure depicts the evolution of the maximum distance between any two nodes.
- Convergence to a cluster can be identified when there is little oscillation in this metric.
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Simulation Results (2/2)

Setup: 200-node network randomly distributed over a 50m × 50m square with two antennae (length and width) used in a periodic scheduling. Around-robin service is used for each antenna using an offset to cover 10 partitions of the terrain.

- A typical run achieves consensus for a single cluster.
- The maximum ball around the nodes has radius equal to the maximum uncertainty $\epsilon_{\text{max}}$.
- The maximum difference between two nodes converges to a value smaller than $\epsilon_{\text{max}}$. 

![Graph showing simulation results](image)
Simulation Results (2/2)

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Concluding Remarks

Contributions:

- the use of SVOs to update the set representing the uncertainty about the position of the nodes;

- Two scenarios are addressed:
  - Broadcast - nodes use the positions for the other nodes;
  - Unicast - nodes obtain information in the shared medium and estimate the position for the other nodes.

- the positions of the nodes are shown to converge to the vicinity of the remaining nodes dependent on a measure of the uncertainty.

- In Simulation, it is observed that the policy for the communication influences the number of clusters.

Thank you for your time.
Set-Consensus using Set-Valued Observers

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