Presentation on the application of set-based fault detection to Cyber-physical Systems

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3 authors:

Daniel Silvestre
Instituto Superior Técnico
39 PUBLICATIONS 162 CITATIONS

Joao P. Hespanha
University of California, Santa Barbara
521 PUBLICATIONS 40,129 CITATIONS

Carlos Silvestre
University of Macao and University of Lisbon
455 PUBLICATIONS 6,585 CITATIONS

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Fault Detection for Cyber-Physical Systems: Smart Grid case

D. Silvestre, J. Hespanha and C. Silvestre

23rd Mathematical Theory of Networks and Systems
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Motivation

- Sensor Smart Grids - Attacks or errors at the communication network can severely impact on the physical component.

- Robot Coordination - Formations of robots can also be seen as another example of a system with a communication network on top.
There are various physical components with their own dynamics.

A communication network to manage the devices.

We study the particular example of smart grids.

Main issue: it is required a fast and distributed strategy to detect faults or attacks.
Smart grid model

- A smart grid is composed of:
  - $n$ generator buses;
  - $m$ load buses;
- network can be incorporated using its Laplacian matrix
- Using the dynamic linearized swing equation and the algebraic DC power flow equation, the model becomes:

$$N_c \dot{x}(t) = A_c x(t) + p(t)$$  \hspace{1cm} (1)

- state $x = [\delta^T \omega^T \theta^T]^T \in \mathbb{R}^{2n+m}$ with:
  - $\delta \in \mathbb{R}^n$ - generator rotor angles;
  - $\omega \in \mathbb{R}^n$ - frequencies;
  - $\theta \in \mathbb{R}^m$ - bus voltage angles.
Problem Statement

- Given that the network is connected, $\theta(t)$ can be written using the other variables;
- The system is rewritten from an algebraic differential model to a kron-reduced version;
- After discretization, it becomes a Linear Time-Invariant (LTI) model:

$$
\begin{align*}
    x(k+1) &= Ax(k) + Bu(k) + Ff(k) + Ed(k) \\
    y(k) &= Cx(k) + Du(k) + Lf(k) + Nd(k)
\end{align*}
$$

Fault detection problem in Cyber-physical systems

*How to perform fault detection without knowledge of the fault inputs? Is it a fast and distributed algorithm?*
Centralized solution 1/2

- A node estimates the subnetwork of interest;
- No uncertainty in the model;
- New estimate for the state can be obtained by the inequality:

\[
\begin{bmatrix}
M(k)A^{-1} - M(k)A^{-1}E \\
\bar{C} & 0 \\
0 & \bar{I}
\end{bmatrix}
\begin{bmatrix}
x \\
d
\end{bmatrix}
\leq
\begin{bmatrix}
m(k) + \tilde{u}(k) \\
\bar{y}(k + 1) + \nu^*1
\end{bmatrix}
\]

(3)

- Propagation equation
Centralized solution 1/2

- A node estimates the subnetwork of interest;
- No uncertainty in the model;
- New estimate for the state can be obtained by the inequality:

\[
\begin{bmatrix}
M(k)A^{-1} - M(k)A^{-1}E \\
\bar{C} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
x \\
d
\end{bmatrix}
\leq
\begin{bmatrix}
m(k) + \tilde{u}(k) \\
\bar{y}(k + 1) + \nu*1 \\
1
\end{bmatrix}
\]

(3)

- Intersection with measurements
Centralized solution 1/2

- A node estimates the subnetwork of interest;
- No uncertainty in the model;
- New estimate for the state can be obtained by the inequality:

\[
\begin{bmatrix}
M(k)A^{-1} & -M(k)A^{-1}E \\
\bar{C} & 0 \\
0 & \bar{I}
\end{bmatrix}
\begin{bmatrix}
x \\
d
\end{bmatrix}
\leq
\begin{bmatrix}
m(k) + \tilde{u}(k) \\
\bar{y}(k+1) + \nu^*1
\end{bmatrix}
\]

(3)

- Bounds on disturbances
The generalized solution exists for singular matrices $A$

We can include previous time instants

If we use a coprime factorization providing $P(z) = G^{-1}(z)Q(z)$ represented in

![Schematic representation of the two coprime systems.](image)

**Figure:** Schematic representation of the two coprime systems.
Decentralized solution

- Replace the known matrix $A$ in the centralized version by:

$$A = A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell A_\ell \quad (4)$$

- Uncertainties parameters $\Delta_\ell$ are used to represent the unknown dynamics.
- The set can be obtained by computing the convex hull for each of the uncertainty vertex:

$$\tilde{X}(k + 1) = \text{co}\left( \bigcup_{\theta \in \mathcal{H}} \text{Set}(M_{\theta}(k + 1), m_{\theta}(k + 1)) \right) \quad (5)$$
Results

- **Centralized solution**
  - If the system with $n$ states is observable, convergence of the estimates is achieved in $n$ time instants.

- **Distributed solution**
  - Convergence is governed by the slowest mode.

- In both cases, maximum magnitude for the attacker can be found by solving:

  $$
  \gamma_{\text{min}} \geq \max_{A_H x \leq b_H} x^T P_A x. \quad (6)
  $$

- $P_A$ defining all the quadratic weights for the fault signals;
- $A_H$ and $b_H$ define the polytope containing all possible states.
Simulation Results (1/2)

Setup: Testbed network of 14 buses from IEEE.

- The average of the fault magnitude decreases with the number of past measurements.
- Attackers have a limited possibility to compromise the state without being detected.
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- Attackers have a limited possibility to compromise the state without being detected.
The centralized solution detects faults of smaller magnitude.

Detection was performed at most in $n$ time instants.

Detection for one of the observers in the network.

Decentralized solution required a higher magnitude fault to ensure detection.
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Detection for one of the observers in the network.

Decentralized solution required a higher magnitude fault to ensure detection.
Concluding Remarks

Contributions:

- We have shown how to perform worst-case fault detection
  - centralized - one node with full knowledge of the network;
  - distributed - various node with a partial view.

- It is possible to give theoretical results about the convergence time;

- Finally, under the framework of distinguishability of models, it was possible to give worst-case bounds on the attacker signal.
References


Thank you for your time.
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